Detection of cyclostationarity in the presence of temporal or spatial structure with applications to cognitive radio

PADERBORN UNIVERSITY

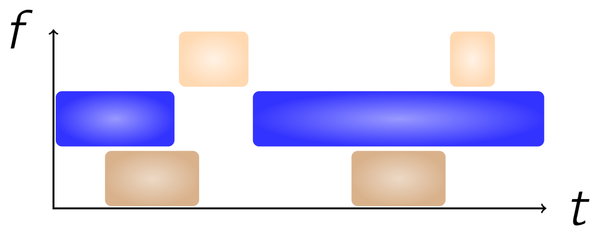
Universidad Carlos III de Madrid

Aaron Pries[1], David Ramírez[2], and Peter J. Schreier[1]

[1]Paderborn University, Germany, [2]University Carlos III of Madrid, Spain

Introduction

- Spectrum sensing scenario:



  - Observations $\mathbf{x}[n]$ from L antennas
  - Test whether or not a primary user is transmitting
  - Digital communication signals are cyclostationary (CS)

$$\mathrm{E}\left[\mathbf{x}[n]\mathbf{x}^H[n-k]\right] = \mathbf{M}_1[n,k] = \mathbf{M}_1[n+P,k]$$

  - Noise is modeled as wide-sense stationary (WSS)

$$\mathrm{E}\left[\mathbf{x}[n]\mathbf{x}^H[n-k]\right] = \mathbf{M}_0[k]$$

- Existing test: CS vs. WSS [1]
- Contribution: Tests for more specific noise models:
  - I temporally colored and spatially uncorrelated
  - II temporally white and spatially correlated
  - III temporally white and spatially uncorrelated

Problem formulation

- Cycle period P is integer-valued and known
- NP samples of $\mathbf{x}[n]$ are collected in

$$\mathbf{y} = \left[\mathbf{x}^T[0]\dots\mathbf{x}^T[NP-1]\right]^T$$

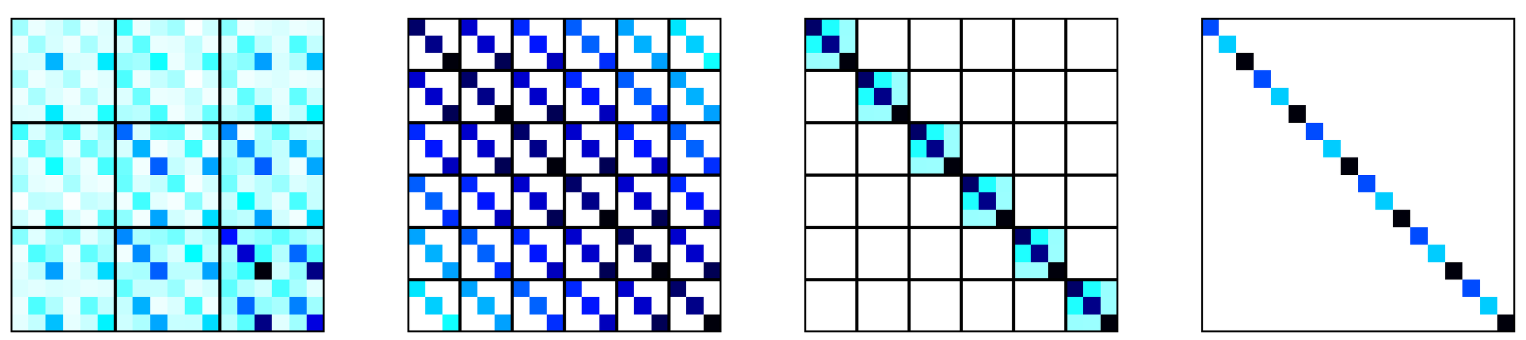
- Assuming $\mathbf{x}[n] \sim \mathcal{CN}$, the hypotheses are

$$\mathcal{H}_1 : \mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_1)$$
$$\mathcal{H}_0 : \mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_0)$$

- Relation $\mathbf{M}[n,k] \leftrightarrow \mathbf{R}$

$$\mathbf{R} = \begin{bmatrix} \mathbf{M}[0,0] & \dots & \mathbf{M}[0,-NP+1] \\ \dots & \ddots & \dots \\ \mathbf{M}[NP-1,NP-1] & \dots & \mathbf{M}[NP-1,0] \end{bmatrix}.$$

- Test about the structure of the covariance matrix



$\mathbf{R}_1$ | $\mathbf{R}_0$ (case I) | $\mathbf{R}_0$ (case II) | $\mathbf{R}_0$ (case III)

block-Toeplitz | block-diagonal with repeating blocks

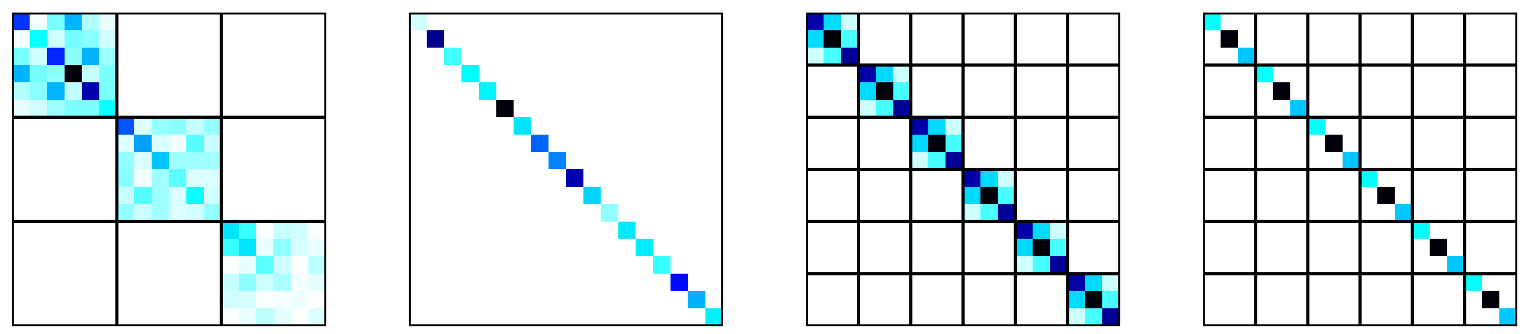
- The exact values of the matrices are unknown
  - Composite hypothesis test: UMPIT, LMPIT, GLRT,...
  - no closed-form ML estimates of block-Toeplitz matrices

Approximation

- Approximation ($N \to \infty$): block-Toeplitz $\simeq$ block-circulant
- Linear transformation $\mathbf{y} \mapsto \mathbf{z}$
- Asymptotically equivalent hypotheses

$$\mathcal{H}_1 : \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_1)$$
$$\mathcal{H}_0 : \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_0)$$

- Covariance matrices are block-diagonal
  - closed-form ML estimates exist

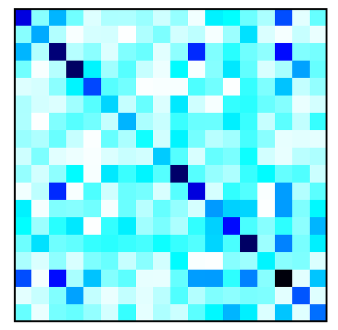


$\mathbf{S}_1$ | $\mathbf{S}_0$ (case I) | $\mathbf{S}_0$ (case II) | $\mathbf{S}_0$ (case III)

repeating blocks

ML estimates

- $M \geq LP$ i.i.d. realizations $\mathbf{z}_i$
- Sample covariance matrix $\hat{\mathbf{S}} = \frac{1}{M}\sum_{i=1}^{M} \mathbf{z}_i\mathbf{z}_i^H$



$\hat{\mathbf{S}}$

  - $\mathrm{diag}_B(\hat{\mathbf{S}})$ returns the diagonal blocks of size $B \times B$
  - the kth diagonal block of $\hat{\mathbf{S}}$ with size $L \times L$ is $\hat{\mathbf{S}}_k$
- Then the estimates are
  - $\hat{\mathbf{S}}_1 = \mathrm{diag}_{LP}(\hat{\mathbf{S}})$
  - case I: $\hat{\mathbf{S}}_0 = \mathrm{diag}_1(\hat{\mathbf{S}})$
  - case II: $\hat{\mathbf{S}}_0 = \mathbf{I}_{NP} \otimes \left[\frac{1}{NP}\sum_{k=1}^{NP} \hat{\mathbf{S}}_k\right]$
  - case III: $\hat{\mathbf{S}}_0 = \mathbf{I}_{NP} \otimes \left[\frac{1}{NP}\sum_{k=1}^{NP} \mathrm{diag}_1(\hat{\mathbf{S}}_k)\right]$
- Sample coherence matrix:

$$\hat{\mathbf{C}} = \hat{\mathbf{S}}_0^{-1/2}\hat{\mathbf{S}}_1\hat{\mathbf{S}}_0^{-1/2}$$

Asymptotic GLRTs

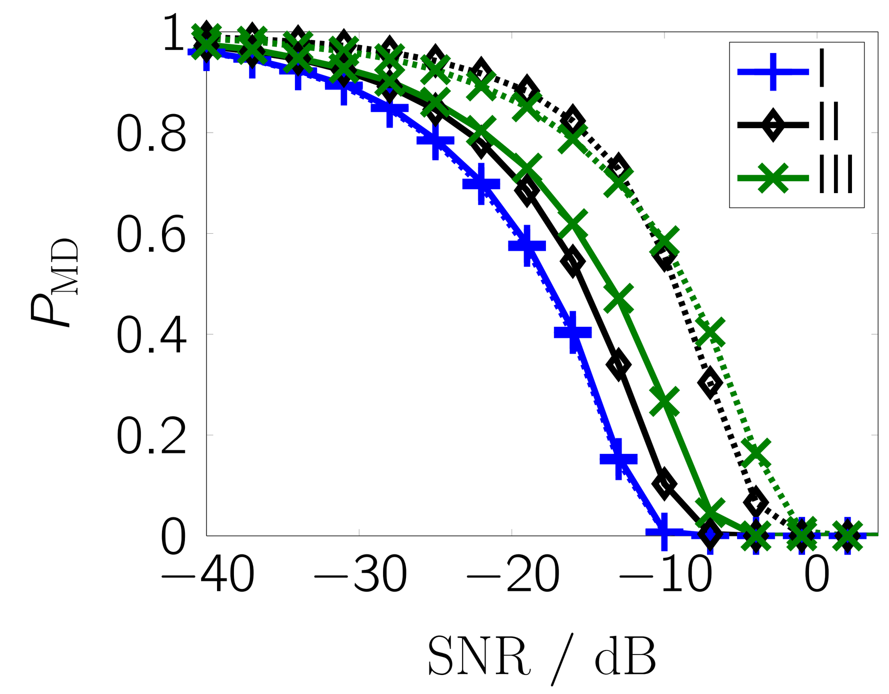
$$\det(\hat{\mathbf{C}}) \underset{\mathcal{H}_1}{\overset{\mathcal{H}_0}{\gtrless}} \eta$$

Numerical results

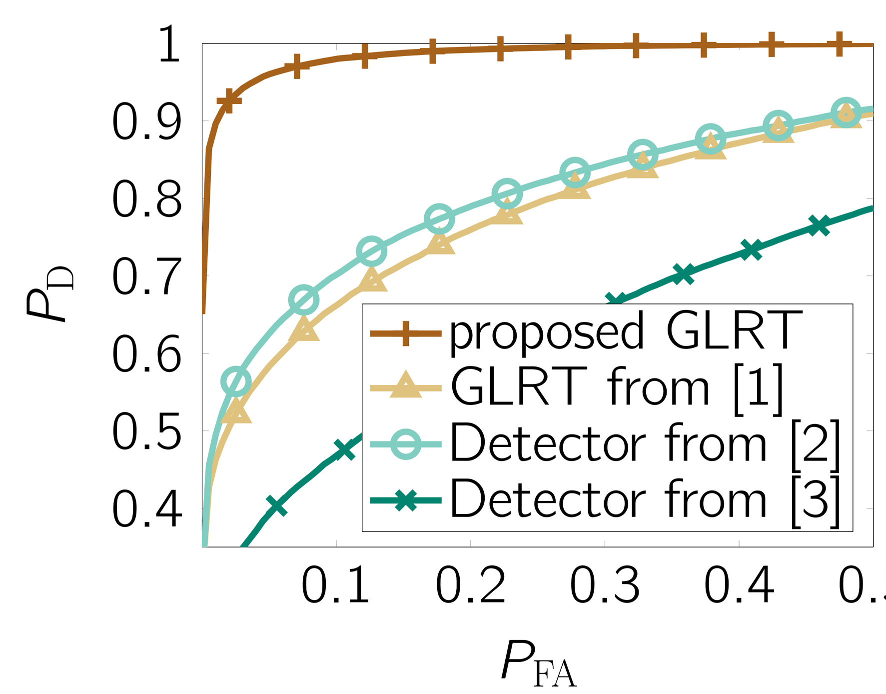
- Observations are simulated as

$$\mathcal{H}_1 : \mathbf{x}[n] = (\mathbf{H} * \mathbf{s})[n] + \mathbf{w}[n]$$
$$\mathcal{H}_0 : \mathbf{x}[n] = \mathbf{w}[n],$$

- QPSK signal $\mathbf{s}[n]$ with symbol length $P = 4$
- $N = 16$, $M = 20$
- Rayleigh fading channel $\mathbf{H}[n]$ with exponential power delay profile
- Noise $\mathbf{w}[n]$
  - if temporally colored: white noise filtered through moving average filter
- Probability of missed detection (P_{MD}) at a false alarm rate of 10^{-3}
  - solid lines: proposed GLRT
  - dashed lines: GLRT from [1]



- ROC curve for case III at an SNR of $-8\,\mathrm{dB}$



Bibliography

[1] D. Ramírez, P. J. Schreier, J. Vía, I. Santamaría, and L. L. Scharf, "Detection of Multivariate Cyclostationarity," in *TSP 2015*

[2] J. Lundén, V. Koivunen, A. Huttunen, and H. V. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," in *TSP 2009*.

[3] P. Urriza, E. Rebeiz, and D. Cabric, "Multiple antenna cyclostationary spectrum sensing based on the cyclic correlation significance test," *J-SAC 2013*