DoA Estimation and Capacity Analysis for 3D Massive-MIMO/FD-MIMO OFDM System

Rubayet Shafin[‡], Lingjia Liu[‡], and Jianzhong (Charlie) Zhang[*]

[‡]Department of EECS, University of Kansas, Lawrence, Kansas-66045, USA.

[*]Standard Research Laboratory, Samsung Research America - Dallas, Richardson, Texas-75082, USA

Outline

- Motivation
- FD-MIMO System Model
- Analytical Results
- Simulation Results
- Summary

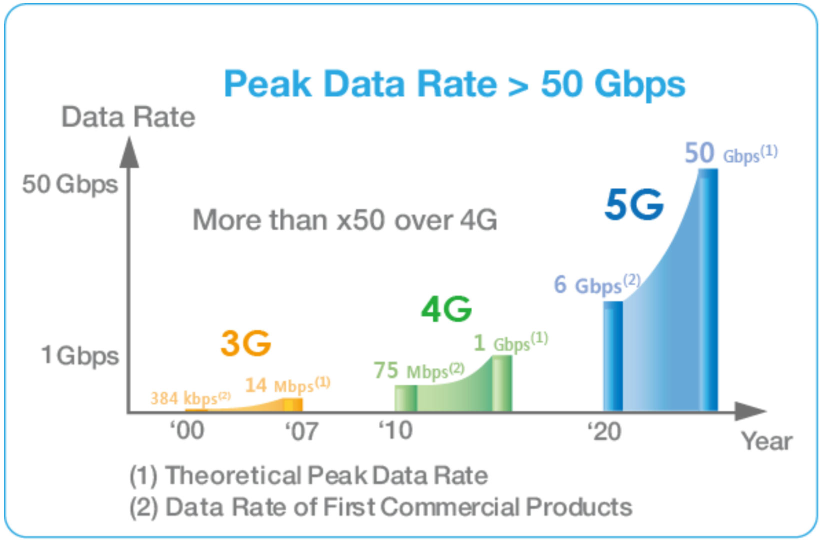
Introduction

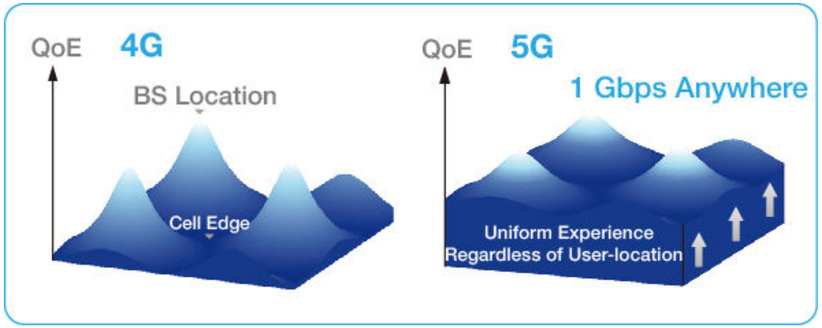
Massive MIMO: An Enabling Technology for 5G

Increased Spatial Resolution

Reduced MU Interference

Simplified Signal Processing





*Samsung's 5G Vision: Whitepaper, 2015.

**T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., Nov. 2011.

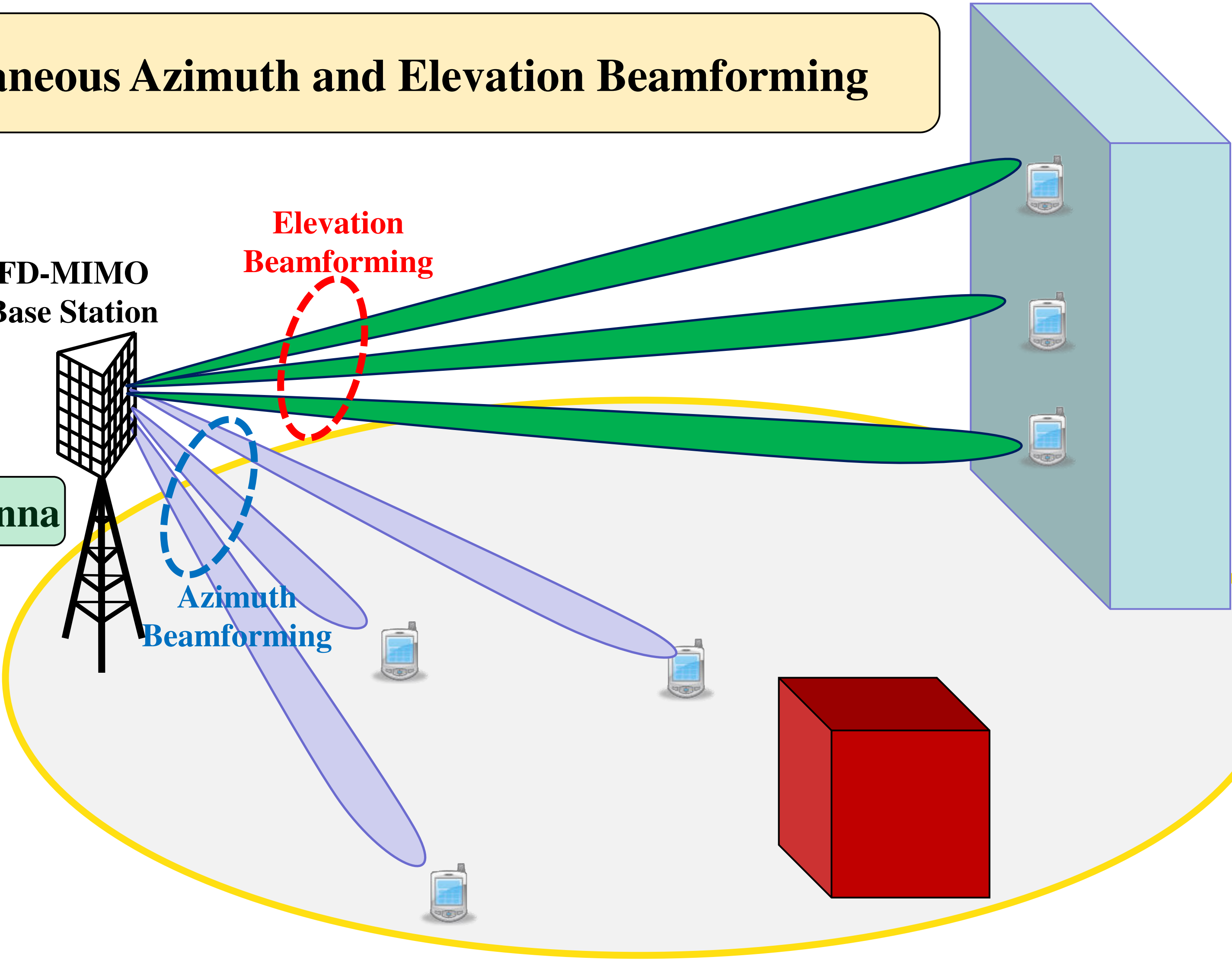
FD-MIMO

Enables Simultaneous Azimuth and Elevation Beamforming

2D Array at BS

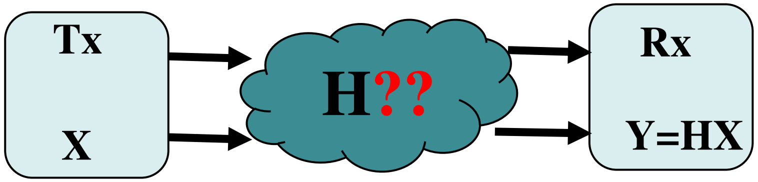
Large Number of Antenna

~10s of UEs



Motivation

CSI is crucial for extracting all benefits of FD-MIMO

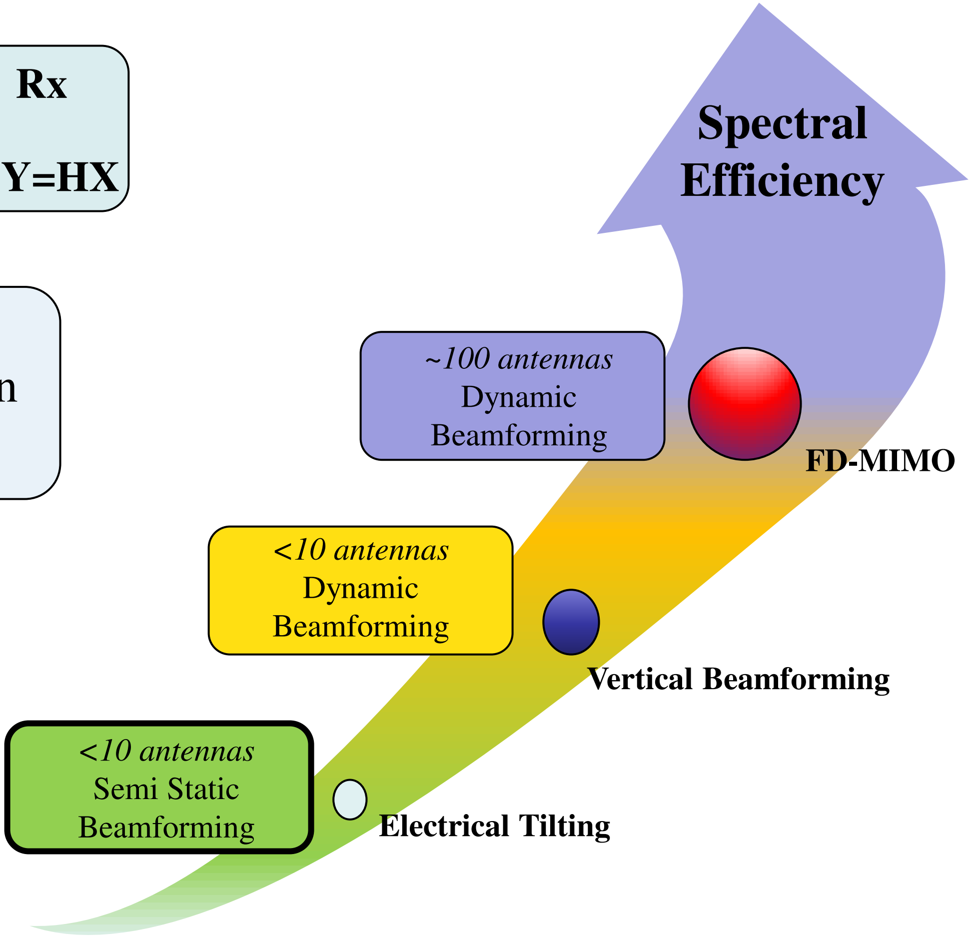


Channel Estimation Approaches:

- Estimating Channel Transfer Function
- Estimating DoA/DoD (Parametric)

Our Approach: Parametric

- BS estimates DoA in Uplink
- TDD Operation
- BS uses uplink DoA information in Downlink Precoding

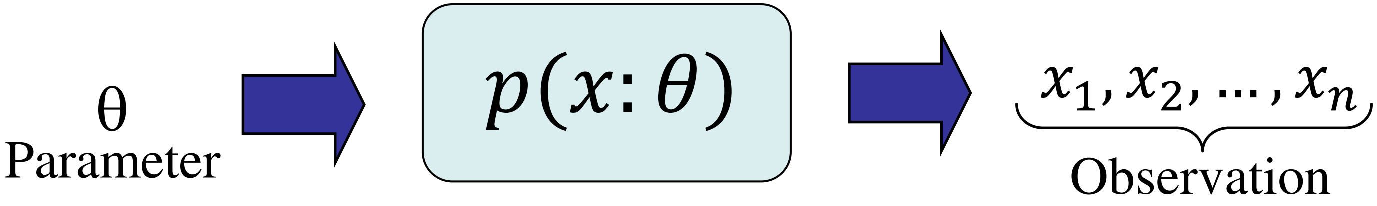


DoA-based Channel Estimation

Estimating CTF:

- Statistical Channel Model.
- Number of coefficients need to be estimated goes large with dimensionality of the channel.

$$\mathrm{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1t} \\ h_{21} & h_{22} & \dots & h_{2t} \\ . & & & \\ h_{r1} & h_{r2} & \dots & h_{rt} \end{bmatrix}$$



θ

Parameter

$p(x\colon\theta)$

$\underbrace{x_1, x_2, \dots, x_n}$

Observation

Parameter Estimation:

- Based on physical channel model.
- Captures more accurate propagation environment.
- Fewer parameters to be estimated.

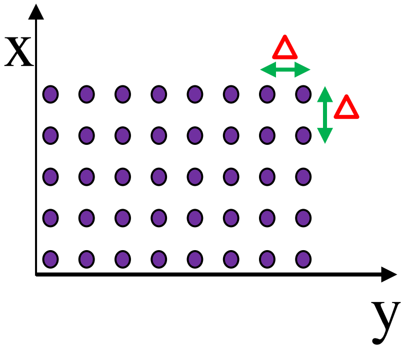
FD-MIMO Scenario:

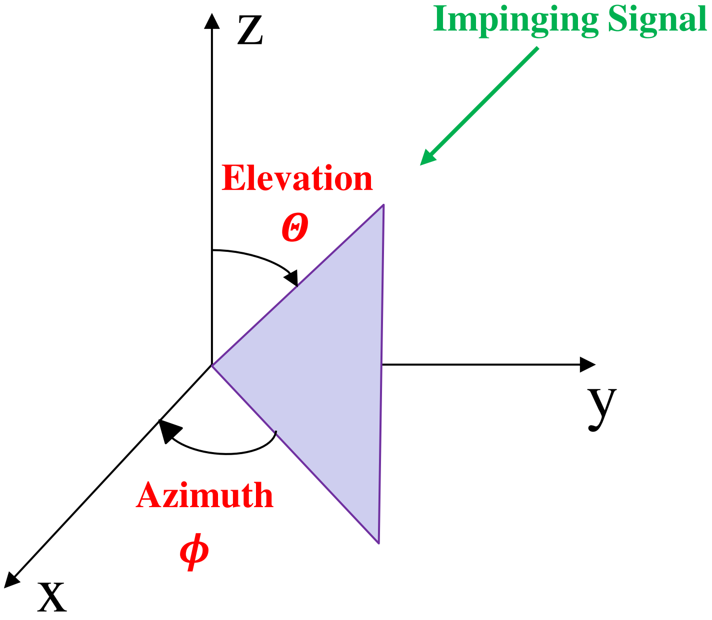
- DoA can help downlink beamforming.
- Massive MIMO: Increased spatial resolution and narrow-beam transmission.
- Hence, DoA is very crucial for FD-MIMO.

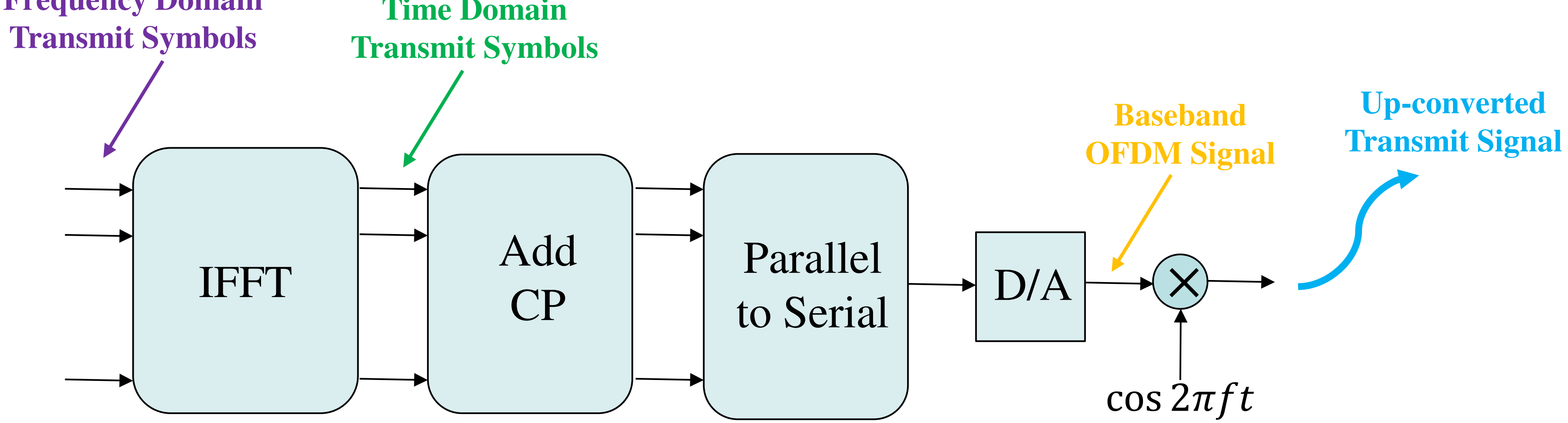
System Model

Consider a 3D Massive MIMO OFDM System:

- ULA placed at the UE
- URA placed at the BS







System Model

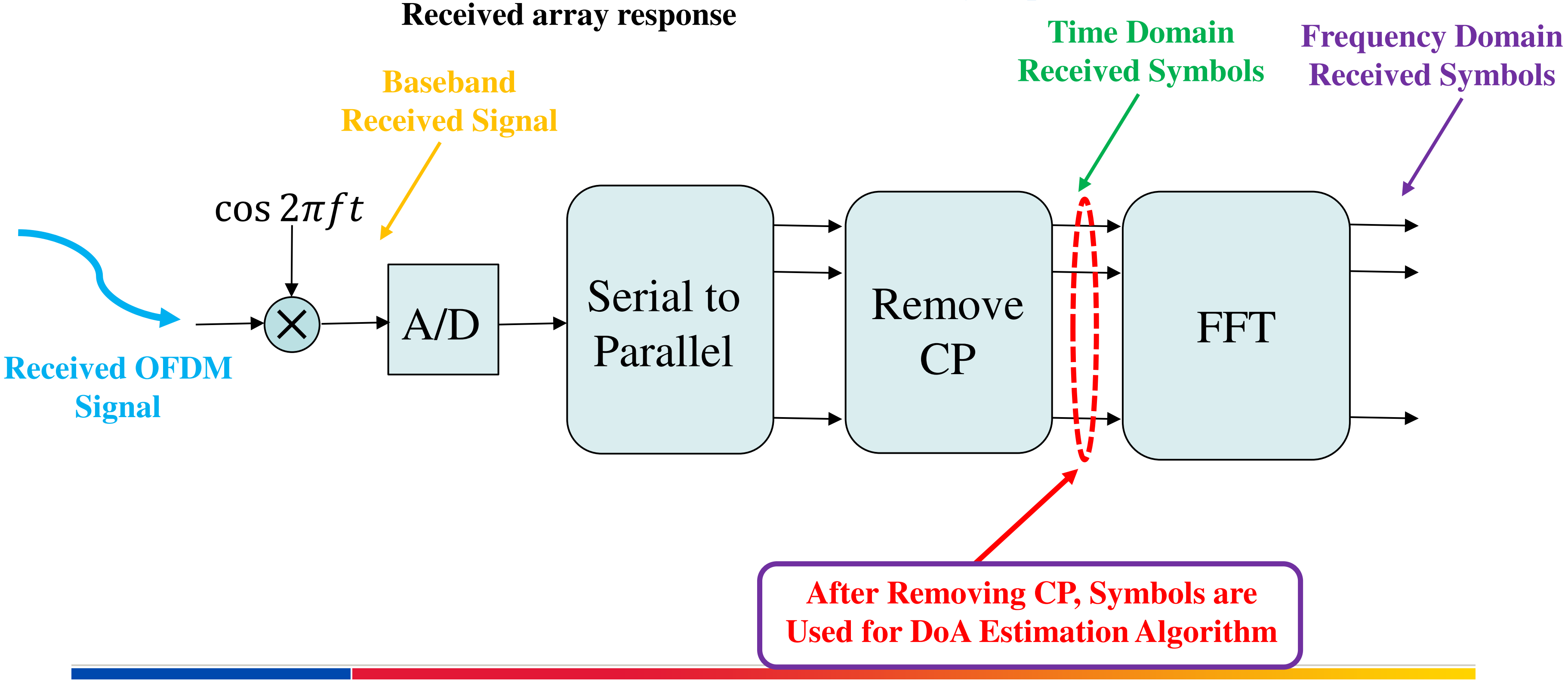
Channel impulse response for ℓ-th path:

$$\mathbf{C}(\ell) = \alpha(\ell)\mathbf{e}_r(\ell)\mathbf{e}_t^H(\ell)$$

Complex channel gain

Received array response

Transmit array response



Time Domain Received Symbols

Frequency Domain Received Symbols

Baseband Received Signal

$\cos 2\pi f t$

Received OFDM Signal

A/D

Serial to Parallel

Remove CP

FFT

After Removing CP, Symbols are Used for DoA Estimation Algorithm

System Model

After removing CP, time domain received signal:

$$\mathrm{Y} = \mathrm{A}\,\mathrm{diag}\{\mathrm{b}\}\tilde{\mathrm{E}}_t\tilde{\mathrm{S}} + \mathrm{W}$$

Array Steering Matrix:
in Vandermonde structure

Transmitted Signal

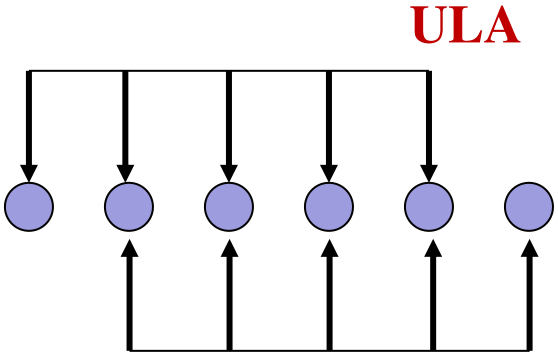
Noise Matrix

b = vector containing complex channel gains

$$\mathbf{A} = \begin{bmatrix} \mathbf{e}_r(n) & \mathbf{e}_r(n-1) & \dots & \mathbf{e}_r(n-N_c+1) \end{bmatrix}$$

$$\tilde{\mathbf{E}}_t = \begin{bmatrix} \mathbf{e}_t^H(n) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{e}_t^H(n-1) & \dots & \mathbf{0} \\ & & \vdots & \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{e}_t^H(n-N_c+1) \end{bmatrix}$$

DoA Estimation



Shift invariance property:

Choose two subarrays with the maximum overlap:

$$\mathbf{a}(\theta) = [1 \quad e^{j\mu} \quad \dots \quad e^{j(N-1)\mu}]^T$$

$$J_1\mathbf{a}(\theta)e^{j\mu} = J_2\mathbf{a}(\theta)$$

ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) utilizes this shift invariance property for parameter estimation.

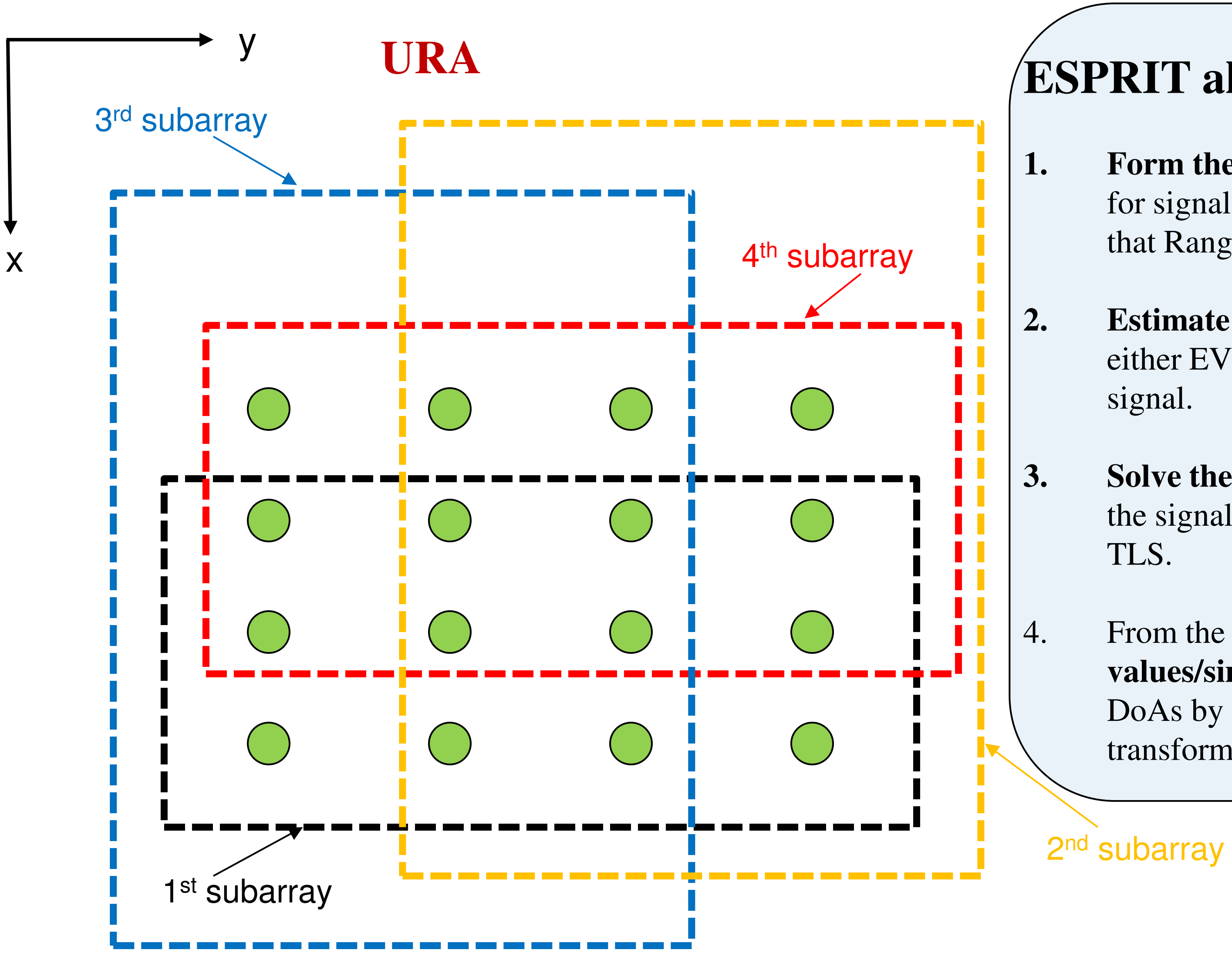
Our noisy received signal: $\mathbf{Y} = \mathbf{A}\mathbf{S} + \mathbf{W}$

Equivalent Transmit Signal

The array steering matrix **A,** has the shift invariance property.

Hence, DoAs at the BS can be estimated using ESPRIT algorithm

DoA Estimation



ESPRIT algorithms summary:

1. **Form the shift invariance equation** for signal subspace, U_s: by observing that Range{A}= Range{U_s}.

2. **Estimate signal subspace, $\boldsymbol{U_s}$**: by either EVD or SVD from the received signal.

3. **Solve the shift invariance operator** of the signal subspace by either LS or TLS.

4. From the solution, **calculate eigen values/singular values**, and extract DoAs by simple parameter transformation.

Analytical Results

The frequency-domain uplink channel transfer function at the k-th subcarrier:

$$\mathbf{H}(k) = \sum_{\ell=0}^{N_c-1} \mathbf{C}(\ell) e^{\frac{-j2\pi k\ell}{N_c}} = \sum_{\ell=0}^{N_c-1} \alpha(\ell)\mathbf{e}_r(\ell)\mathbf{e}_t^H(\ell) e^{\frac{-j2\pi k\ell}{N_c}} = \sum_{\ell=0}^{N_c-1} \alpha(\ell)\mathbf{e}_r(\ell,k)\mathbf{e}_t^H(\ell)$$

This channel transfer function can be rearranged as:

$$\boxed{\mathbf{H}(k) = \mathbf{A}(k)\mathbf{D}\mathbf{B}^H}$$

where

$$\mathbf{A}(k) = [\mathbf{e}_r(0,k), \mathbf{e}_r(1,k), \ldots, \mathbf{e}_r(N_c-1,k)]$$

$$\mathbf{e}_r(\ell,k) = e^{\frac{-j2\pi k\ell}{N_c}} \mathbf{e}_r(\ell)$$

$$\mathbf{D} = \operatorname{diag}\{\alpha(0), \alpha(1), \ldots, \alpha(N_c - 1)\}$$

$$\mathbf{B} = [\mathbf{e}_t(0), \mathbf{e}_t(1), \ldots, \mathbf{e}_t(N_c - 1)]$$

Then the frequency domain downlink channel transfer function:

$$\boxed{\mathbf{H}^{dl}(k) = [\mathbf{H}(k)]^T = \mathbf{B}^*\mathbf{D}\mathbf{A}^T(k)}$$

Analytical Results

The Mutual information for the downlink channel in the absence of DoA estimation error:

$$\mathcal{I}_k = \log_2 \det \left[\mathbf{I}_{N_t} + \frac{\mathbf{H}^{dl}(k)\mathbf{Q}_k\mathbf{H}^{dl^H}(k)}{\sigma^2} \right]$$

where $\boldsymbol{Q}_k$ is the covariance matrix of the downlink transmit signal. System capacity then

$$\mathcal{C} = E\left\{ \frac{1}{N_c} \sum_{k=0}^{N_c-1} \mathcal{I}_k \right\} = E\left\{ \frac{1}{N_c} \sum_{k=0}^{N_c-1} \log_2 \det \left[\mathbf{I}_{N_t} + \frac{\mathbf{H}^{dl}(k)\mathbf{Q}_k\mathbf{H}^{dl^H}(k)}{\sigma^2} \right] \right\}$$

We now consider the following Lemma:

Lemma 1: For a uniform rectangular array (URA) with azimuth and elevation DoAs drawn independently from a continuous distribution, the normalized frequency-domain array response vectors are orthogonal, that is, $\bar{\mathrm{e}}_r(i,k) \perp \textbf{\textit{span}}\{\bar{\mathrm{e}}_r(j,k)|\forall i \neq j\}$ when N_r goes large.

Analytical Results

Using Lemma-1, optimum downlink precoding matrix, in the absence of DoA estimation error:

$$\boxed{\mathbf{V}^{opt}(k) = \frac{1}{N_r}\mathbf{A}^*(k) = \frac{1}{N_r}[\mathbf{e}_r^*(0,k), \mathbf{e}_r^*(1,k), \ldots, \mathbf{e}_r^*(N_c-1,k)]}$$

Then the mutual information is simplified:

$$\mathcal{I}_k = \log_2 \prod_{\ell} \left(1 + \frac{N_t|\alpha(\ell)|^2 p_\ell(k)}{\sigma^2}\right) = \sum_{\ell=0}^{N_c-1} \log_2\left(1 + \gamma_\ell p_\ell(k)\right)$$

where

$$p_\ell(k) = [\mu_\ell(k) - 1/\gamma_\ell]^{\diamond}$$

Power allocation follows the traditional water-filling algorithm.

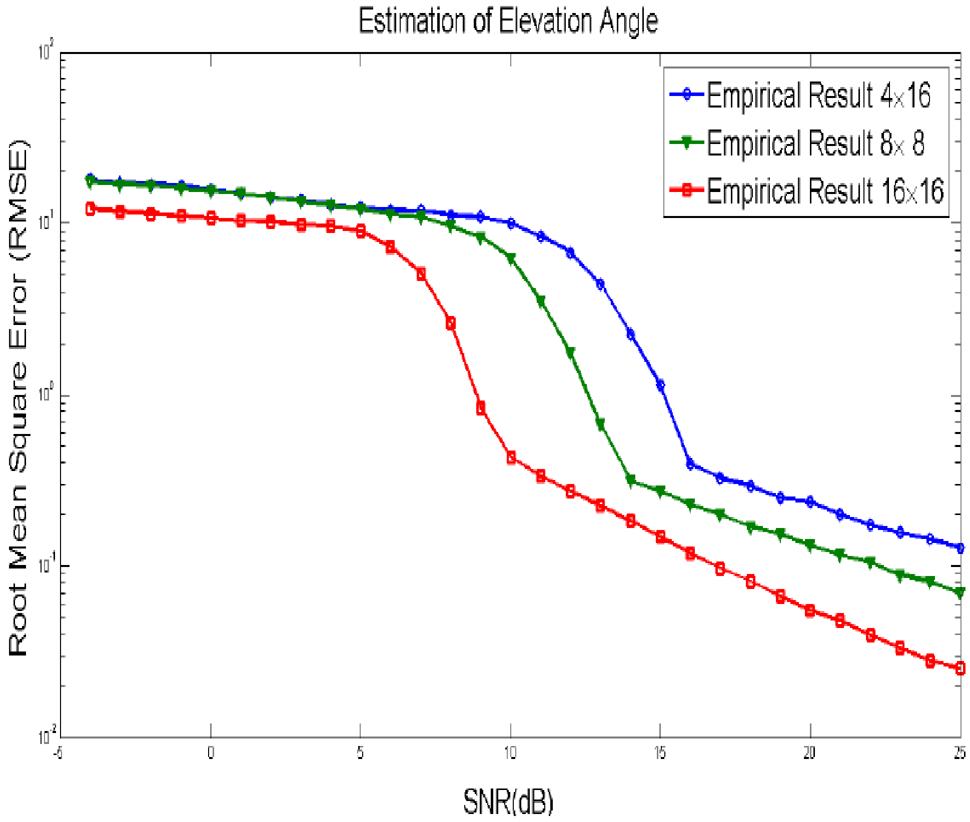
Simulation Results

Simulation parameters

- Far Field Assumptions: Planer wavefront impinging on the antenna array.
- Isotropic and linear transmission medium

Simulation parameters

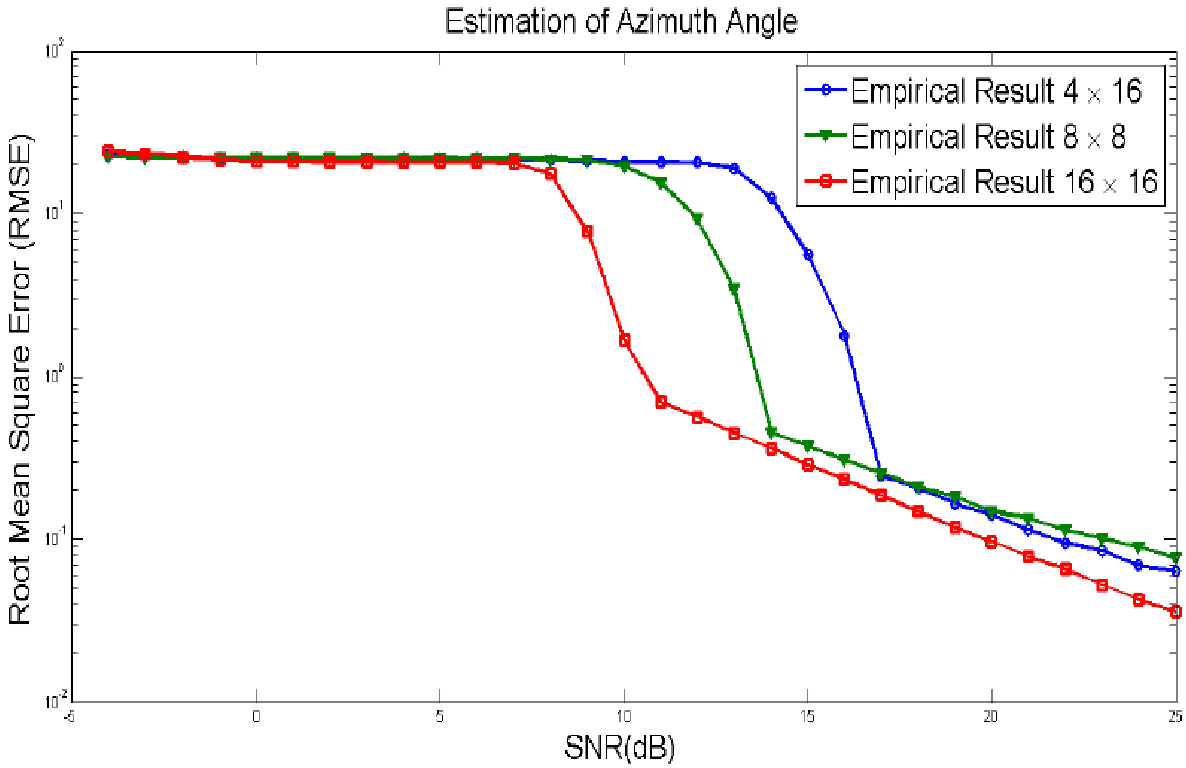
- Four resolvable paths
- Number of subcarrier = 32
- Adjacent antenna spacing = 0.5λ
- Elevation and Azimuth DoA's are uniformly chosen from $U[-\pi, \pi]$
- Number of transmit antennas= 2



Simulation Results

Simulation parameters

- As the SNR increases, the DoA estimation performance improves.
- For the same 64 antennas, in azimuth angle estimation, 8 x 8 antenna pattern performs better than the 4 x 16 antenna pattern.
- As expected, 16 x 16 antenna pattern outperforms both 8 x 8 and 4 x 16 arrays both in elevation and azimuth.



Summary

- **Accurate CSI is critical for FD-MIMO systems. In this work,**
  - A DoA- based channel estimation method has been presented.
  - A capacity analysis of the channel based on DoA based channel estimation has been carried out.

- **Results show that:**
  - Optimum downlink precoding matrix can be constructed in terms of only DoA vectors.
  - Antenna configuration plays vital role in DoA estimation performance.

- **Future Work:**
  - Formulating optimal precoding matrix in the presence of DoA estimation error.
  - Extending the work to MU-MIMO scenario.

Thank You