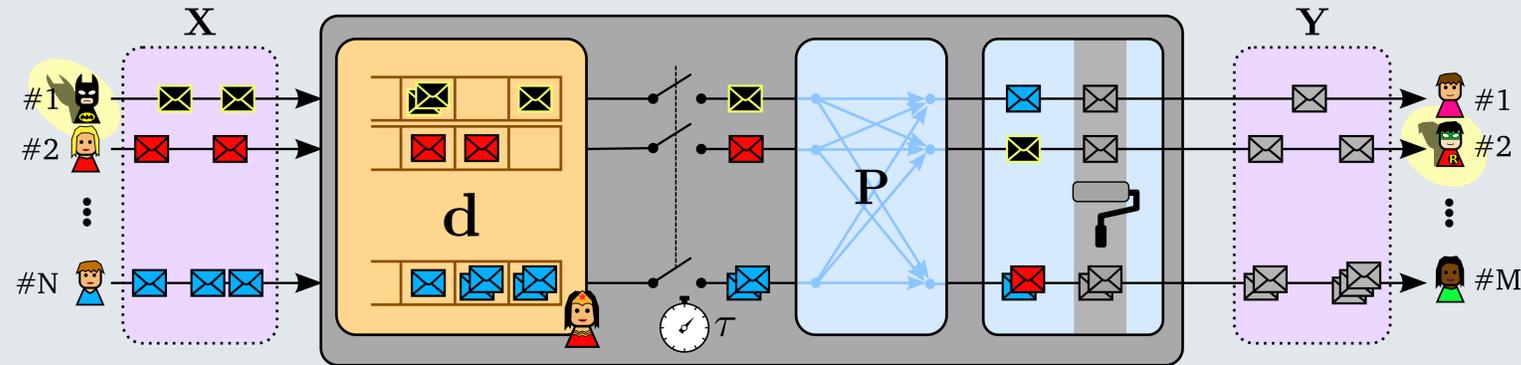


Delay-Based Anonymous Communication System

Some users send messages through the anonymous communication system

The anonymous communication system (Timed Mix) works in four steps

The recipients receive the messages after some time



Attacker
 Observes inputs and outputs

Defender
 Designs the probability mass function of the delay to stop the attacker...

Another way of looking at the problem: linear filtering

From the model:

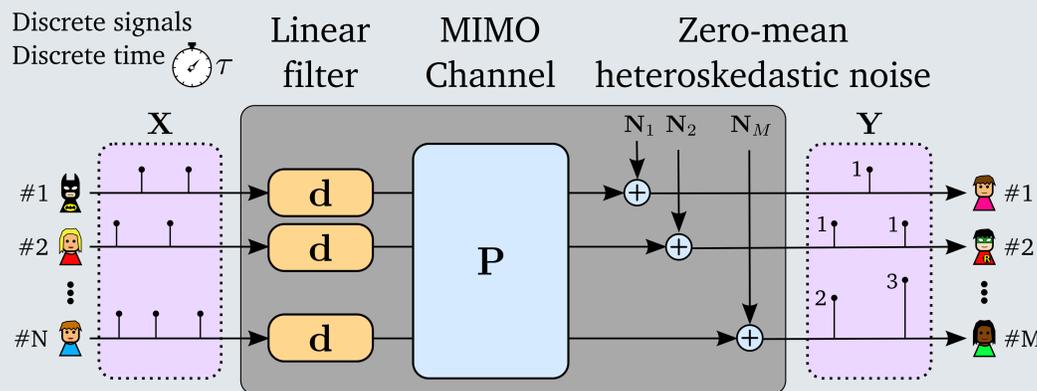
$$Y = D \cdot X \cdot P + N$$

Convolution matrix that contains **d**

Zero-mean noise with covariance

$$\Sigma_{N|X} = \text{diag}\{DX1_N\} - D\text{diag}\{Xv\}D^T$$

This is like a MIMO system:

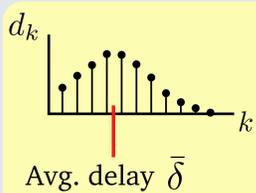


Attacker
 The optimal adversary performs a least-squares estimation

$$\hat{P} = (X^T D^T D X)^{-1} X^T D^T Y$$

Defender
 Designs the linear filter that worsens the least-squares estimation...

This is a filter design problem... with some constraints



Positive taps

$$d_k \geq 0$$

Adds up to one

$$\sum_k d_k = 1$$

Average delay

$$\sum_k d_k \cdot k \leq \bar{\delta}$$

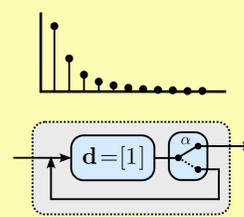
It is easier to achieve attenuation at high frequencies; and the first DFT coefficient is larger than the others.

The first DFT coefficient is one.

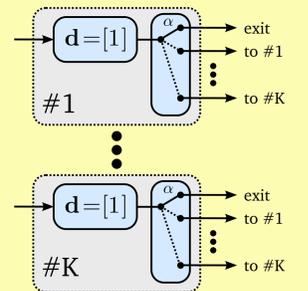
We prefer the minimum-phase solution.

Applications of filter design

An exponential delay is an IIR filter:



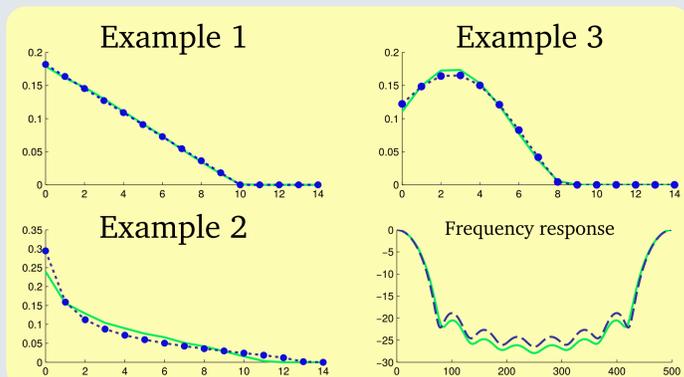
Distributed implementation of the exponential delay



Evaluation

Depending on the system parameters, the shape of the "optimal filter" changes.

- Numerical, from real data.
- Analytical, from the filter design problem.

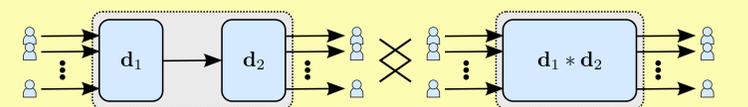


The **analytical** designs are very close to the **numerical** solutions with real-data experiments!



In real systems, we have networks of mixes

Cascade of mixes = convolution of filters



Mixes in parallel = sum of filters

