Practical Concentric Open Sphere Cardioid Microphone Array Design for Higher Order Sound Field Capture

M. R. P. Thomas

Dolby Laboratories, San Francisco, USA

Motivation

- Many spherical array designs employ omnidirectional microphones mounted on the surface of a rigid sphere.
  - Plane wave density estimated at origin by radial mode strength inversion.
  - Rigid scatterer necessary to avoid spectral nulls in radial filters.
- Design tradeoff
  - Small radius increases spatial aliasing frequency.
  - Large radius improves white noise gain at low frequencies
- Cardioid microphones beneficial [1]
  - Improved white noise gain in 1st order.
  - Open-sphere designs allow embedding at multiple radii.
- Aims
  - Specify design criteria for practical open-sphere designs.
  - Investigate white noise gain performance.

Theory

- Microphone signal at $\Omega = (\theta, \phi)$ in field $\breve{S}_l^m(\omega)$ at radius r

$$P(r,\Omega,\omega) = \sum_{l=0}^{N} \sum_{m=-l}^{l} b_l\left(\frac{\omega}{c}r\right) \breve{S}_l^m(\omega) Y_l^m(\Omega)$$

with degree l, order m and mode strength $b_l\left(\frac{\omega}{c}r\right)$ (**Fig. 1**).

$$b_l\left(\frac{\omega}{c}r\right) = \begin{cases} j_l\left(\frac{\omega}{c}r\right) & \text{Open omni} \\ j_l\left(\frac{\omega}{c}r\right) - \dfrac{j_l'\left(\frac{\omega}{c}a\right)}{h_l'\left(\frac{\omega}{c}a\right)} h_l\left(\frac{\omega}{c}r\right) & \text{Rigid omni} \\ j_l\left(\frac{\omega}{c}r\right) - i j_l'\left(\frac{\omega}{c}r\right) & \text{Radial Cardioid} \end{cases}$$

- In matrix form: $\mathbf{P}(\omega) = \mathbf{\Psi}(\omega)\,\breve{\mathbf{S}}(\omega) \implies \breve{\mathbf{S}}(\omega) = \boldsymbol{\zeta}(\omega)\mathbf{P}(\omega)$
  - Formulation permits multiple radii, constrained to limit WNG in practice.

Proposed Design

Criteria

1. Near-uniformity to aid conditioning.
2. Few unique edge lengths/vertex types to simplify manufacture.
3. Multiples of 8 mics for common microphone amplifiers.
4. $> (N+1)^2$ microphones, preferably $= (N+1)^2$.
5. Ease of mounting to a microphone stand.
6. Acoustic transparency in the audible frequency range.
7. Unoccluded backport for pressure gradient operation.
8. Easy suspension of smaller spheres from larger outer spheres.

Distributions

- Place microphones on polyhedral vertices $\mathbf{p}$ that minimize [2].

$$J = \sum_{i=1}^{P} \sum_{j=i+1}^{P} \frac{1}{\left\|\mathbf{p}_i - \mathbf{p}_j\right\|_2}$$

- Uniform distributions ($P = 4, 6, 12$) yield good metrics.
- $P = 25$ (critical sampling, $N = 4$) poorly conditioned & hard to construct.
- $P = 32$ good except for criterion 4.
- $P = 16$ (critical sampling, $N = 3$), well conditioned, triangle at base.

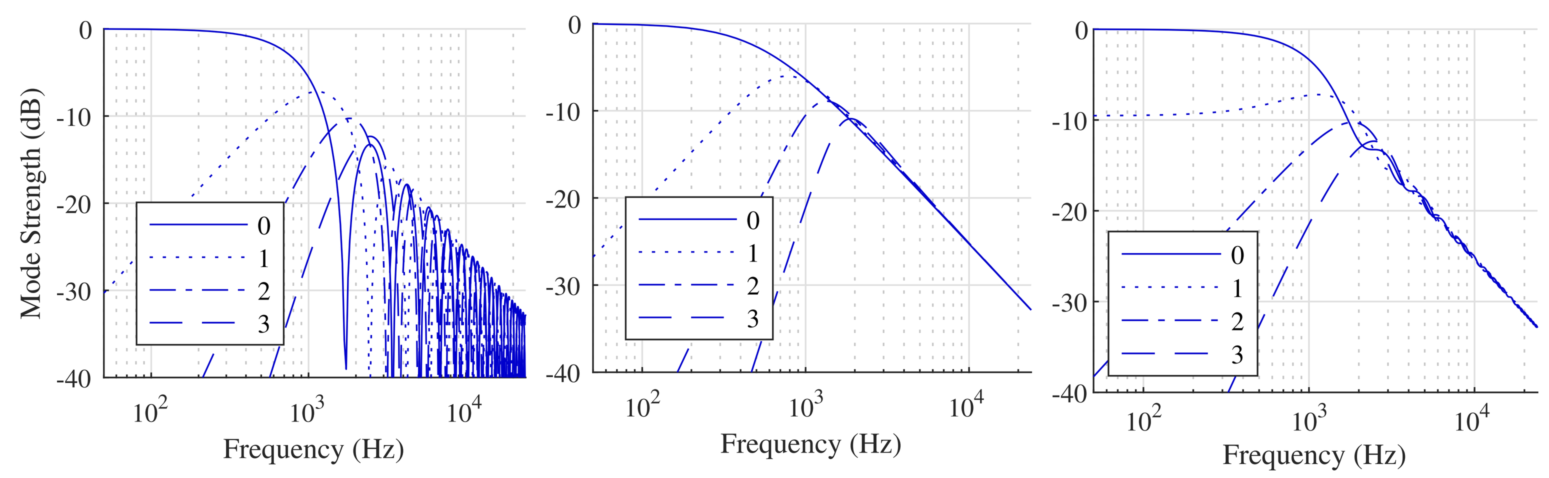

Fig. 1. Mode strengths for r=100 mm, orders 0 to 3.

Evaluation & Conclusions

- Example dual sphere, radii 42 mm and 420 mm, 16 mics per sphere.

Fig. 2. Full assembly

- 16 vertices (12 x v_5, 4 x v_6), 28 faces, 42 edges. v_6 vertices form a tetrahedron.
- White noise gain $1/\mathrm{diag}\{\boldsymbol{\zeta}(\omega)\boldsymbol{\zeta}^H(\omega)\}$

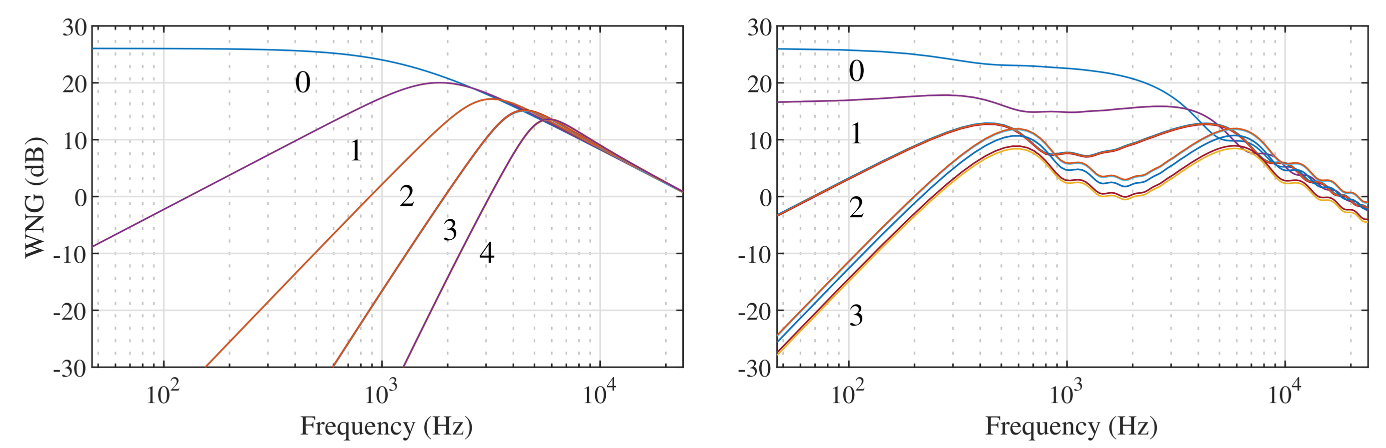

Fig. 3. White noise gains.

- 32-microphone 42 mm rigid omni sphere and 2x16 42 + 420 mm dual cardioid sphere break unity around 2 kHz and 200 Hz respectively.
- Improved 1st order WNG due to cardioid pressure gradient response.
- Dual sphere exhibits dips where inner & outer sphere cross over.

[1] I. Balmages and B. Rafaely, "Open-sphere design for spherical mirophone arrays," *IEEE/ACM Trans. on Audio, Speech and Lang. Process,* vol. 15, no. 2, pp. 727–732, Feb. 2007.
[2] M. R. P. Thomas, "Fast computation of cubature formulae for the sphere," in *Proc. Joint Workshop on Hands-Free Speech Communication and Microphone Arrays (HSCMA)*, Mar. 2017.