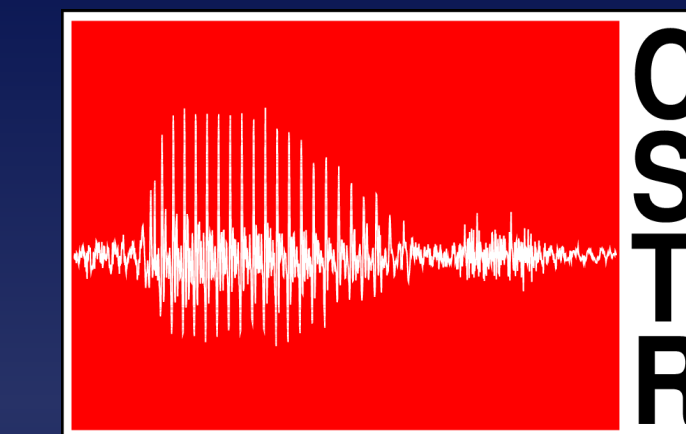




# ON THE USEFULNESS OF STATISTICAL NORMALISATION OF BOTTLENECK FEATURES FOR SPEECH RECOGNITION



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## Abstract

**GOAL:** An attempt to understand the DNNs from a statistical perspective

**HOW:** Statistical properties of bottleneck (BN) layer pre-activations (Z) and activations (Y) are studied

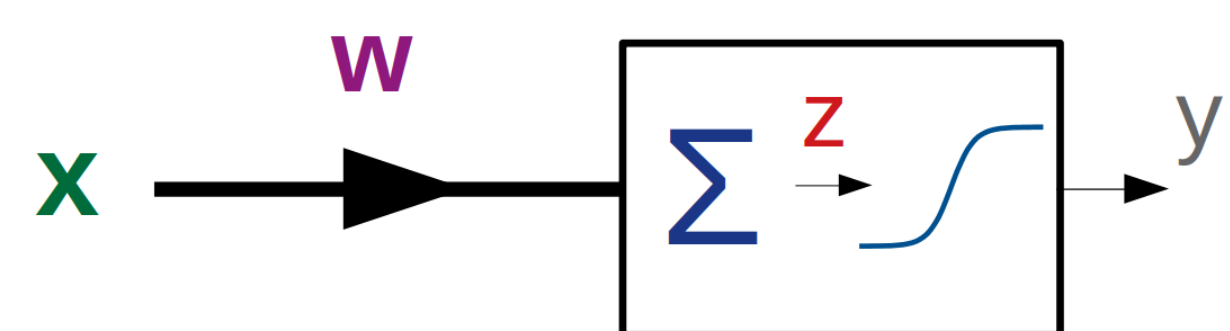
### CONTRIBUTIONS:

1. Distribution of the NN activation in the BN layer was analytically derived
2. Statistical properties of the BN features were empirically studied and compared with analytic pdf
3. Sparsity of ReLU was (re-)explained
4. Post-processing of the BN features through statistical normalisation for ASR were investigated

**EXPERIMENTS:** Aurora-4, train by clean/additive

**RESULTS:** Up to 2% absolute (9% relative) performance gain (WER reduction) was achieved in mismatch condition

## STATISTICAL DISTRIBUTION OF BOTTLENECK FEATURES



$$y = f(\mathbf{w}^T \mathbf{x}) = f(\mathbf{z}) \Rightarrow \mathbf{z} = \mathbf{f}^{-1}(\mathbf{y})$$

$$P_Y(y) = \left| \frac{d}{dy} f^{-1}(y) \right| P_Z(f^{-1}(y))$$

$$P_Y^{\tanh}(y) = \frac{1}{1-y^2} P_Z\left(\frac{1}{2} \log \frac{1+y}{1-y}\right)$$

## Assumptions for Approximating $P_Z(z)$

1. Central Limit Theorem (CLT)

$$z \sim \mathcal{N}(z; \mu_z, \sigma_z^2)$$

2.  $\text{Prob}(z > 0) \approx \text{Prob}(z < 0)$

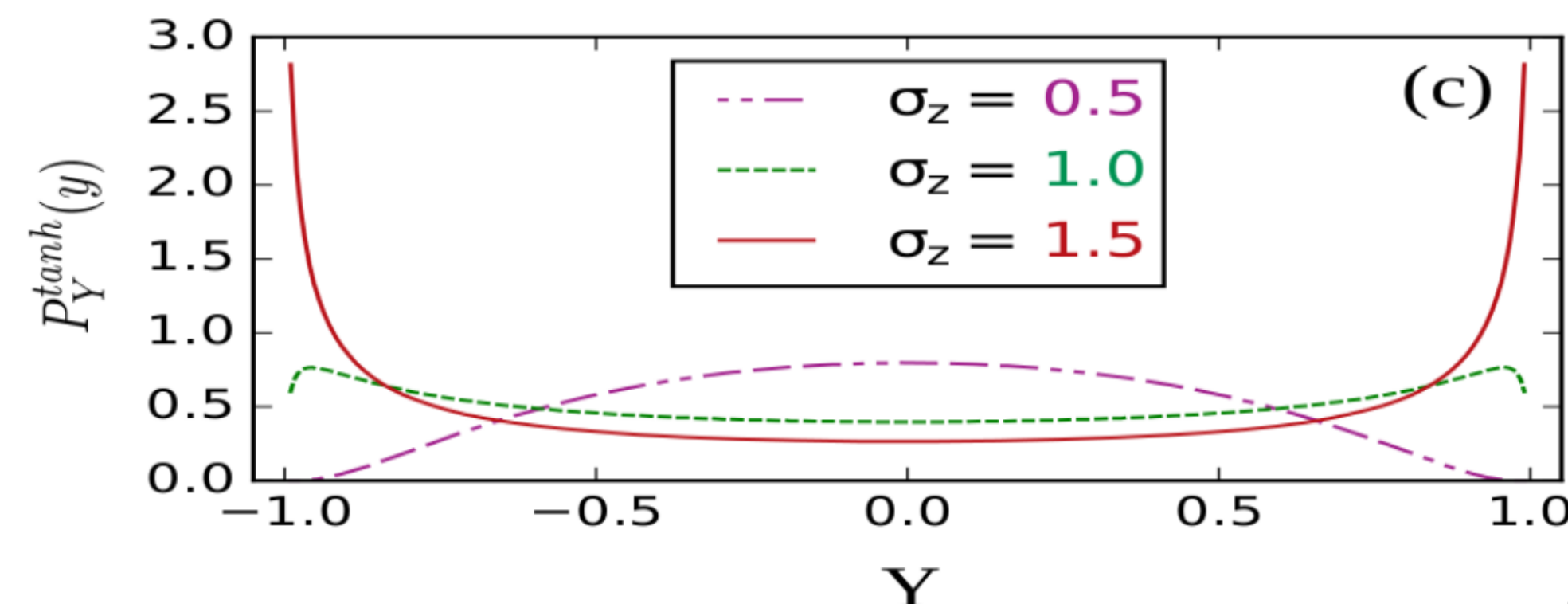
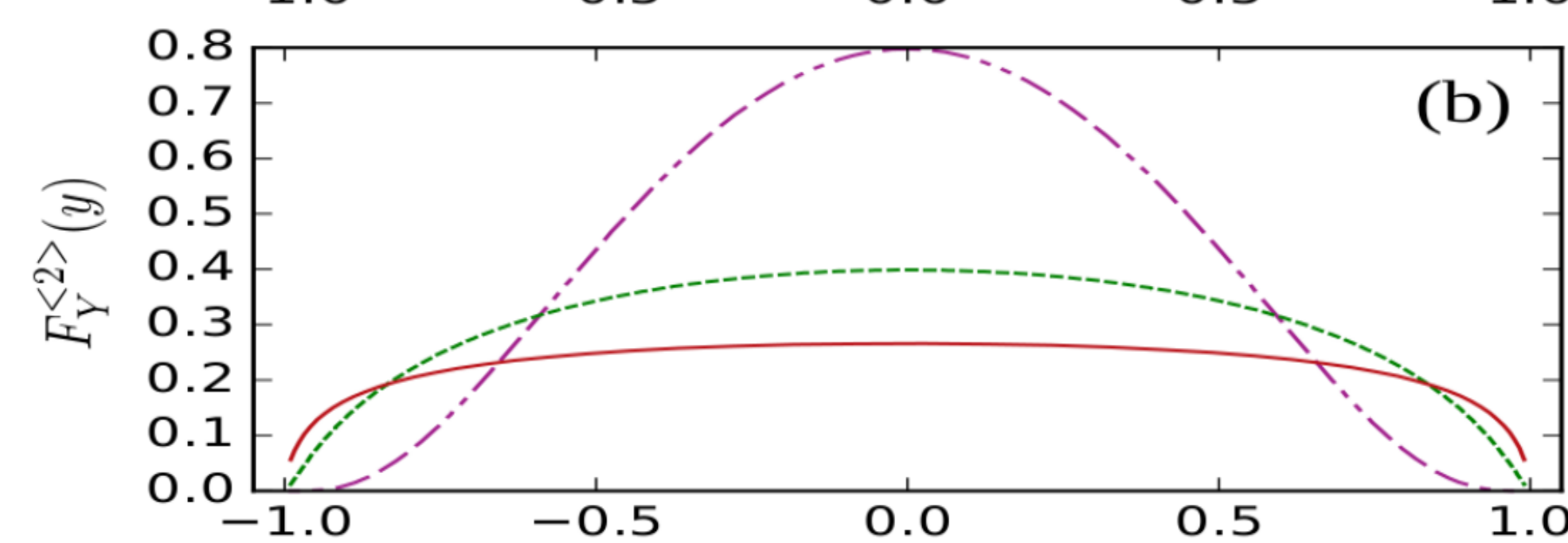
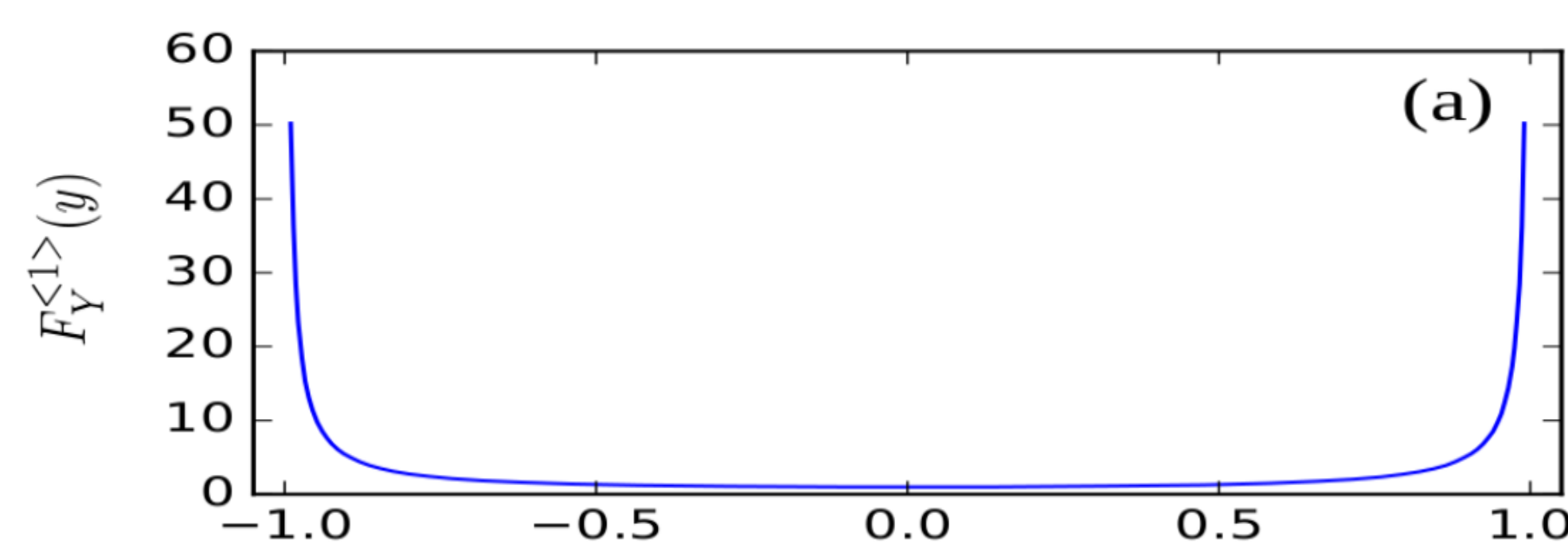
$$\mu_z \rightarrow 0 \quad z \sim \mathcal{N}(z; 0, \sigma_z^2)$$

## Density Estimating for Nodes with tanh Activation

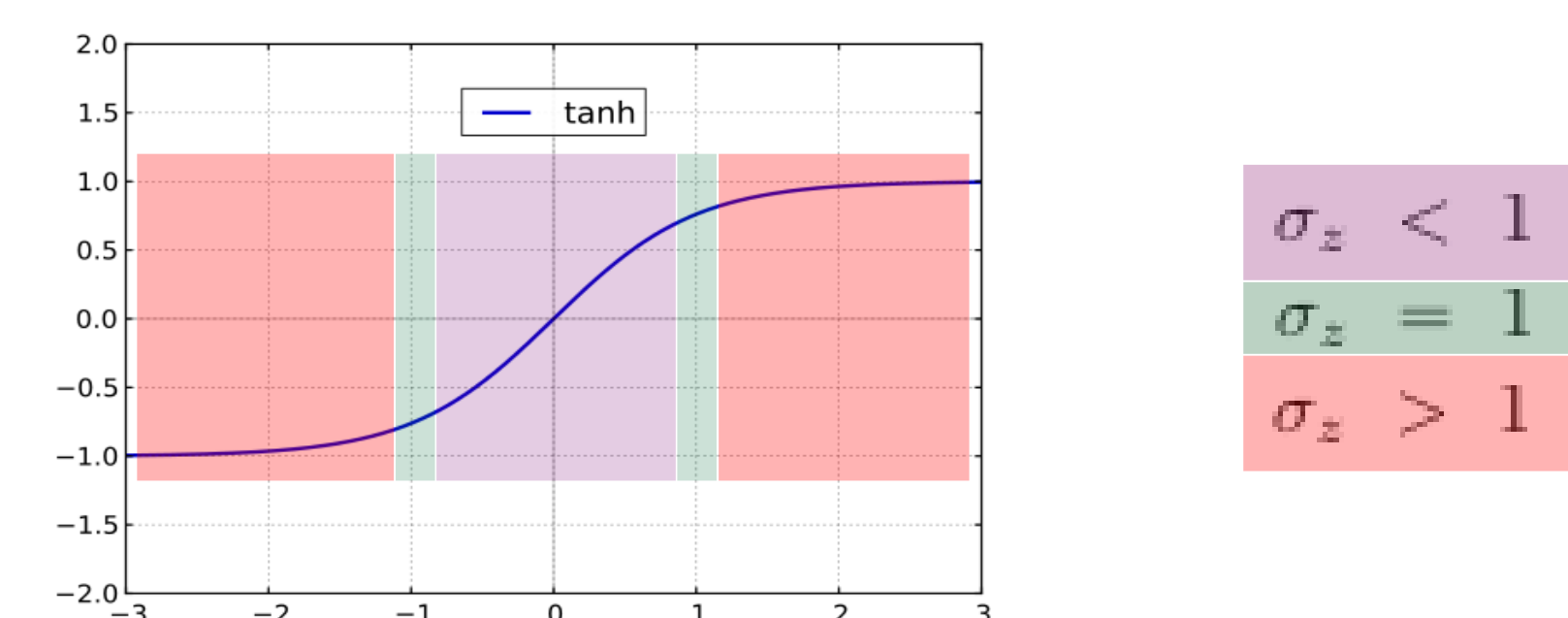
$$P_Y^{\tanh}(y) = \frac{1}{1-y^2} \mathcal{N}\left(\frac{1}{2} \log \frac{1+y}{1-y}; 0, \sigma_z^2\right)$$

$$= \underbrace{\frac{1}{1-y^2}}_{F_Y^{<1>}(y)} \underbrace{\frac{1}{\sqrt{2\pi\sigma_z}} \left(\frac{1+y}{1-y}\right)^{-\frac{1}{8\sigma_z^2}} \log \frac{1+y}{1-y}}_{F_Y^{<2>}(y, \sigma_z)}$$

← Factor 1      ← Factor 2 → a function of  $\sigma_z$



## Non-linearity of NNs and Density Shape Parameter

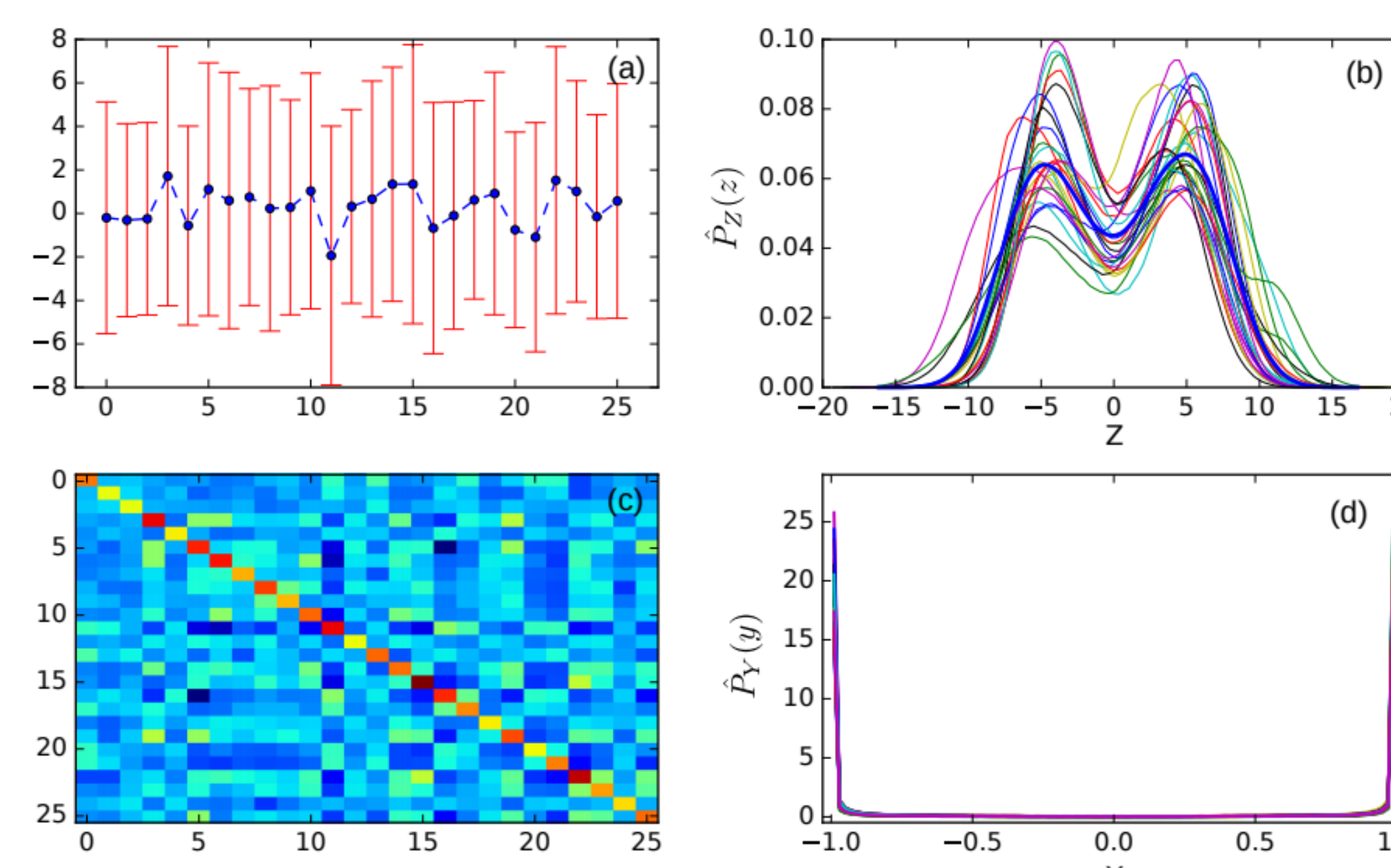


$\sigma_z < 1 \rightarrow$  Nodes/NN operates in **linear** mode

$\sigma_z > 1 \rightarrow$  Nodes/NN operates in **non-linear** mode

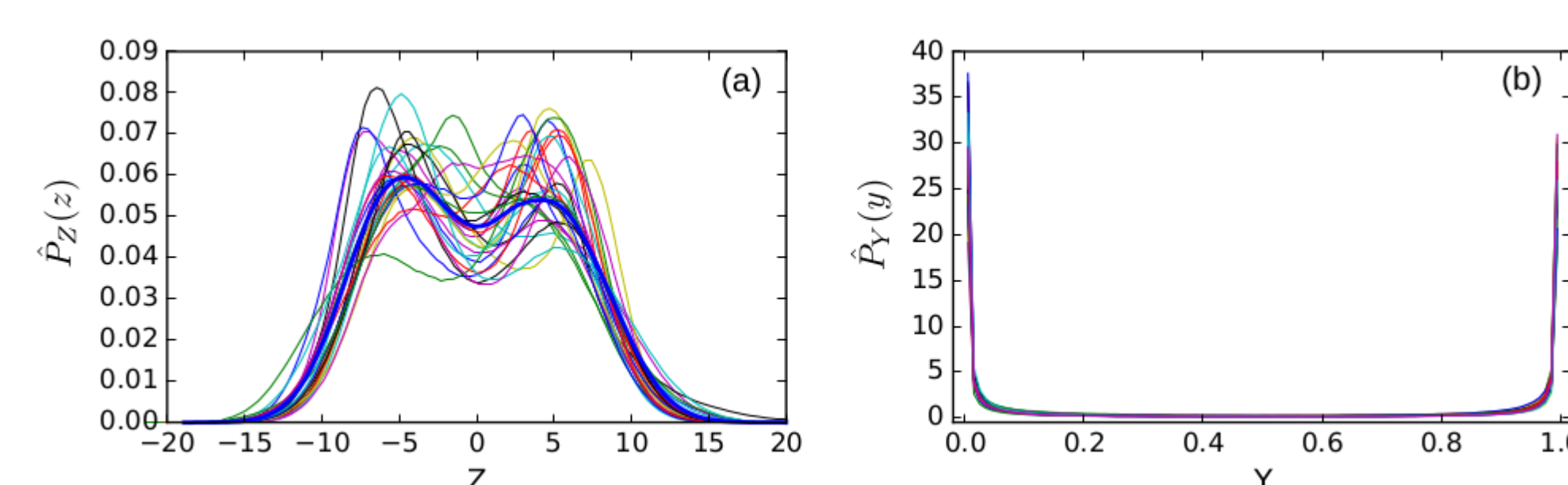
## EMPIRICAL STUDIES

### Density of Tanh

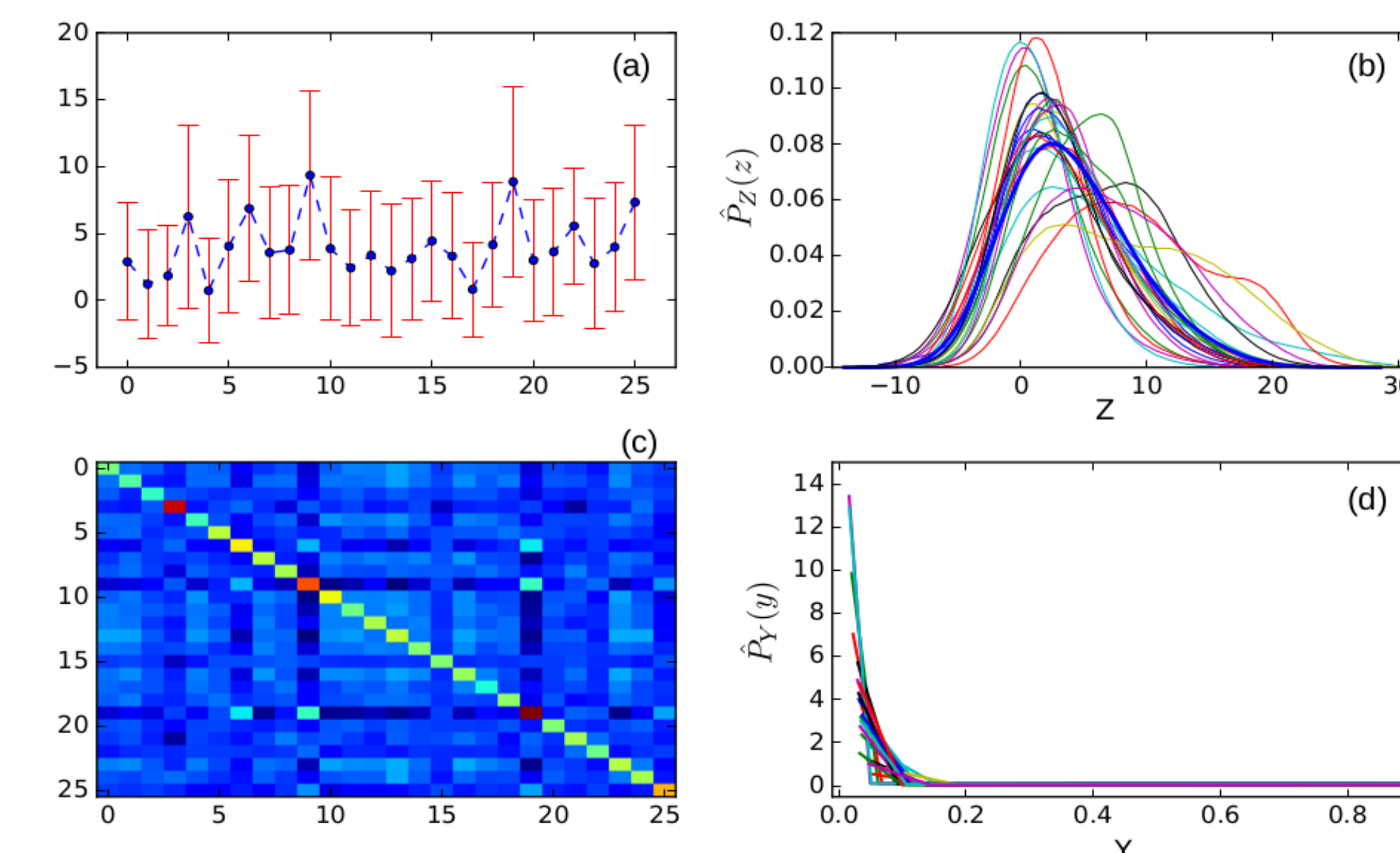


- (1) Zero mean approximation for Z is reasonable
- (2)  $\sigma_z > 1 \implies$  DNN operates in the non-linear mode
- (3) Distribution of Z can be easily fitted by a GMM
- (4) Distribution of Y **may NOT** be fitted by a GMM
- (5) DNN decorrelates the features in the BN layer
- (6) Distribution of Y matches the derived equation

### Density of Sigmoid



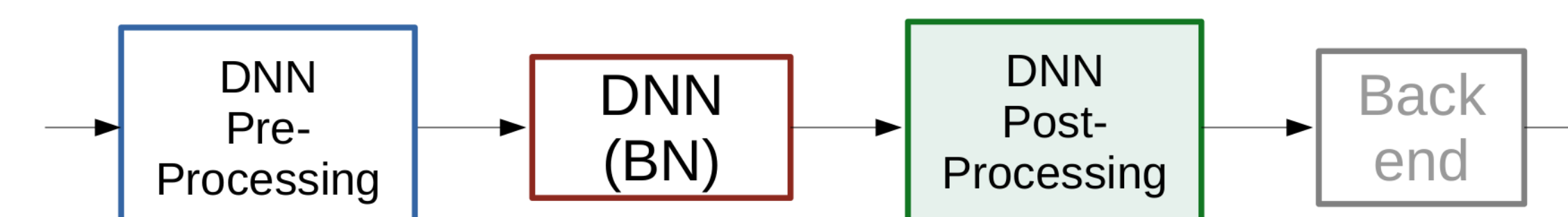
## Sparsity of ReLU



\* Glorot, et al, "Deep Sparse Rectifier Neural Networks", 2011  
 – 50% negative preactivations  $\rightarrow$  50% of activations are 0

\* Our argument: Coincidence of the positive zero ( $0^+$ ) activation with the non-linear operating mode regions  
 – Before zero  $\rightarrow$  Blocked; After zero  $\rightarrow$  Linear

## STATISTICAL NORMALISATION OF THE BOTTLENECK (BN) FEATURES



## Experimental Results

Table 1: WER for Aurora-4 (Kaldi-LDA-MLLT).

Feature	A	B	C	D	Ave4
BN (baseline)	3.87	7.96	21.80	32.72	16.58
BN+MN	3.64	7.66	21.02	32.20	16.13
BN+MVN	4.07	8.31	20.34	33.04	16.44
BN+Gauss	4.15	8.12	20.18	32.67	16.28
BN+HEQ	3.96	7.43	19.76	30.87	15.50
BN+PCA	3.75	7.88	21.56	32.46	16.41
BN+DCT	3.77	7.77	21.76	32.49	16.44