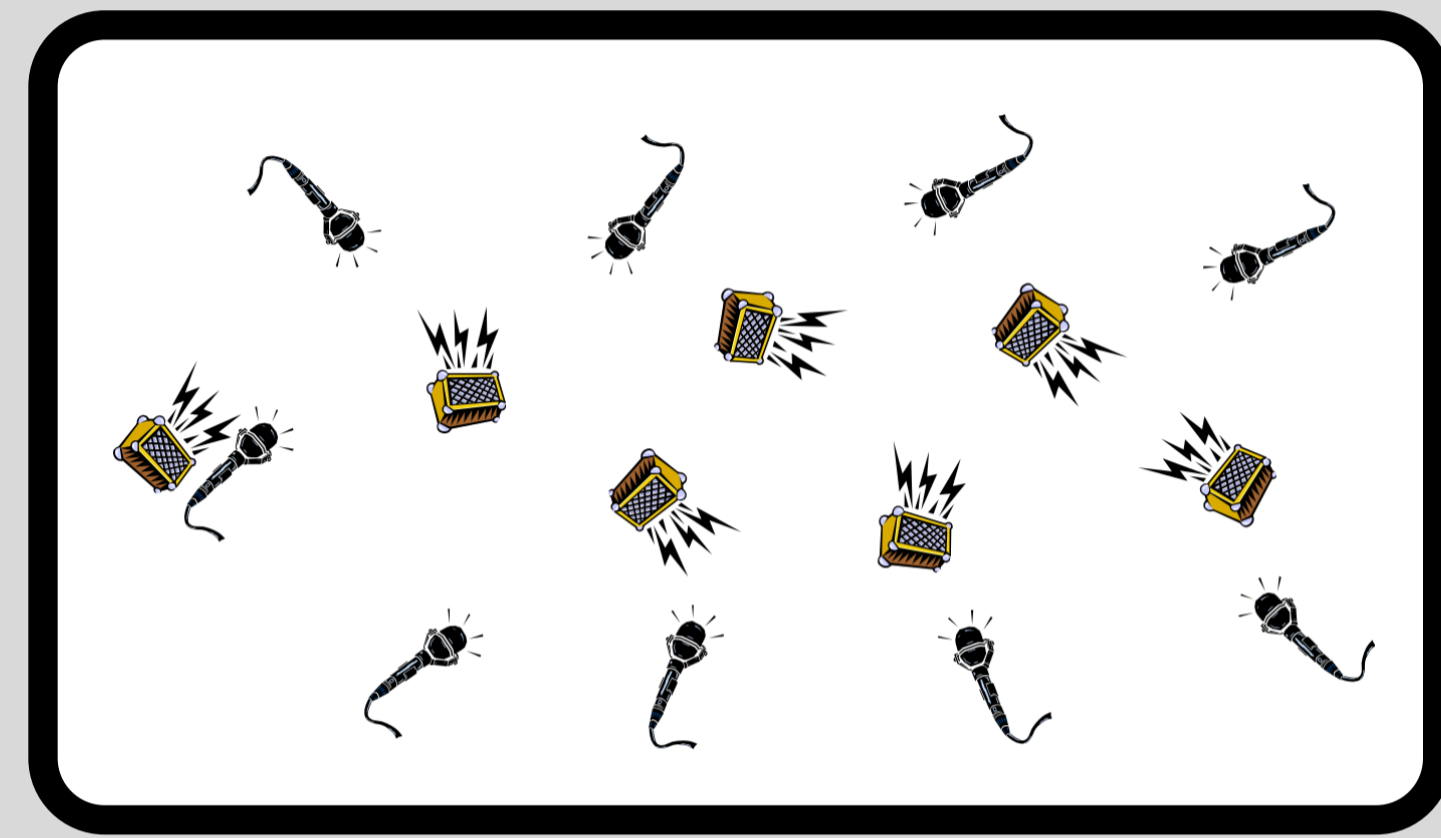
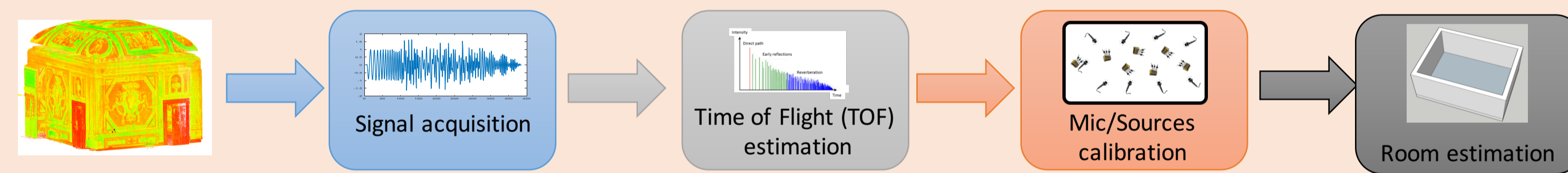


A method to extract Time Difference of Arrivals (TDOAs) for room reflections:

- Estimation of TDOAs with **no a priori** information (statistic of signals, walls/reflectors positions)
- **Few parameters** to tune.
- Fast, iterative solution that promotes **sparsity and non-negativity**.
- Tested with **real signals** (speech and non-speech).



Room Reconstruction Problem



The estimation of reliable TDOAs is a necessary step for room reconstruction to recover the 3D position and orientation of reflectors and the position of sound sources/microphones.

The cross-relation property of a Multi-Output system:

$h_{m_i}(k)$ is the Room Impulse Response (RIR) between source location and microphone m_i location. For every couple of microphones we have that:

$$h_{m_1}(k) * h_{m_2}(k) * s(k) = h_{m_2}(k) * h_{m_1}(k) * s(k),$$

Thus giving:

$$h_{m_1}(k) * y_{m_2}(k) = h_{m_2}(k) * y_{m_1}(k)$$

$$y_{m_1}(k) * h_{m_2}(k) = Y_{m_1} \mathbf{h}_{m_2} = Y_{m_2} \mathbf{h}_{m_1}$$

Shift to matrix form

$$\text{Leading to: } J(\mathbf{h}) = \sum_{m_1 \neq m_2} \|Y_{m_1} \mathbf{h}_{m_2} - Y_{m_2} \mathbf{h}_{m_1}\|_2^2 = \mathbf{h}^T \mathbf{Q} \mathbf{h}$$

Solution of the quadratic cost function $J(\mathbf{h})$:

Include penalty terms in the cost function to avoid trivial solutions and to promote properties of the RIR: **Sparsity and non-negativity**.

Sparse: $\min_{\mathbf{h}} J(\mathbf{h}) + \lambda \|\mathbf{h}\|_0 \quad s.t. \quad \|\mathbf{h}\|_2^2 = 1$

\downarrow

$$\min_{\mathbf{h}} J(\mathbf{h}) + \lambda \|\mathbf{h}\|_1 \quad s.t. \quad \|\mathbf{h}\|_2^2 = 1$$

NP-hard

Drawbacks:

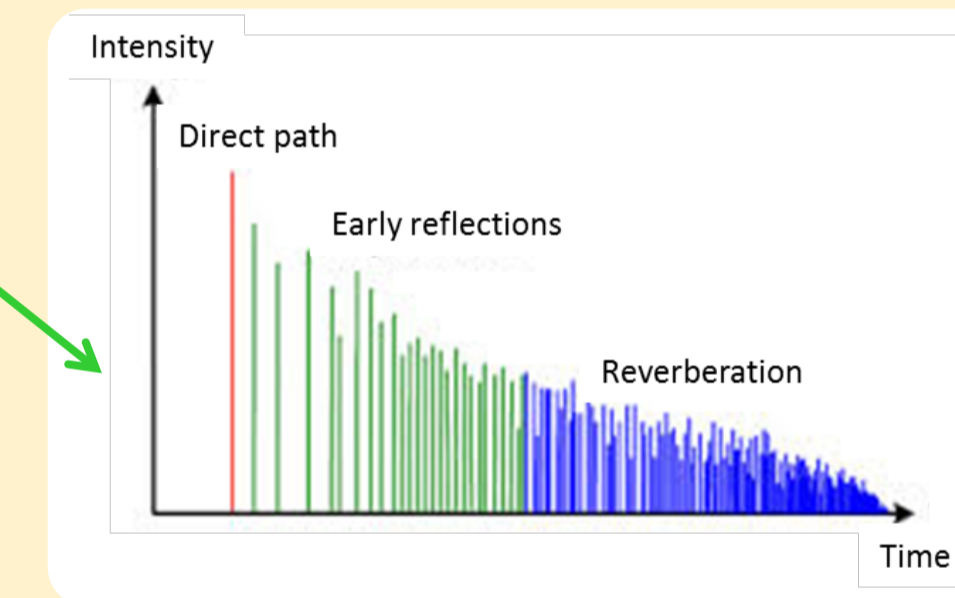
- Non-convex problem due to the quadratic equality constraint;
- L_1 norm penalizes larger coefficients more: the solution might be not equal to the L_0 norm.

Kowalczyk et al. "Blind System Identification Using Sparse Learning for TDOA Estimation of Room Reflections." Sig. Proc. Letters, 2013.

Non-negative:

$$\min_{\mathbf{h}} J(\mathbf{h}) + \lambda \|\mathbf{h}\|_1 \quad s.t. \quad h_1(a) = 1, \quad \mathbf{h} \geq 0$$

Convex formulation



Drawbacks:

- amplitude distortion, peak of the anchor overly enhanced;
- does not solve the L_1 penalty limitations.

Lin et al. "Blind sparse-nonnegative (BSN) channel identification for acoustic time-difference-of-arrival estimation." IEEE Workshop on applications of Signal Processing to Audio and Acoustics, 2007

Iterative weighted L_1 constraint (IL1C)

IL1C promotes **sparsity and non-negativity** by removing the previous drawbacks. It is an iterative method that at **each iteration re-weights to improve sparsity**.

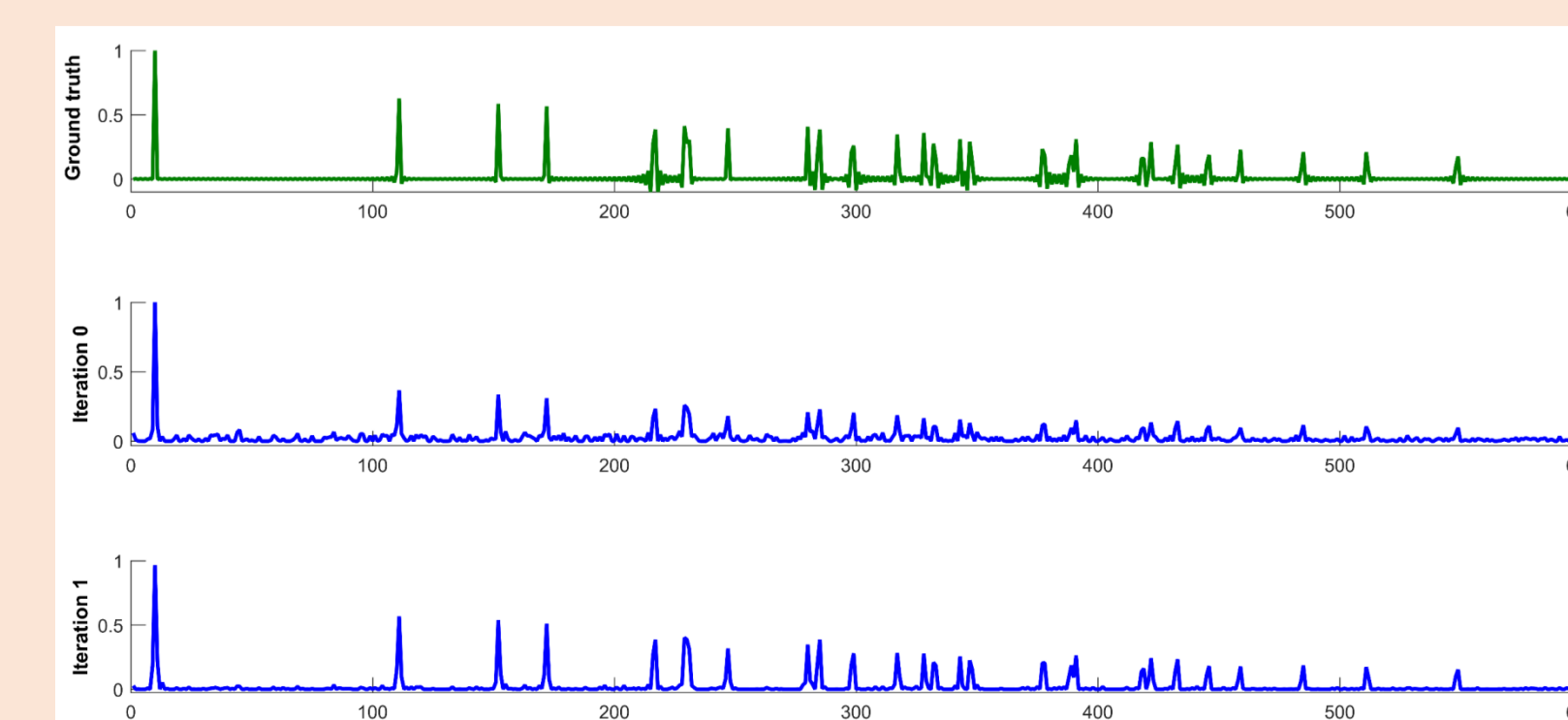
Solve a sequence of sub-problems for $z = 1, \dots, Z$:

$$\mathbf{h}^z = \arg \min_{\mathbf{h}} J(\mathbf{h}) + \|\mathbf{h}\|_1, \quad s.t. \quad \mathbf{p}^z \mathbf{h} = 1, \quad \mathbf{h} > 0$$

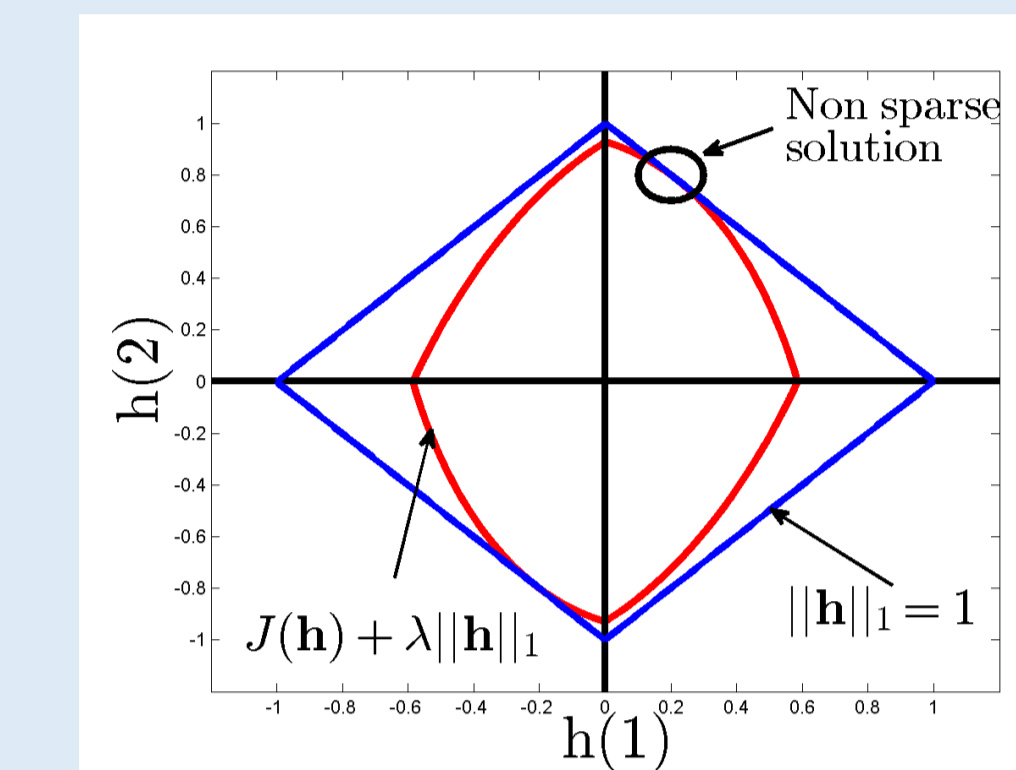
Weight update rule
 $\mathbf{p}^z = \mathbf{h}^{z-1}$

No anchor constraint and penalizing more the small components

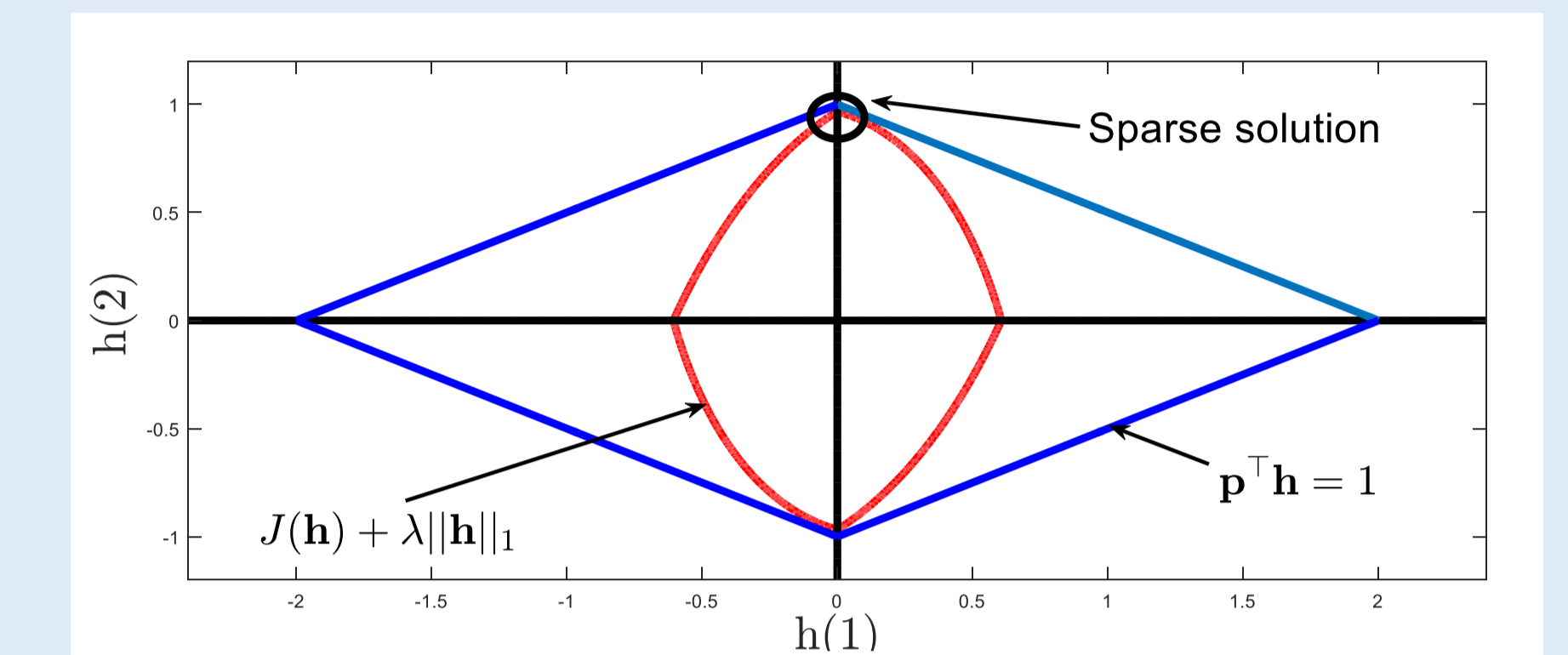
Each iteration consists of the minimization of a **quadratic cost function + linear constraints: convex problem** easily solved by standard optimization methods.



Geometrical interpretation



Quadratic cost function with L_1 penalty

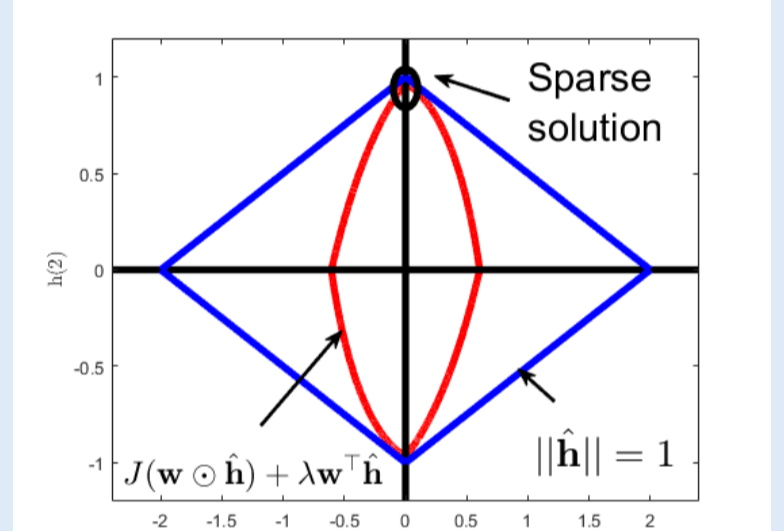


Quadratic cost function with L_1 penalty and weighted L_1 constraint

Let us make a variable change: $\mathbf{h} = \mathbf{w} \odot \hat{\mathbf{h}}$

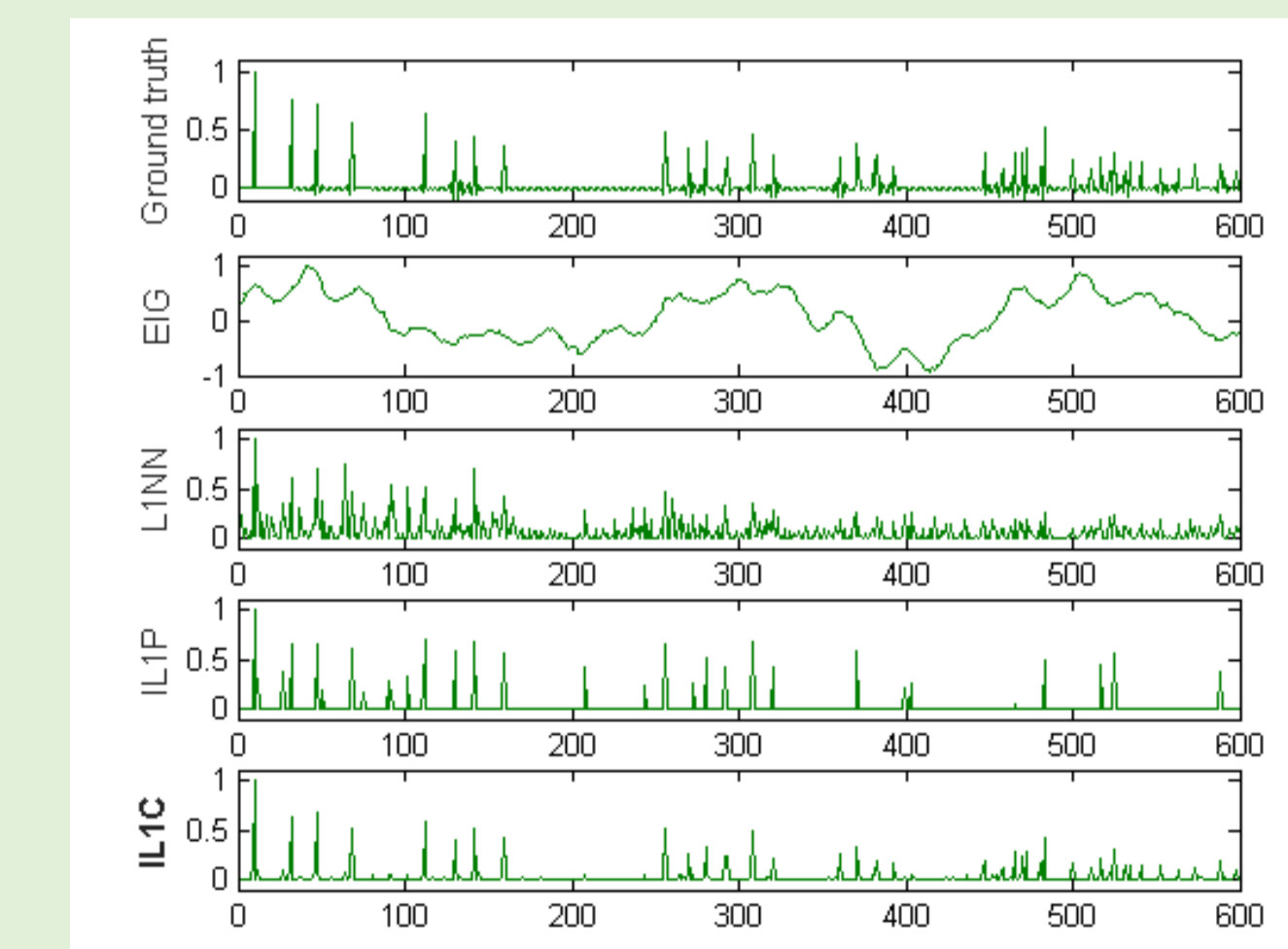
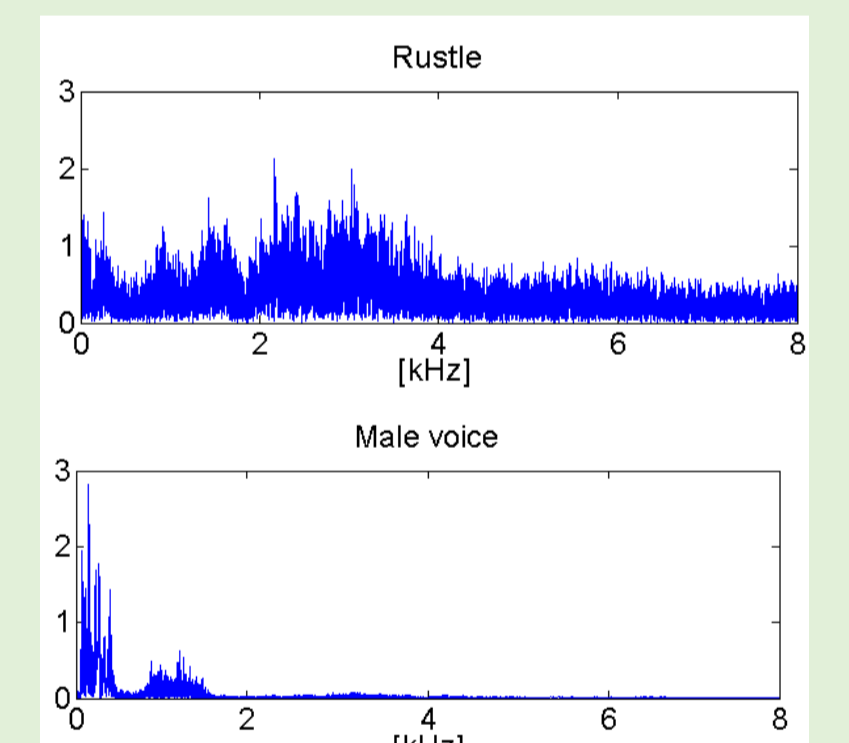
with: $w(k) = \frac{1}{p(k)}$

$$J(\mathbf{w} \odot \hat{\mathbf{h}}) + \mathbf{w}^T \hat{\mathbf{h}}, \quad s.t. \quad \|\hat{\mathbf{h}}\|_1 = 1, \quad \mathbf{h} > 0$$



Experiments

- Simulated room of 5 x 4 x 6 m
- 2 microphones and a source randomly placed
- RIRs simulated with the image method [Allen & Berkley, 1979]
- Synthetic and **real** signals: white noise, **rustle**, **male voice**
- **Variable SNR: 0, 6, 14, 20, 40 dB**
- 50 Monte Carlo simulations for each SNR



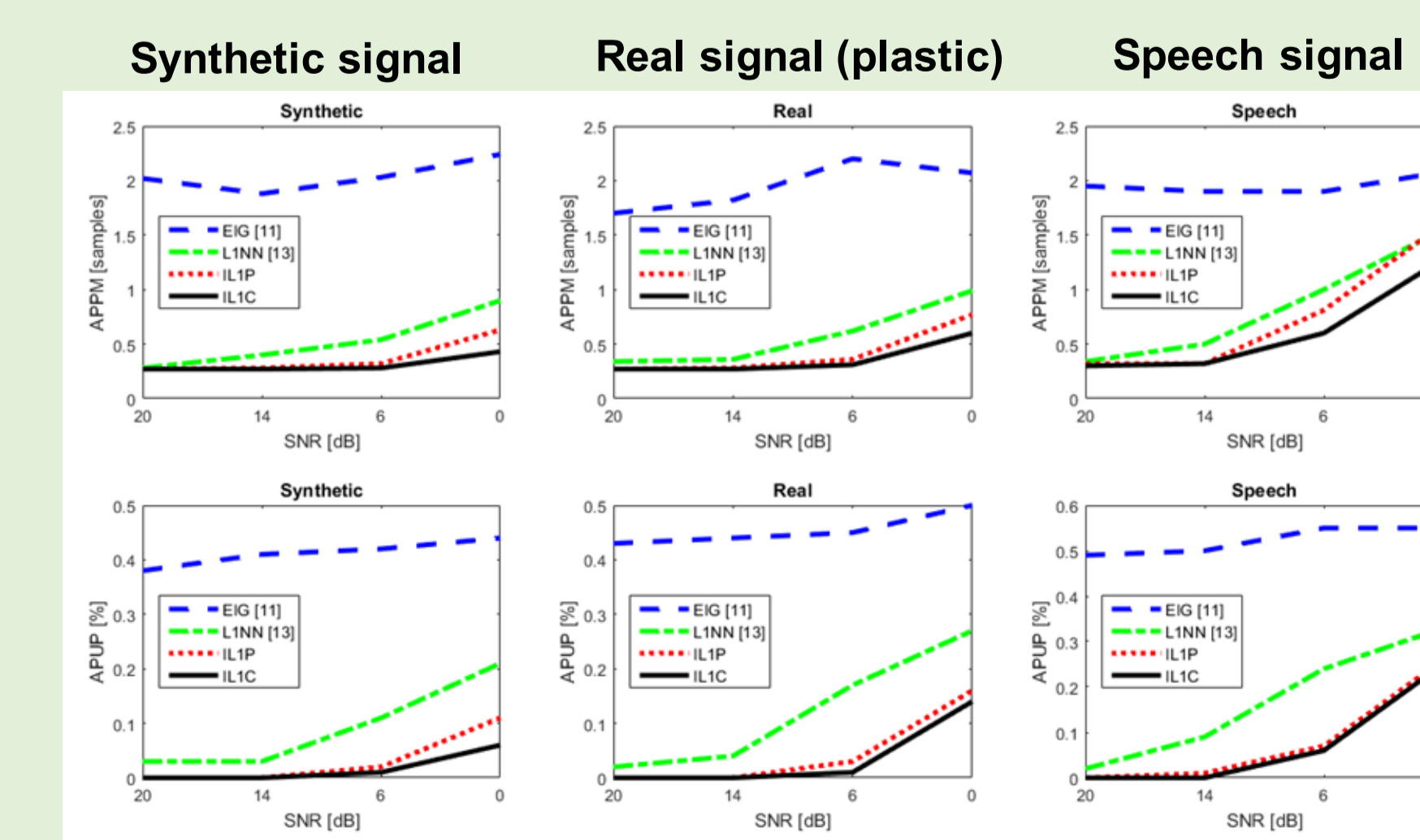
EIG: eigenvalue problem [Tong et al. 1994]

L1NN: anchor constraint [Lin et al. 2007]

IL1P: iterative L1 penalty [Crocco and Del Bue, 2015]

IL1C: iterative L1 constraint [Crocco and Del Bue, 2016]

Results show that SVD based solution fails while anchor constraints do not achieve desired sparsity. IL1P solution is a penalty based solution over the L_1 regulariser while IL1C achieves the desired level of sparsity



Average peak position mismatch

$$APPM = \sum_{i=1}^{50} \sum_{p=1}^{P_i} \frac{|r_{p,i}^g - r_{p,i}^e|}{50P_i}$$

Peak position accuracy over the inliers

Average percentage of unmatched peaks

$$APUP = \sum_{i=1}^{50} \frac{K - P_i}{50K}$$

Percentage of outliers (> 20 samples)