OPTIMAL TRANSMIT STRATEGY FOR MIMO CHANNELS WITH JOINT SUM AND PER-ANTENNA POWER CONSTRAINTS

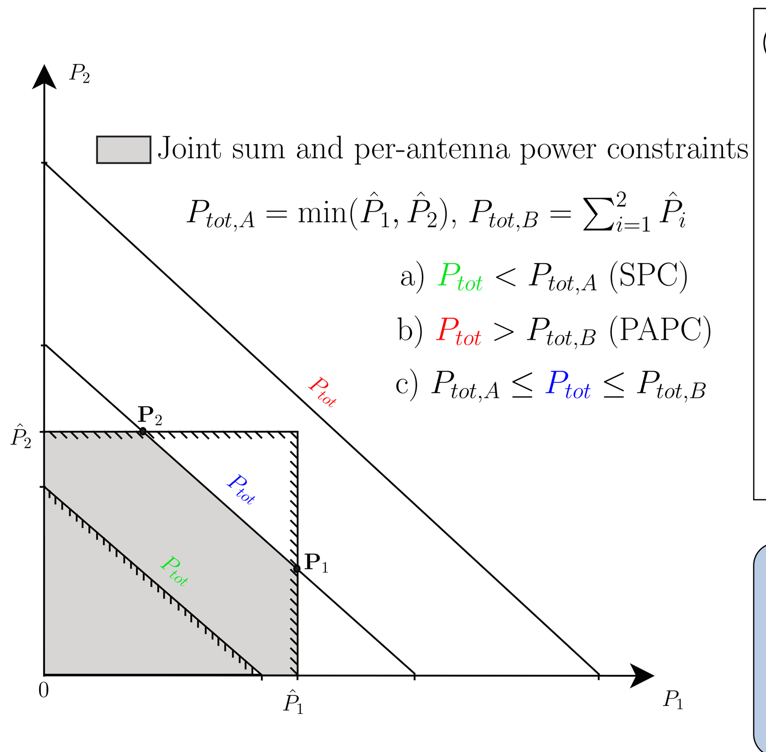
Phuong Le Cao and Tobias J. Oechtering

School of Electrical Engineering and ACCESS Linnaeus Centre,
KTH Royal Institute of Technology, Stockholm, Sweden

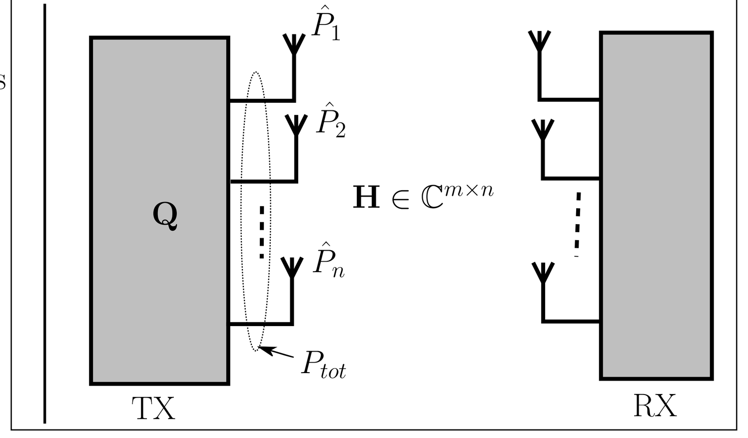
Why Joint Sum and Per-antenna Power Constraints?

- **Sum power constraint (SPC)** is imposed by regulations or to limit the energy consumption.
- **Per-antenna power constraints (PAPC)** are imposed by hardware limitation of each RF chain
- Problems with **SPC** and **PAPC** have been studied by many researchers before for both point-to-point and multi-user channels
- **Both motivations are simultaneously relevant** for practical systems, thus we consider a system with a **joint sum and per-antenna power constraints (JSPC)**[1]

MIMO Channels with JSPC



MIMO Transmission



Definition

- $\mathcal{S}(\mathcal{A}) := \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \le P_{tot}, P_i = \mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i \le \hat{P}_i, \forall i \in \mathcal{A} := \{1 \dots n\}\}$
- $f(\mathbf{Q}) := \log |\mathbf{I}_m + \mathbf{H}\mathbf{Q}\mathbf{H}^H|$

? Optimization Problem

$$\max_{\mathbf{Q}} f(\mathbf{Q}), \quad \text{s.t.} \mathbf{Q} \in \mathcal{S}(\mathcal{A}).$$

PROPERTIES

Propisition 1

The maximum transmission rate $R^\star$ can be achieved when the optimal transmit strategy $\mathbf{Q}^\star$ uses full power P_{tot}

Lemma 1

Let $\mathcal{A}' \subseteq \mathcal{A}$, $\mathcal{S}(\mathcal{A}') := \{\mathbf{Q} \succcurlyeq 0 : \mathrm{tr}(\mathbf{Q}) \le P_{tot}, P_j^{\mathcal{S}(\mathcal{A}')} = \mathbf{e}_j^T \mathbf{Q} \mathbf{e}_j \le \hat{P}_j, j \in \mathcal{A}'\}$, and $\mathcal{P} := \{i \in \mathcal{A}^c : P_i^{\mathcal{S}(\mathcal{A}')} > \hat{P}_i\}$ with $\mathcal{A}^c = \mathcal{A} \setminus \mathcal{A}'$. Then the optimal power can be allocated as

$$\begin{cases} P_i^\star = P_i^{\mathcal{S}(\mathcal{A}')}, \forall i \in \mathcal{A}^c & \text{if } \mathcal{P} = \emptyset, \\ P_i^\star = \hat{P}_i, \quad \forall i \in \mathcal{P} & \text{otherwise.} \end{cases}$$

APPROACHING METHOD

- Sequence of optimization problems:

$$\begin{aligned} \max_{\mathbf{Q}\in\mathcal{S}(\emptyset)} f(\mathbf{Q}) &= \max_{\mathbf{Q}\in\mathcal{S}(\emptyset)\cap\{[\mathbf{Q}]_{ii}\le\hat{P}_i:\forall i\in\mathcal{P}(1)\}} f(\mathbf{Q}) \\ &\ge \max_{\mathbf{Q}\in\mathcal{S}(\emptyset)\cap\{[\mathbf{Q}]_{ii}\le\hat{P}_i:\forall i\in\mathcal{P}(2)\}} f(\mathbf{Q}) \\ &\dots \\ &\ge \max_{\mathbf{Q}\in\mathcal{S}(\emptyset)\cap\{[\mathbf{Q}]_{ii}\le\hat{P}_i:\forall i\in\mathcal{P}(K)\}} f(\mathbf{Q}) = \max_{\mathbf{Q}\in\mathcal{S}(\mathcal{A})} f(\mathbf{Q}), \end{aligned}$$

- Re-assign the antenna coefficient order, form $\mathbf{Q}^\star = \begin{bmatrix} \mathbf{Q}_P & \mathbf{q}^H \\ \mathbf{q} & \mathbf{Q}_S \end{bmatrix}$
- Finding the remaining optimal power allocation in $\mathbf{Q}_S$ by solving the reduced optimization problem

$$\max_{\mathbf{Q}(k)} f(\mathbf{Q}(k)) \text{ s.t. } \mathbf{Q}(k) \in \mathcal{S}(\mathcal{P}(k))$$

using the generalized water-filling solution[2]:

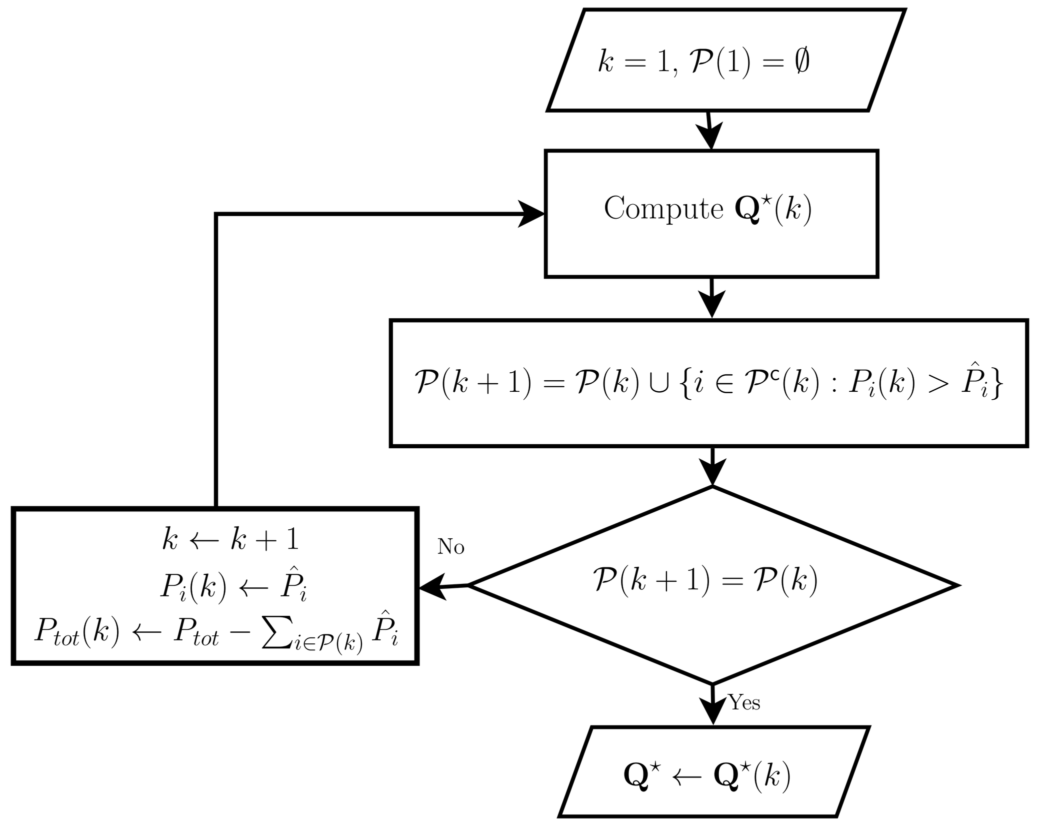
Lemma 2

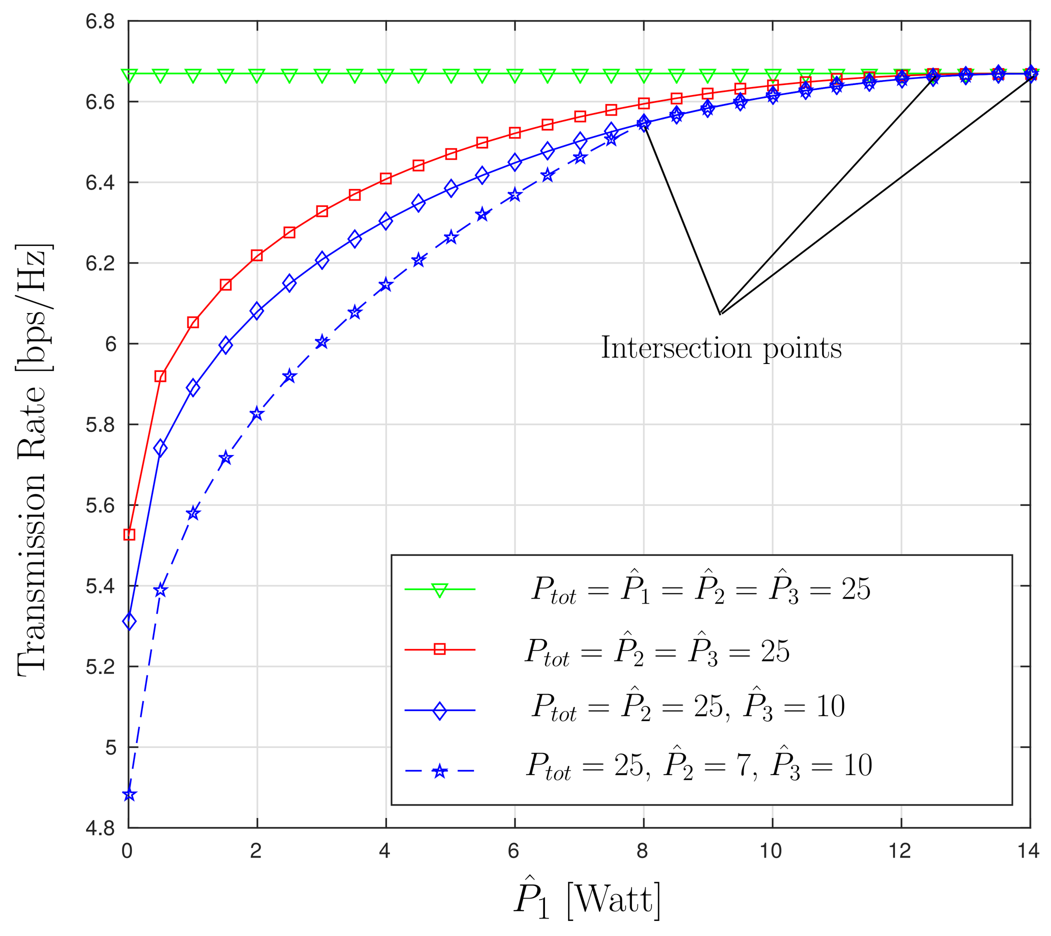
The optimal solution of the transmit strategy $\mathbf{Q}^\star(k)$ in k-th iteration

$$\mathbf{Q}^\star(k) = (\mathbf{D}^{-\frac{1}{2}}[\mathbf{U}]_{:,1:L}[\mathbf{U}]_{:,1:L}^H\mathbf{D}^{-\frac{1}{2}} - [\mathbf{U}]_{:,1:L}\mathbf{\Lambda}^{-1}[\mathbf{U}]_{:,1:L}^H)^+,$$

where $\mathbf{\Lambda}$ and $[\mathbf{U}]_{:,1:L}$ are obtained from eigenvalue decomposition $\mathbf{H}^H\mathbf{H}$. The diagonal elements of $L \times L$, $L = \min(n, m)$, diagonal matrix $\mathbf{\Lambda}$ are positive real values in decreasing order.

ALGORITHM





- The more restricted per-antenna power constraint, the less optimal transmission rate since adding more per-antenna power constraints, we have less freedom to allocate the optimal transmit power
- Intersection points: points where the power constraints on some antennas change their state from active to inactive.

CONCLUSIONS

- Optimal transmit strategy in closed-form using generalized water-filling
- An unconstraint optimal power allocation of an antenna exceeds a per-antenna power constraint, then it is optimal to allocate the maximal power in the constraint optimal transmit strategy including the per-antenna power constraints
- Highly relevant and interesting for massive MIMO since the results might be extended to have power constraints for groups of antennas which are driven by one amplifier which has an own power constraint.

References

[1] P. L. Cao, T.J Oechtering, R.Schaefer and M. Skoglund, "Optimal Transmit Strategy for MISO Channels with Joint Sum and Per-antenna Power Constraints," *IEEE Transactions on Signal Processing*, vol.64, no.16, pp.4296-4306, Aug 2016.

[2] C. Xing, Z. Fei, Y. Zhou, and Z. Pan, "Matrix-field Water-filling Architecture for MIMO Transceiver Designs with Mixed Power Constraints," in *PIMRC*, Sep. 2016.