Multichannel Random Discrete Fractional Fourier Transform

Xuejing Kang, Feng Zhang, Member, IEEE, and Ran Tao, Senior Member, IEEE

Abstract: We propose a multichannel random discrete fractional Fourier transform (MRFrFT) with random weighting coefficients and partial transform kernel functions. First, the weighting coefficients of each channel are randomized. Then, the kernel functions, selected based on a choice scheme, are randomized using a group of random phase-only masks (RPOMs). The proposed MRFrFT can be carried out both electronically and optically, and its main features and properties have been given. Numerical simulation about one-dimensional signal demonstrates that the MRFrFT has an important feature that the magnitude and phase of its output are both random. Moreover, the MRFrFT of two-dimensional image can be viewed as a security enhanced image encryption scheme due to the large key space and the sensitivity to the private keys.

The Proposed MRFrFT

We propose the MRFrFT by implementing the following three steps.:

1) Randomize the weighting coefficients.;
2) Randomize the transform kernel function
3) A choice mechanism for selecting channels.

Discussion of the Computation Cost of the MRFrFT

To make a comparison of running time quantitatively between the MRFrFT and other FrFT expansions, we use the 2D image with size to execute the following operators:

$$\mathbf{Q} = \mathbf{L}^{a} \cdot \mathbf{P} \cdot \mathbf{L}^{b}$$

where , and denote the operators of the FrFT expansions or the MRFrFT with different orders. Table I records the calculating time of all transforms and verifies that the computation cost of the MRFrFT is very much reduced.

TABLE I COMPARISON OF COMPUTATION TIME BETWEEN THE PROPOSED MRFRFT AND FRFT EXPANSIONS

Method	Computation time (s) (128×128)	Computation time (s) (256× 256)	Computation time (s) (512× 512)
DFrFT	0.41761	4.27308	70.75624
MPDFrFT	0.44934	4.31678	70.42953
RFrFT	1.29342	14.9103	219.9258
RDFrFT	3.52712	42.8361	651.79362
MRFrFT	**0.23718**	**0.79624**	**6.35184**

Numerical Simulations of The MRFrFT

We use a rectangle function given as follows to simulate the 1D numerical results.

$$x(n) = \begin{cases} 1, & \text{if } 100 < n \le 155 \\ 0, & \text{otherwise} \end{cases}$$

Here we have taken the number of sample points equal to 256. By applying the process defined in (13), the MRFrFT of 1D rectangle function is calculated and the numerical results are depicted in Fig 3. It demonstrates an important feature of the proposed MRFrFT that the magnitude and phase of its output are both random.

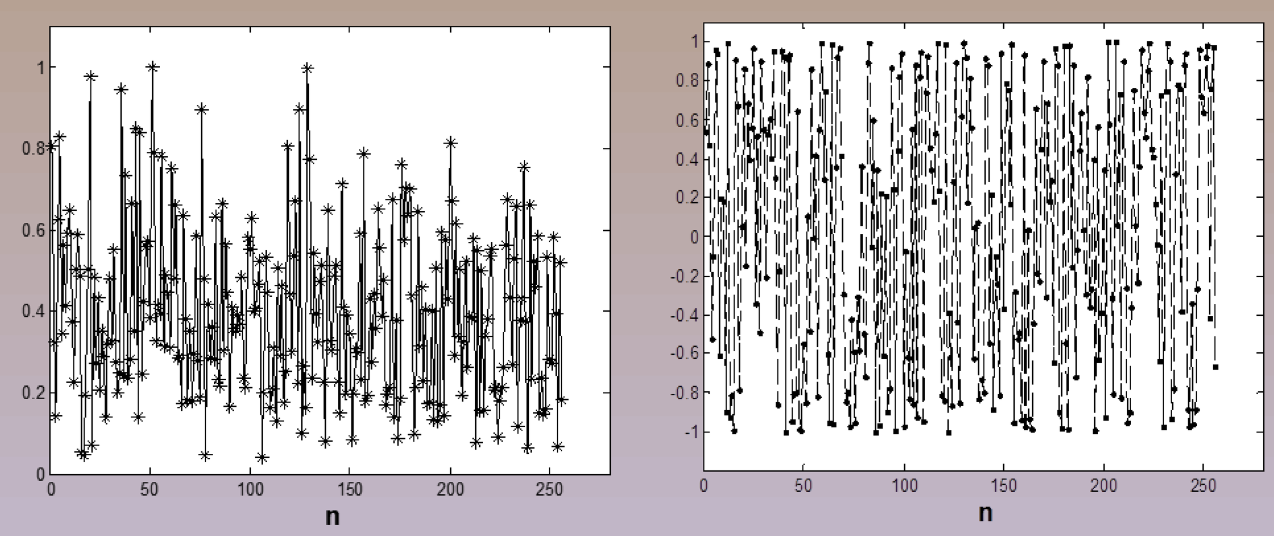

Fig. 3. The MRFrFT of a 1D rectangular function. (a) The output magnitude, (b) The output phase.

The proposed MRFrFT can be directly used in image encryption by executing (15). The security is guaranteed by large key space composed by the random coefficients vector , the logistic map parameters and , and the RPOMs. The decryption schematic is the inverse process of the image encryption. Fig. 4 illustrates the image encryption and decryption results by using a gray image “lena” with 256 256 pixels. It can be seen that slight variations of the keys will cause strong damage to the decryption image and cannot identify them visually.

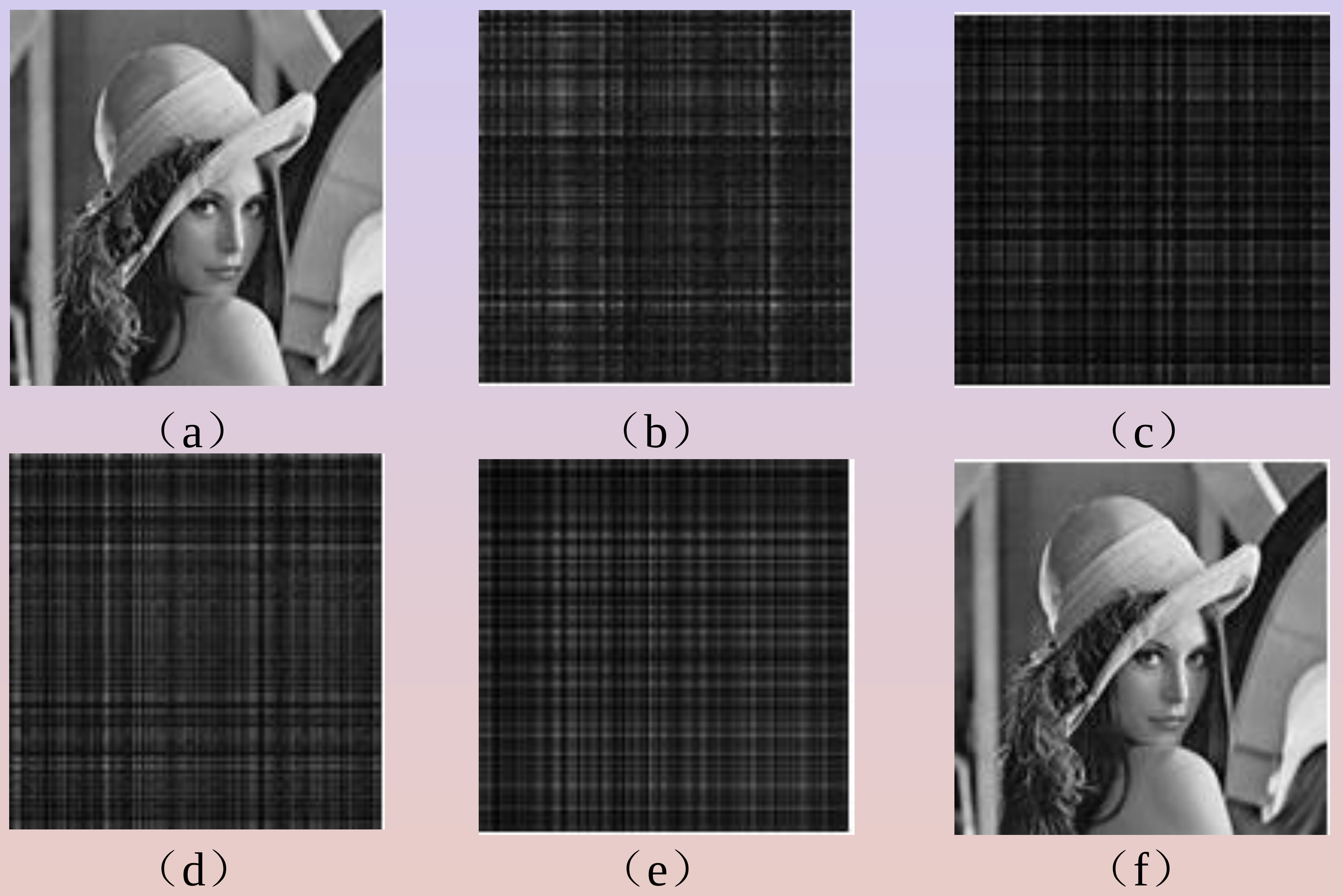

Fig. 4. Results of image encryption and decryption with the MRFrFT. (a) Original image, (b) encrypted image, (c) decrypted image with incorrect weighting coefficients p1=0.9562, (d) decrypted image with incorrect logistic map parameters x02=0.2112, (e) decrypted image with a couple incorrect RPOMs in the first chosen channel, (f) decrypted image with correct keys.

Conclusion In this letter, we propose an efficient MRFrFT with random weighting coefficients and partial channel transform kernel functions. It can be carried out both electronically and optically. This proposed transform is also able to be used in color image encryption and multi-image encryption with high security and efficiency.