

A STUDY ON MIXING SEQUENCES IN MODULATED WIDEBAND CONVERTERS

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0. Short Summary

- The modulated wideband converter (MWC) is a sub-Nyquist sampler exploiting the compressed sensing theory, which employs multiple number of periodic random sequences.
- We scope on choosing the periodic random sequences.
 - The sequences should be well chosen for the lossless sampling.
 - The robustness against noise in the lossless sampling is also influenced by the type of sequences.
- We provide conditions for the best sequences!

1. Background

- The Modulated wideband converter (MWC): A sub-Nyquist sampler employing multiple number of periodic random sequences $p_i(t)$.

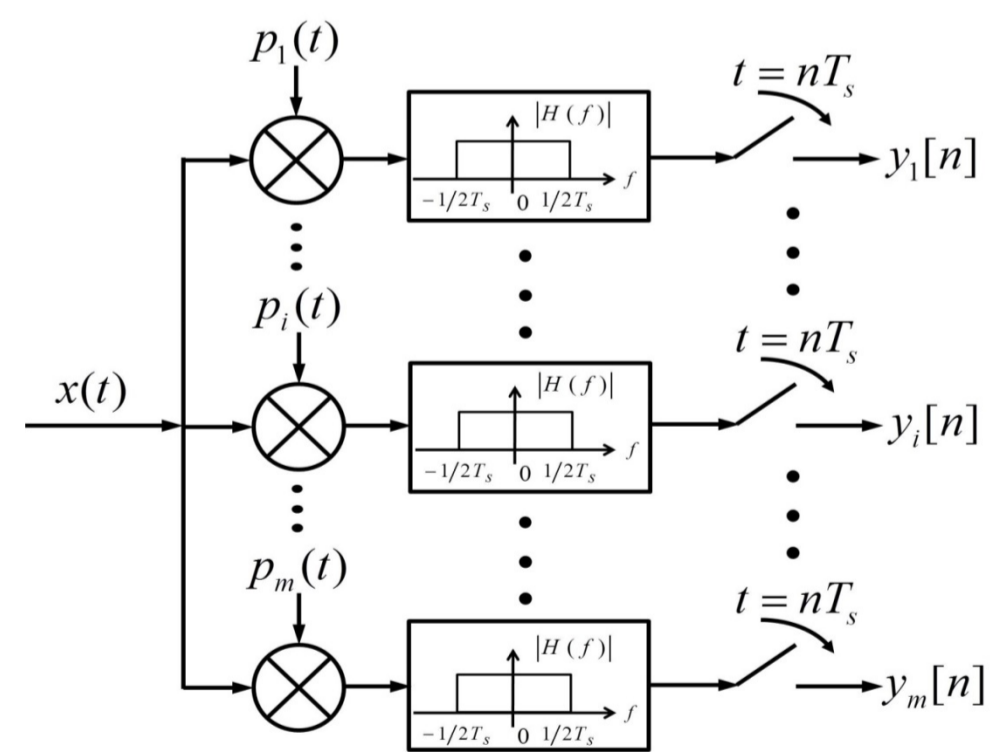


Fig. 1. Schematic of the MWC

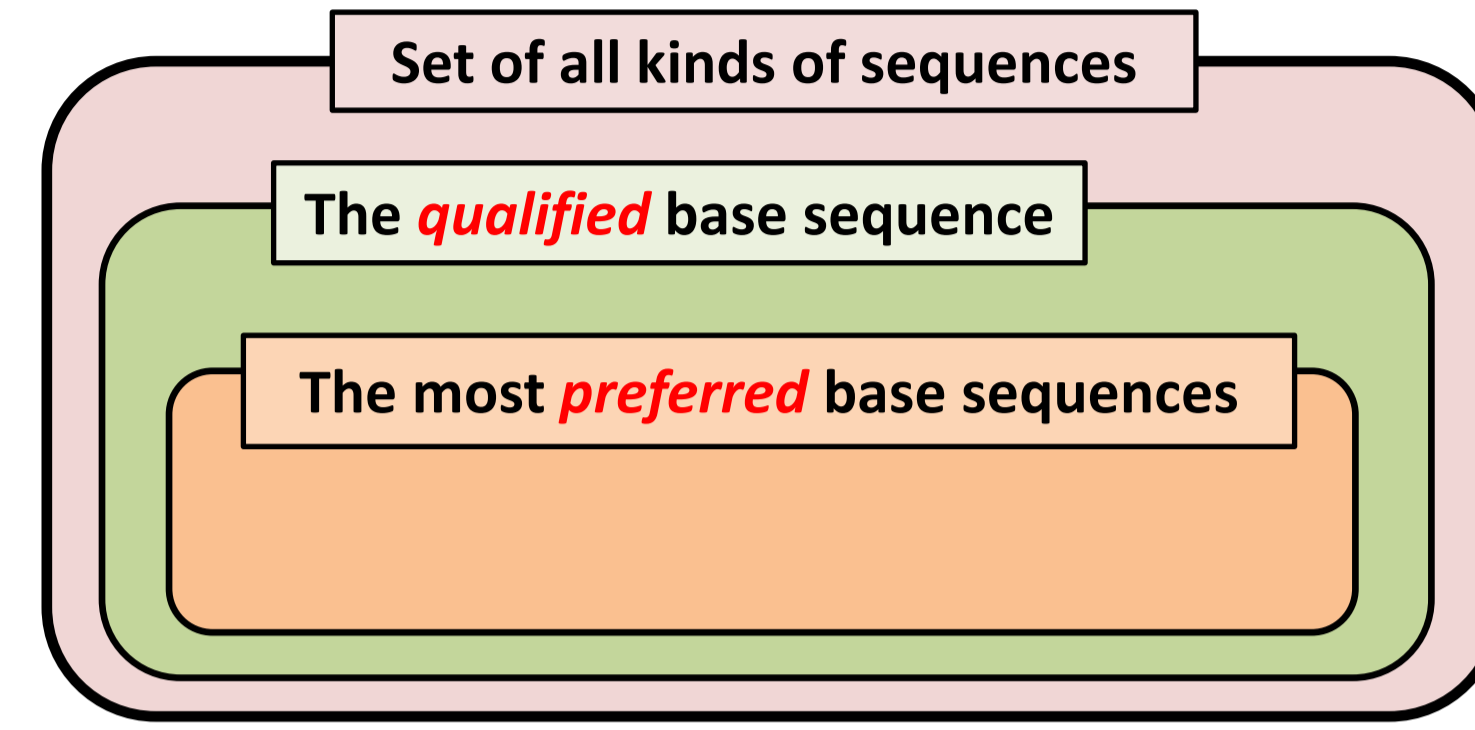
- We scope on choosing the sequences $p_i(t)$.
- The random partial Fourier MWC (RPFMWC): A MWC, whose sequences $p_i(t)$ are random cyclic shifts of a base sequence.
- We scope on choosing the base sequence.

2. Question?

Q. What kinds of sequences are
1. Qualified and 2. Preferred
for the base sequence of RPFMWC?

Find conditions for:

- The **qualified** base sequence: necessary for the lossless sampling of the RPFMWC.
- The more **preferred** base sequence: more robust against noise for the lossless sampling.



3. Existing Results

- The Qualified sequences:** If the discrete Fourier transform (DFT) of a base sequence have uniform magnitudes, the RPFMWC can achieve the lossless sampling. [L. Gan 2013]
 - The sufficient condition is quiet strict.
 - More general conditions are needed.
- The Preferred sequences:** No research efforts have been reported.

4-1. Answer (1)

- Theorem 1:** Sufficient and necessary conditions for the **qualified** sequences.
 - A sequence is qualified if and only if the DFT does not have zero value at all.

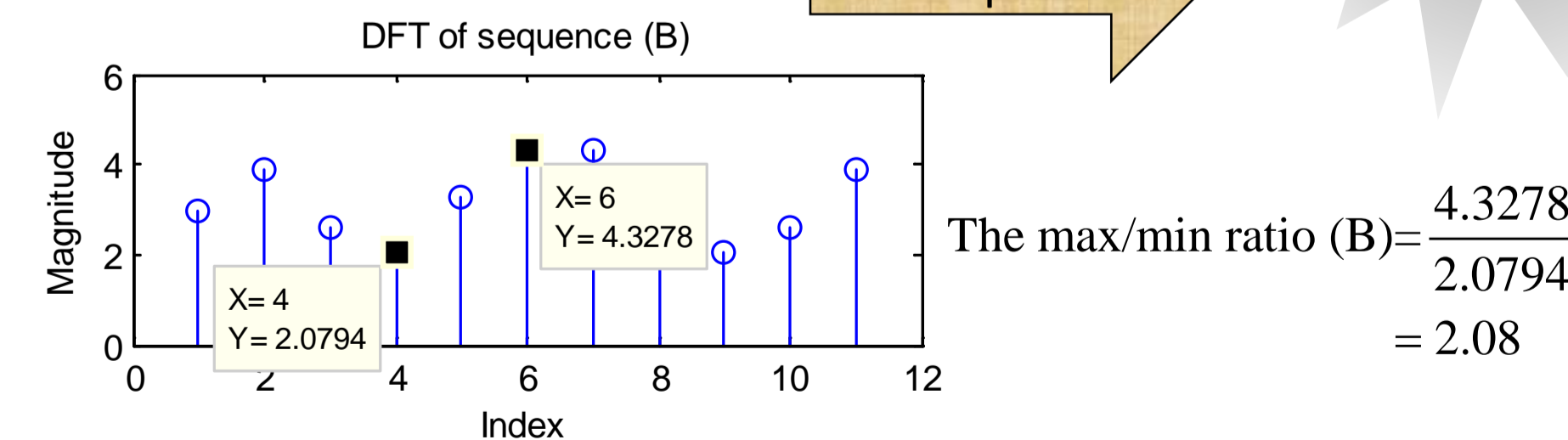
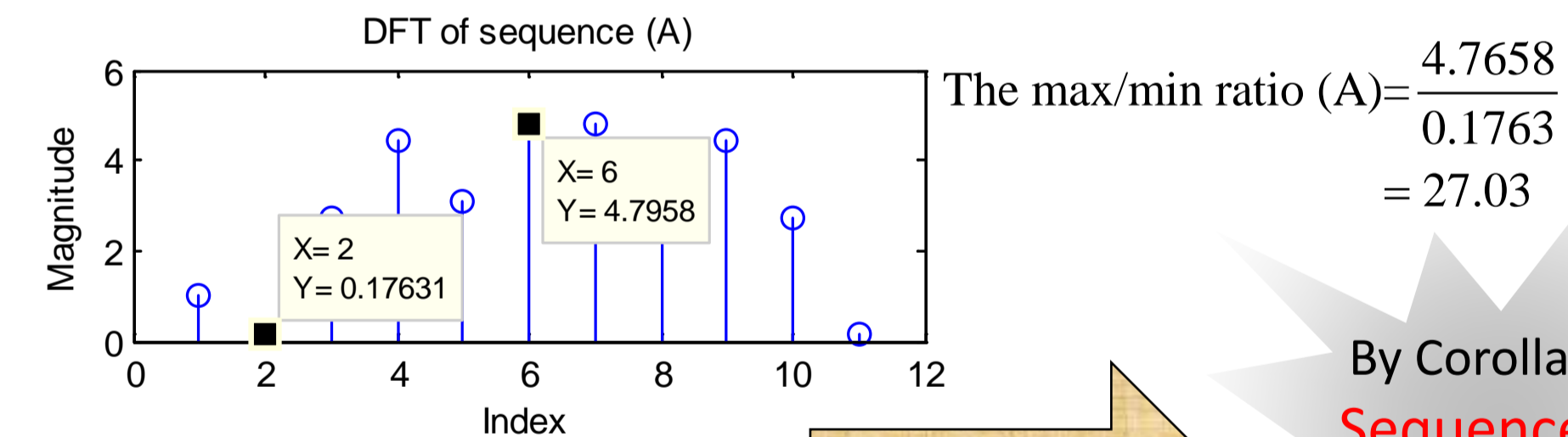
ex 1)
Seq.(A)=[1 -1 -1 1 -1 1 1 -1 -1 -1]
DFT (Mag.)
 $\sigma(\mathbf{A})=[2 \ 2.3 \ 3 \ 3.8 \ 4.3 \ 0 \ 4.3 \ 3.8 \ 3 \ 2.3]$
Zeros in DFT
Sequence A is Not Qualified

ex 2)
Seq.(B)=[-1 1 1 -1 1 1 -1 1 1 1]
DFT (Mag.)
 $\sigma(\mathbf{B})=[4 \ 0.7 \ 1.2 \ 5.2 \ 3.2 \ 2 \ 3.2 \ 5.2 \ 1.2 \ 0.7]$
No zero in DFT
Sequence B is Qualified!!

4-2. Answer (2)

- Corollary 3:** An answer for the **preferred** sequences in terms of robustness against noise.

- SNR required for the lossless sampling is *inversely proportional* to the ratio of max/min magnitudes in the DFT of the base sequence noise.



Max/Min ratios comparison

By Corollary 3,
Sequence (B)
Wins!

- The **Spectral instability:** The ratio of max/min magnitudes in the DFT of a sequence.

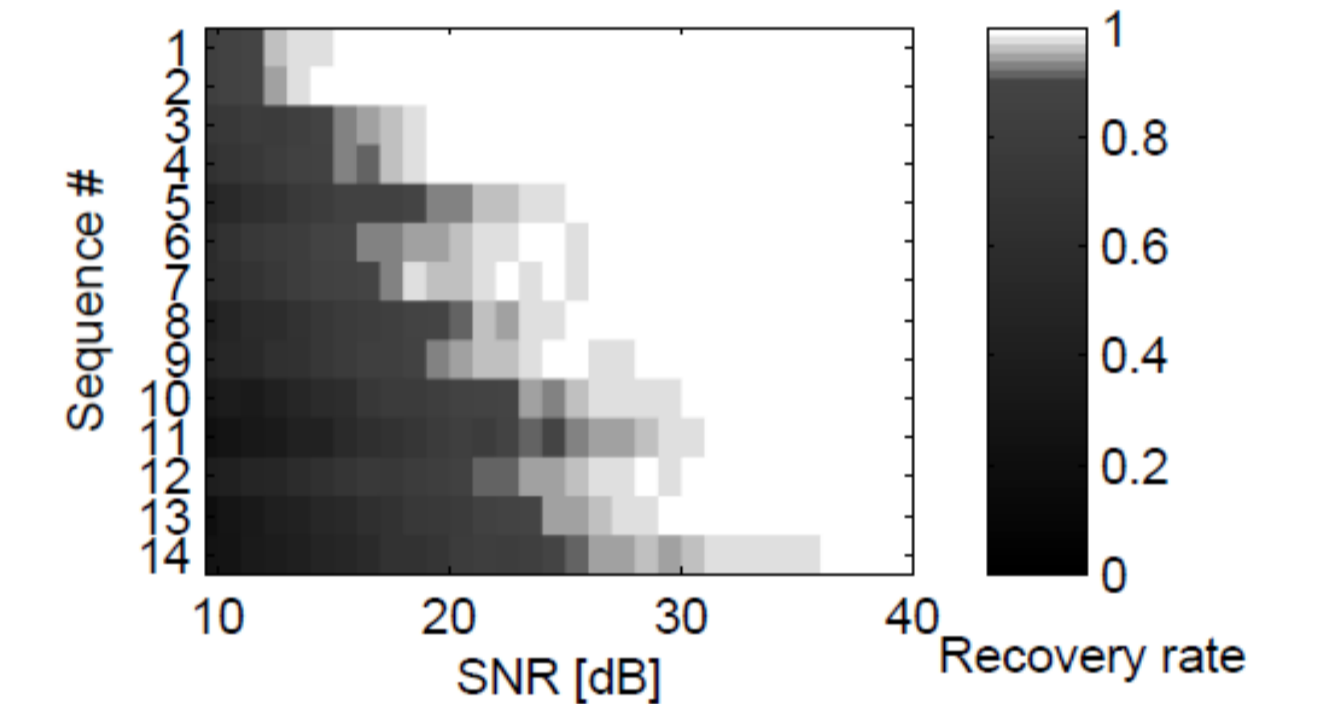
$$\eta := \frac{\max_i |\sigma_i|}{\min_i |\sigma_i|}$$

5. Simulations

The Spectral instability well evaluates the base sequences!

TABLE I: List of Mixing Sequences and Spectral Instabilities for $M = 127$ and $\varphi = 1$

#	Sequence type	$\eta(T)$	#	Sequence type	$\eta(T)$
1	<i>m</i> -sequences	1.000	8	Bernoulli 5	13.443
2	Legendre	1.000	9	Gold 2	16.527
3	Bernoulli 1	3.878	10	Bernoulli 6	19.917
4	Bernoulli 2	4.933	11	Gold 3	26.946
5	Gold 1	6.609	12	Bernoulli 7	33.423
6	Bernoulli 3	9.344	13	Bernoulli 8	41.296
7	Bernoulli 4	10.987	14	Bernoulli 9	54.820



(c) $K = 16$

Fig. 3. Successful support recovery rate of RPFMWC versus SNR.

6. Conclusion

Topic: The **qualification** and the **preference** for the base sequence of the RPFMWC.

- The **Qualification:** The base sequence should not have zero value in the DFT at all.
- The **Preference:** The RPFMWC of a base sequence having the lower Spectral instability is more robust against noise.

The lower spectral instability

The higher noisy robustness