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Localized random sampling for robust compressive beam alignment

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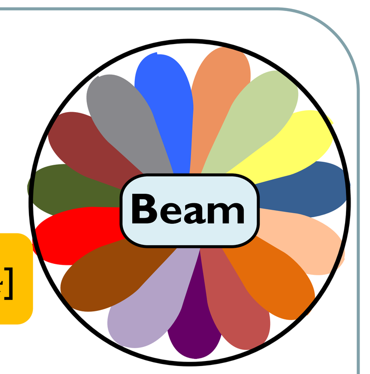
Motivation

Exhaustive beam search can result in a lot of training overhead [1]

Standard compressed sensing is sensitive to carrier frequency offset [2-4]

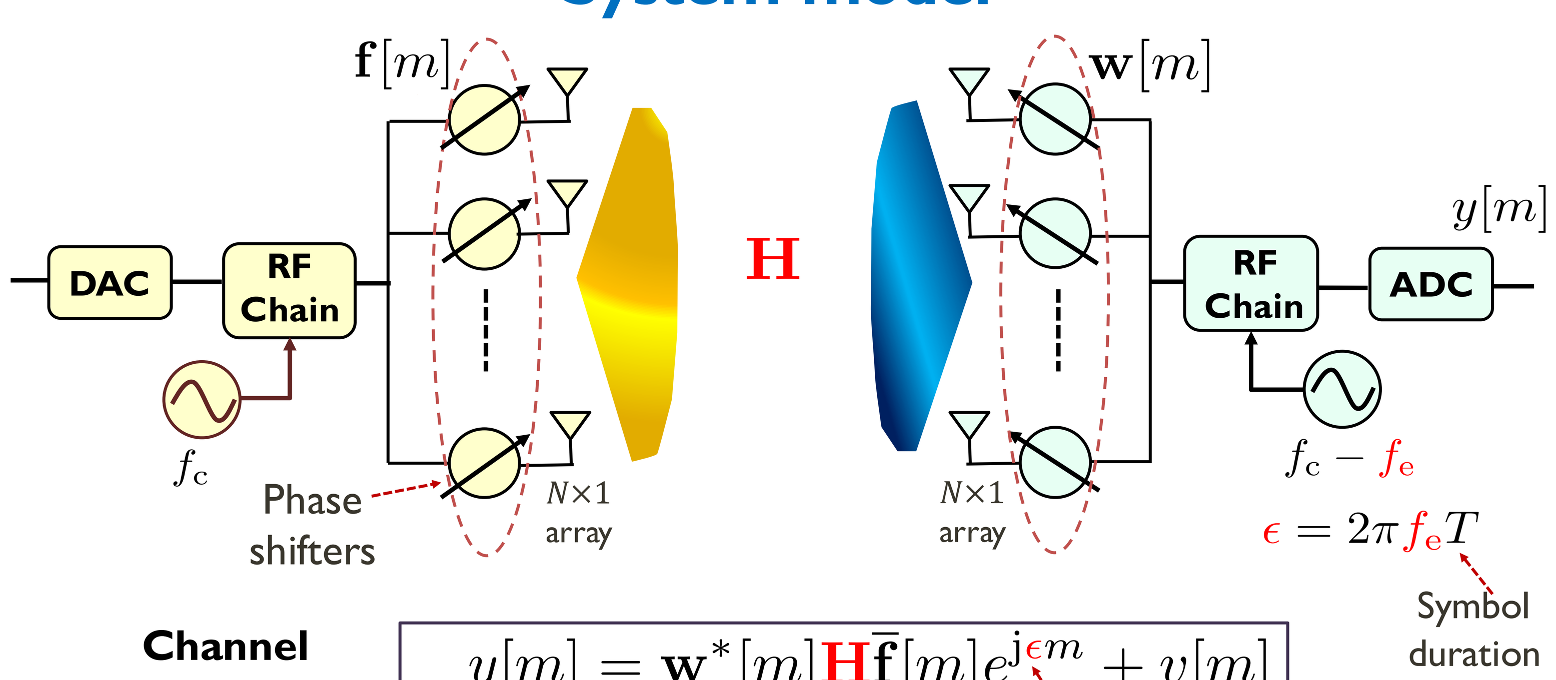


Carrier frequency offset (CFO)

CFO compensation can be hard due to low SNR prior to beamforming [5]

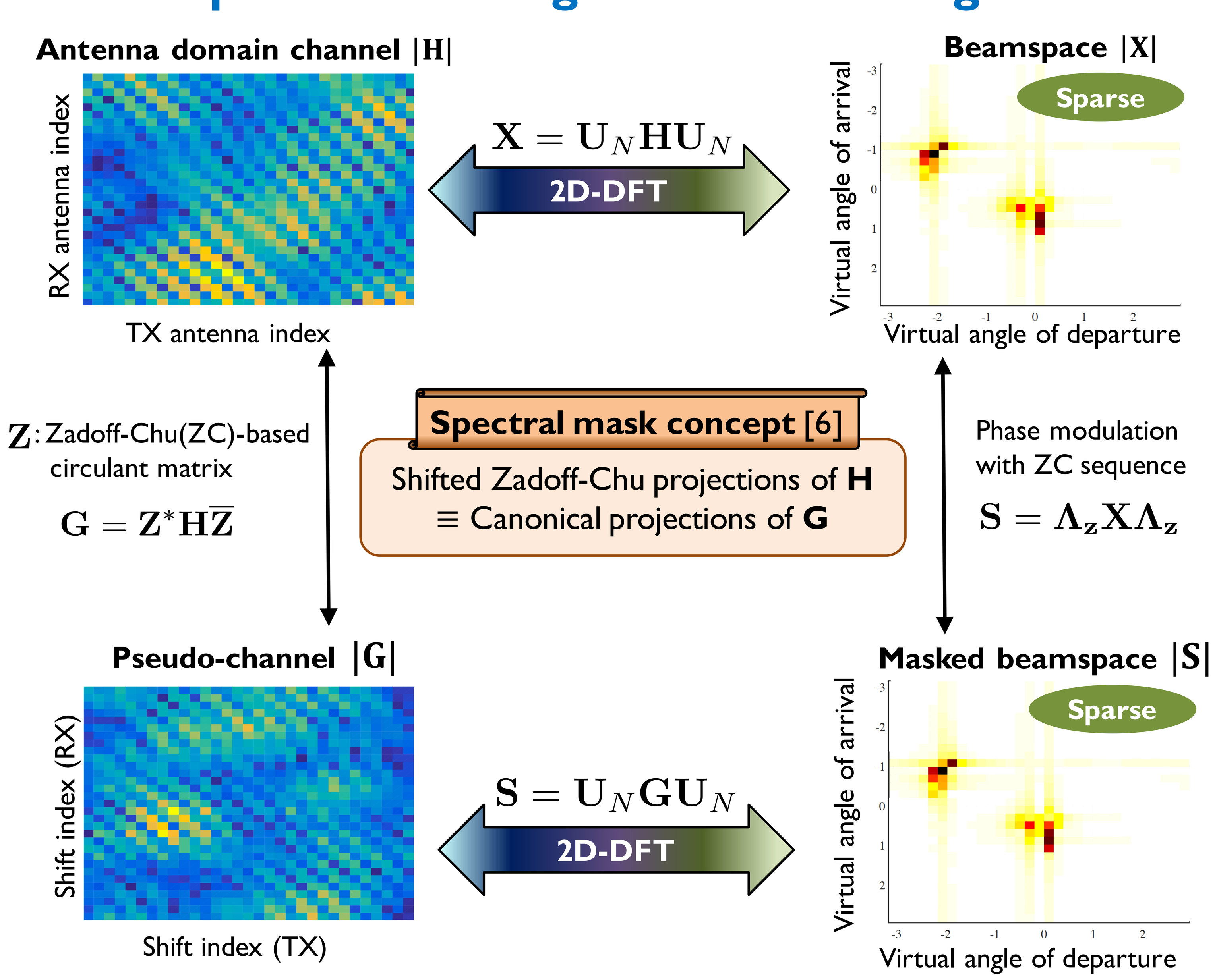
Objective: Design a compressed sensing (CS) strategy that is robust to CFO

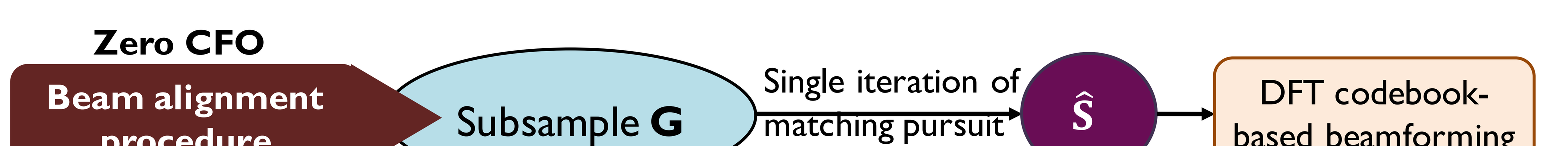
System model



Channel measurement $y[m] = \mathbf{w}^*[m]\mathbf{H}\bar{\mathbf{f}}[m]e^{\mathrm{j}\epsilon m} + v[m]$ (CFO, AWGN)

Compressed sensing-based beam alignment





Phase perturbed CS model

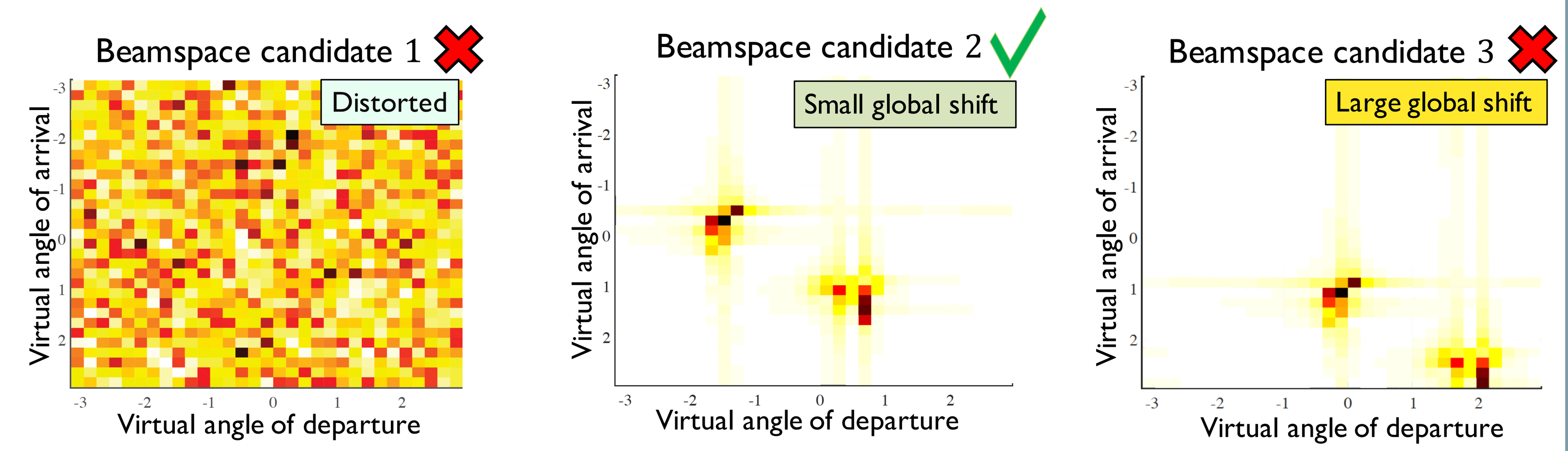
$$\mathbf{y} = \begin{bmatrix} 1 & & & \\ & e^{\mathrm{j}\epsilon} & & \\ & & e^{\mathrm{j}2\epsilon} & \\ & & & \ddots \\ & & & & e^{\mathrm{j}(M-1)\epsilon} \end{bmatrix} \mathbf{A}\mathbf{s} + \mathbf{v}$$

- $\mathbf{y}$: Vector of M channel measurements
- $\mathbf{A}$: CS matrix, Determined by the sampling locations in $\mathbf{G}$
- $\mathbf{s}$: Sparse vector vec(**S**)
- Regime of interest: $M \ll N^2$

Goal: Identify a class of CS matrices that achieve robustness to CFO

Robust CS in the context of beam alignment

Assumption: Maximum carrier frequency offset = Δ rad. Define $u = N\Delta/2\pi$.

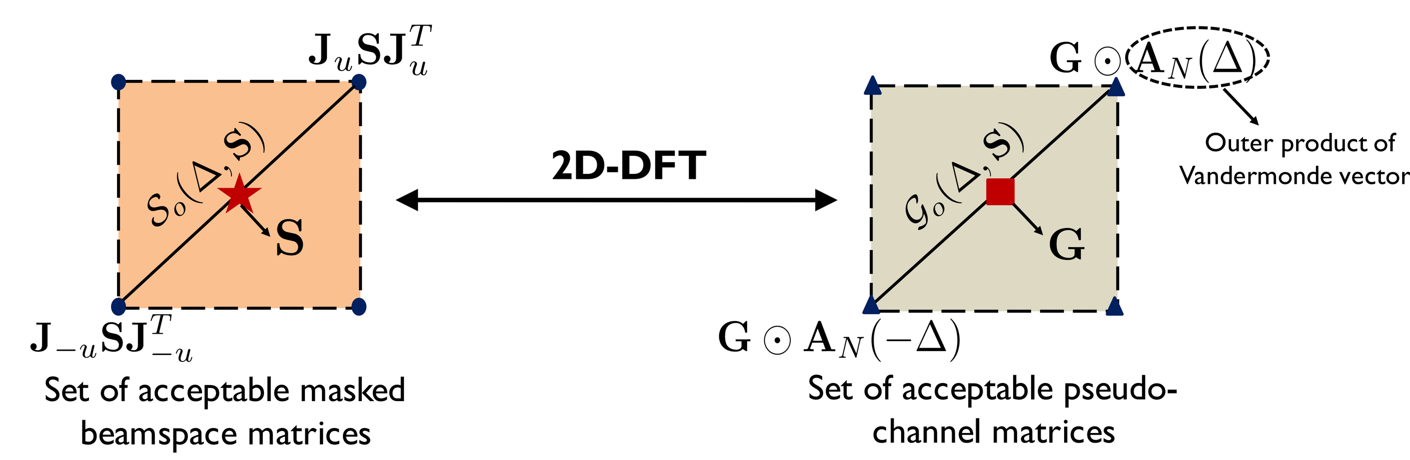


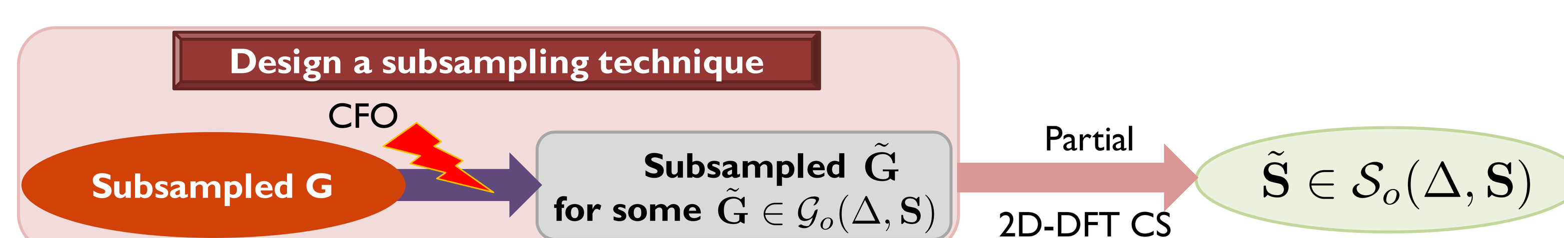
Acceptable set: Set of matrices around **S** that differ from **S** by a maximum shift of (u, u)

Definition of acceptable set

$$\mathcal{S}_o(\Delta, \mathbf{S}) = \{\mathbf{R} : \mathbf{R} = \mathbf{J}_t\mathbf{S}\mathbf{J}_t^T, |t| \leq u\}$$
$$\mathcal{G}_o(\Delta, \mathbf{S}) = \{\mathbf{P} : \mathbf{P} = \mathbf{U}_N^*\mathbf{R}\mathbf{U}_N^* \text{ for } \mathbf{R} \in \mathcal{S}_o(\Delta, \mathbf{S})\}$$

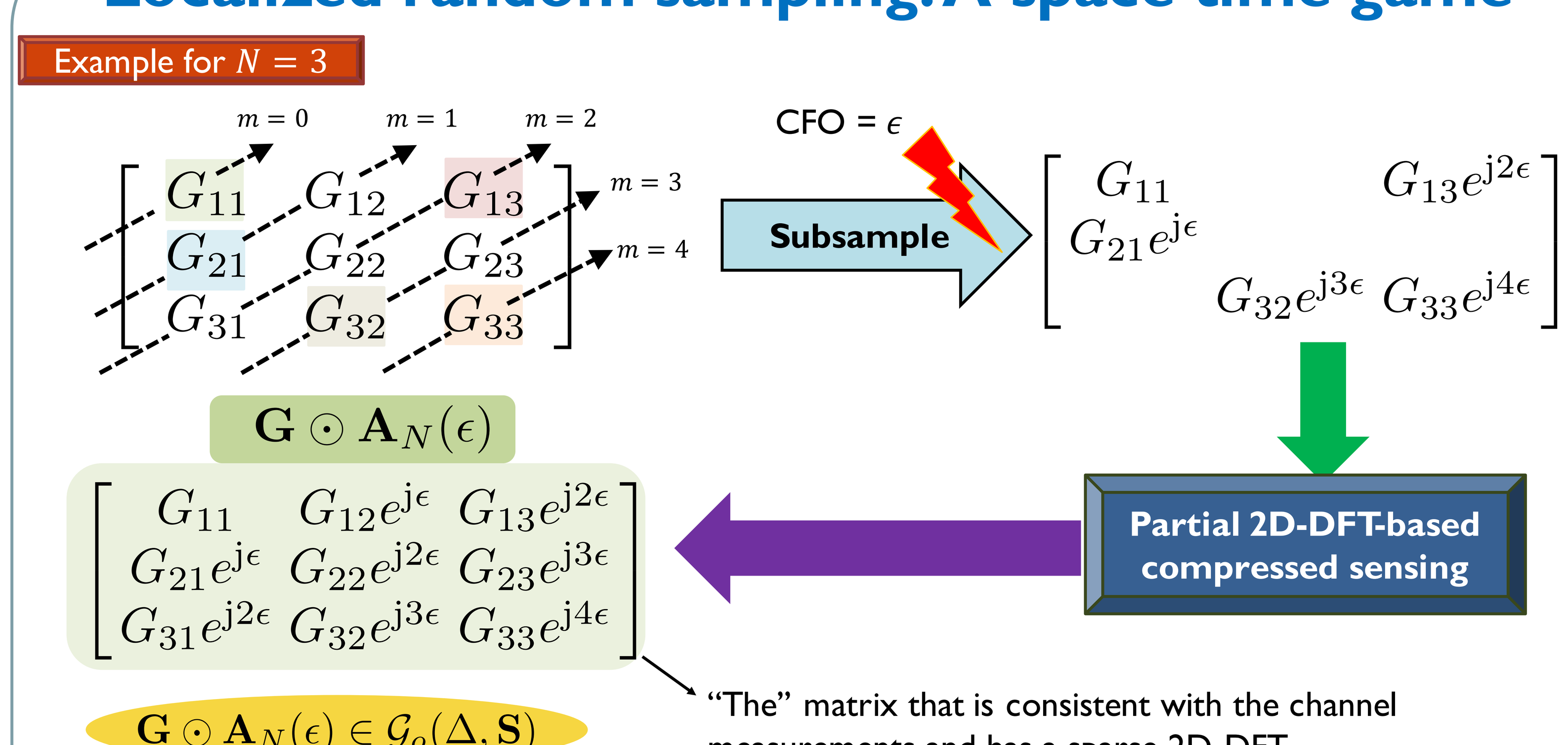
($\mathbf{J}_t$: t circulant delay matrix)





Generic random subsampling of G may result in an estimate outside $\mathcal{S}_o(\Delta, \mathbf{S})$

Localized random sampling: A space time game



Restricting randomness in CS can help achieve robustness to structured errors

Proposed solution for robust beam alignment

Channel measurements with the proposed training

Define $\mathcal{I}_N = \{0, 1, 2, \cdots, N-1\}$

for $m = 0$ to $M-1$ **do**

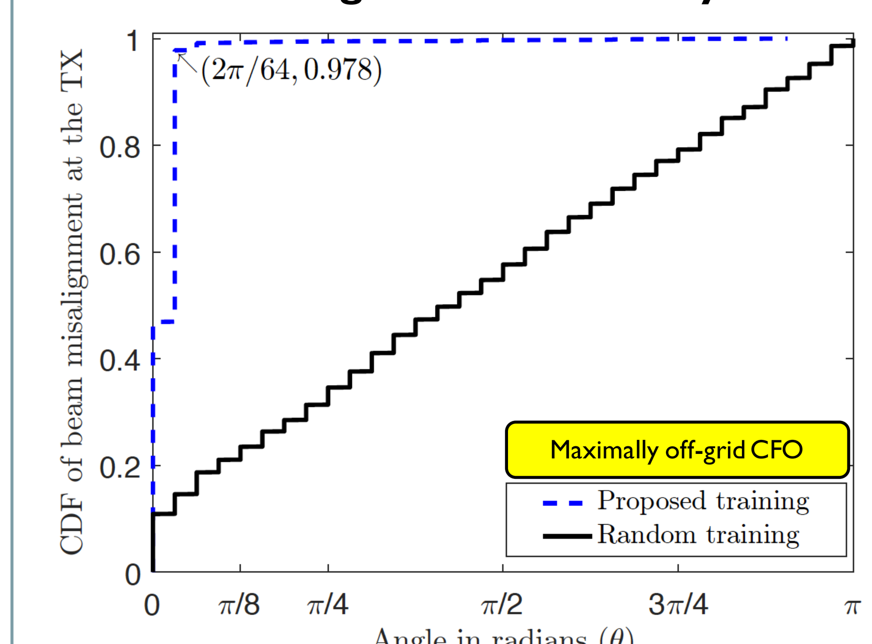
1. Sample a coordinate at random from $\{(a, b) : a + b = m, a \in \mathcal{I}_N, b \in \mathcal{I}_N\}$
2. Define the sampled coordinate as $(\ell_{\mathrm{rx}}[m], \ell_{\mathrm{tx}}[m])$
3. Set $\mathbf{f}[m] = \mathbf{Z}\mathbf{e}_{\ell_{\mathrm{tx}}[m]}$ and $\mathbf{w}[m] = \mathbf{Z}\mathbf{e}_{\ell_{\mathrm{rx}}[m]}$

end for

Channel measurements with the proposed training → (Single iteration of matching pursuit) → Estimate beams → (CAN algorithm [7]) → Beam alignment using broadened beams

Simulation results

Beam misalignment induced by CFO



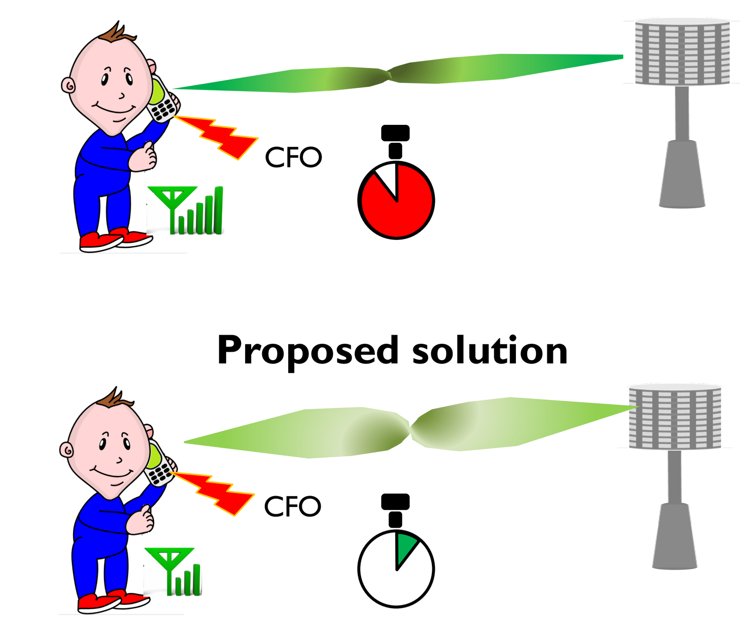
Simulation settings

- $N = 64$ uniform linear array of antennas at the TX and the RX
- 3-bit phase shifters in the transceiver
- Root of the ZC sequence = 11
- 38 GHz carrier, 100 MHz bandwidth
- Narrowband channel model
- NYUSIM channel, UMi-LoS setting
- SNR of channel measurements = 0 dB
- Max. CFO = 40 ppm, $\Delta = 0.096$ rad
- Off-grid CFO = 20.5579 ppm
- $M = 126$, Subsampling rate of ~3%

Performance with CFO

Exhaustive beam search

Proposed solution



Performance guarantees

Result [9]: On an average, the proposed CS matrix satisfies the restricted isometry property (RIP) of order $K > 1$ with parameter δ_K if

$$M \geq 2\frac{K-1}{\delta_K \sin\frac{\pi d_{\min}}{N}}\left[4 + 2\log\left(\frac{N}{N+1-(M-2)/4}\right)\right].$$

In the worst case, our design requires $O(\sqrt{N^2})$ times the samples required by standard CS for robustness to CFO

Code Implementation available at https://github.com/nitinjmyers/ICASSP2019RobustCS

References

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