

## Introduction

### Generalized Frequency Division Multiplexing (GFDM)

- Generalization of traditional OFDM
- Low out-of-band (OOB) emissions
- Relaxed time-frequency synchronization requirements
- Potential inter subcarrier interference (ICI) w/ most prototype filters
- ICI-free  $\leftrightarrow$  frequency-domain (FD) decoupling

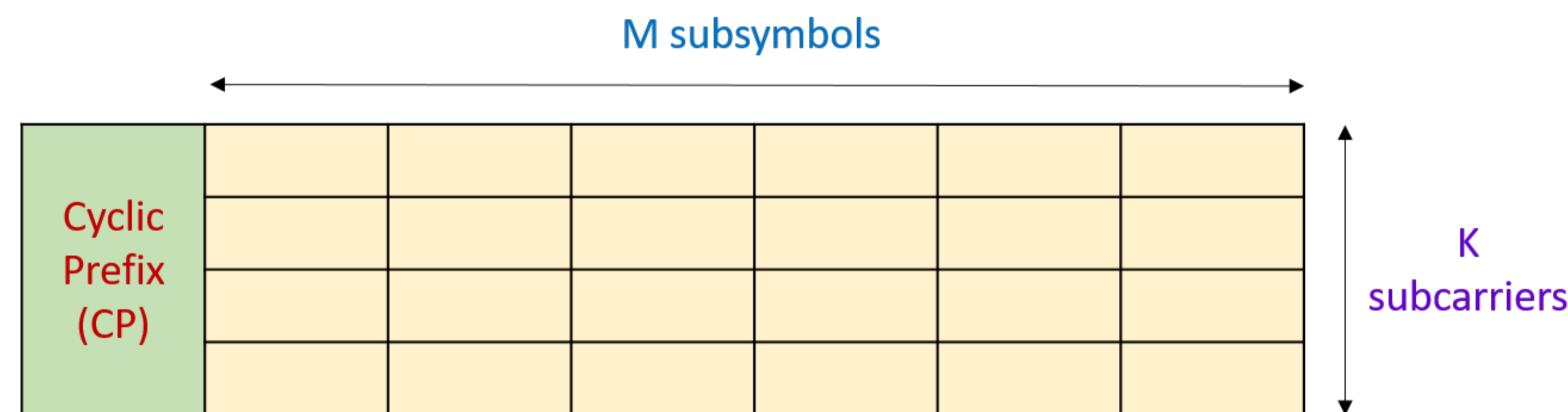


Figure 1: GFDM block

### Contribution

- Propose a class of ICI-free prototype filters for MIMO-GFDM.
- Evaluate the depth-first sphere decoding (DFSD) with this class of prototype filters for MIMO-GFDM spatial multiplexing (SM).

## System Model & Problem Formulation

### MIMO-GFDM SM (w/ $T$ Tx and $R$ Rx antennas)

- GFDM transmitter matrix ( $D = K \times M$ )

$$\mathbf{A} = [\mathbf{g}_{0,0} \dots \mathbf{g}_{K-1,0} \quad \mathbf{g}_{0,1} \dots \mathbf{g}_{K-1,1} \dots \mathbf{g}_{K-1,M-1}]$$

- GFDM prototype filter:  $\mathbf{g} \in \mathbb{C}^D$  (FD prototype filter:  $\mathbf{g}_f = \sqrt{D}\mathbf{W}_D\mathbf{g}$ )
- Pulse-shaping:  $[\mathbf{g}_{k,m}]_n = [\mathbf{g}]_{(n-mK)_D} e^{j2\pi kn/K}$ ,  $n = 0, 1, \dots, D-1$ ,  $m = 0, 1, \dots, M-1$ ,  $k = 0, 1, \dots, K-1$

- The signal at receive antennas

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_R \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{H}_{1,1}\mathbf{A} & \dots & \mathbf{H}_{1,T}\mathbf{A} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{R,1}\mathbf{A} & \dots & \mathbf{H}_{R,T}\mathbf{A} \end{bmatrix}}_{\tilde{\mathbf{H}}} \begin{bmatrix} \mathbf{d}_1 \\ \vdots \\ \mathbf{d}_T \end{bmatrix} + \mathbf{n} \quad (1)$$

- Maximum likelihood (ML) solution to (1):  $\hat{\mathbf{d}} = \arg \min_{\mathbf{d} \in \mathcal{D}} \|\mathbf{y} - \tilde{\mathbf{H}}\mathbf{d}\|^2$  [4]

- \* Exhaustive search is infeasible due to the huge size of  $\mathcal{D}$ .
- \* FD decoupling splits the original problem into  $K$  subproblems.

## Proposed Method

### MIMO-GFDM Detection

#### Theorem (FD Decoupling)

Let  $\mathbf{A}$  be a GFDM matrix derived from its FD prototype filter  $\mathbf{g}_f$  and assume that  $\mathbf{g}_f$  contains at most  $M$  consecutive nonzero entries (i.e., there exist  $\mathbf{g}_1 \in \mathbb{C}^M$  and an integer  $l$ ,  $0 < l \leq D$ ) such that

$$\mathbf{g}_f = \mathbf{\Pi}_D^l [\mathbf{g}_1^T \mathbf{0}_{(K-1)M}^T]^T.$$

Consequently, the matrix  $\tilde{\mathbf{H}}$  as defined in (1) can be decomposed into the form

$$\tilde{\mathbf{H}} = \mathbf{U}^H \text{blkdiag}(\{\mathbf{F}_k\}_{k=0}^{K-1}) \mathbf{P},$$

where  $\mathbf{U} = (\mathbf{\Pi}_{KR} \otimes \mathbf{I}_M)(\mathbf{I}_R \otimes \mathbf{\Pi}_D^{-l}\mathbf{W}_D)$ ,  $\mathbf{P} = (\mathbf{\Pi}_{KT} \otimes \mathbf{I}_M)(\mathbf{I}_T \otimes \mathbf{\Pi}_{KM})$ , and  $\mathbf{F}_k$ ,  $k = 0, \dots, K-1$ , are some  $MR \times MT$  matrices.

- Notations:  $\mathbf{\Pi}_A = \begin{bmatrix} \mathbf{0}^T & \mathbf{1} \\ \mathbf{I}_{A-1} & \mathbf{0} \end{bmatrix}$ ,  $[\mathbf{\Pi}_{AB}]_{mB+p,qA+n} = \delta_{mn}\delta_{pq}$ ,  $\forall m, n \in \{0, 1, \dots, A-1\}$ ,  $\forall p, q \in \{0, 1, \dots, B-1\}$ .
- MIMO-GFDM w/ Dirichlet filter:  $\mathbf{g}_1 = \sqrt{K} \times \mathbf{1}_M$  and  $l = D - \lfloor M/2 \rfloor$
- MIMO-OFDM:  $M = 1$

- Step 1: Multiply both sides of (1) with  $\mathbf{U}$ .

$$\tilde{\mathbf{y}} = \mathbf{U}\mathbf{y} = \text{blkdiag}(\{\mathbf{F}_k\}_{k=0}^{K-1})\tilde{\mathbf{d}} + \tilde{\mathbf{n}}$$

- Step 2: Split into  $K$  subproblems and apply sorted QR decomposition (SQRD).

$$\tilde{\mathbf{y}}_k = \mathbf{F}_k\tilde{\mathbf{d}}_k + \tilde{\mathbf{n}}_k = \mathbf{Q}_k\mathbf{R}_k\mathbf{P}_k^T\tilde{\mathbf{d}}_k + \tilde{\mathbf{n}}_k, k = 0, 1, \dots, K-1 \quad (2)$$

- Step 3: Multiply both sides of (2) with  $\mathbf{Q}_k^H$ .

$$\tilde{\mathbf{y}}_k = \mathbf{Q}_k^H\tilde{\mathbf{y}}_k = \mathbf{R}_k\tilde{\mathbf{d}}_k + \tilde{\mathbf{n}}_k, k = 0, 1, \dots, K-1 \quad (3)$$

- Step 4: Find the ML solution  $\hat{\tilde{\mathbf{d}}}_k$  to (3) with DFSD.

### Complexity Analysis

- Complexity: # of complex multiplications (CMs) required to detect  $KMT$  symbols at the receiver

**Table 1:** Computational Complexity of SQRD and successive interference cancellation (SIC)

Scheme	SQRD	SIC
OFDM	$KMT^2R + KMTR + (KMT^2 - KMT)/2$ (Using SQRD)	0
Near-ML MIMO-GFDM [4]	$K^3M^3T^2R + K^2M^2TR + (2K^3M^3T^3 + 3K^2M^2T^2 + KMT)/6$ (Using MMSE-SQRD)	$K^2T^2M^2$
Proposed	$KM^3T^2R + KM^2TR + (KM^2T^2 - KMT)/2$ (Using SQRD)	0

## Simulation

- Modulation: QPSK, symbol energy:  $E_s = 1$ , and CP length:  $L = D/8$
- # of SM-mode MIMO spatially uncorrelated Rayleigh-fading multipath channel realizations: 500
- Chan. power delay profile: exponential from 0 to -10 dB with  $L$  taps
- # of independent data blocks for each channel realization: 100
- $(T, R) = (2, 2)$

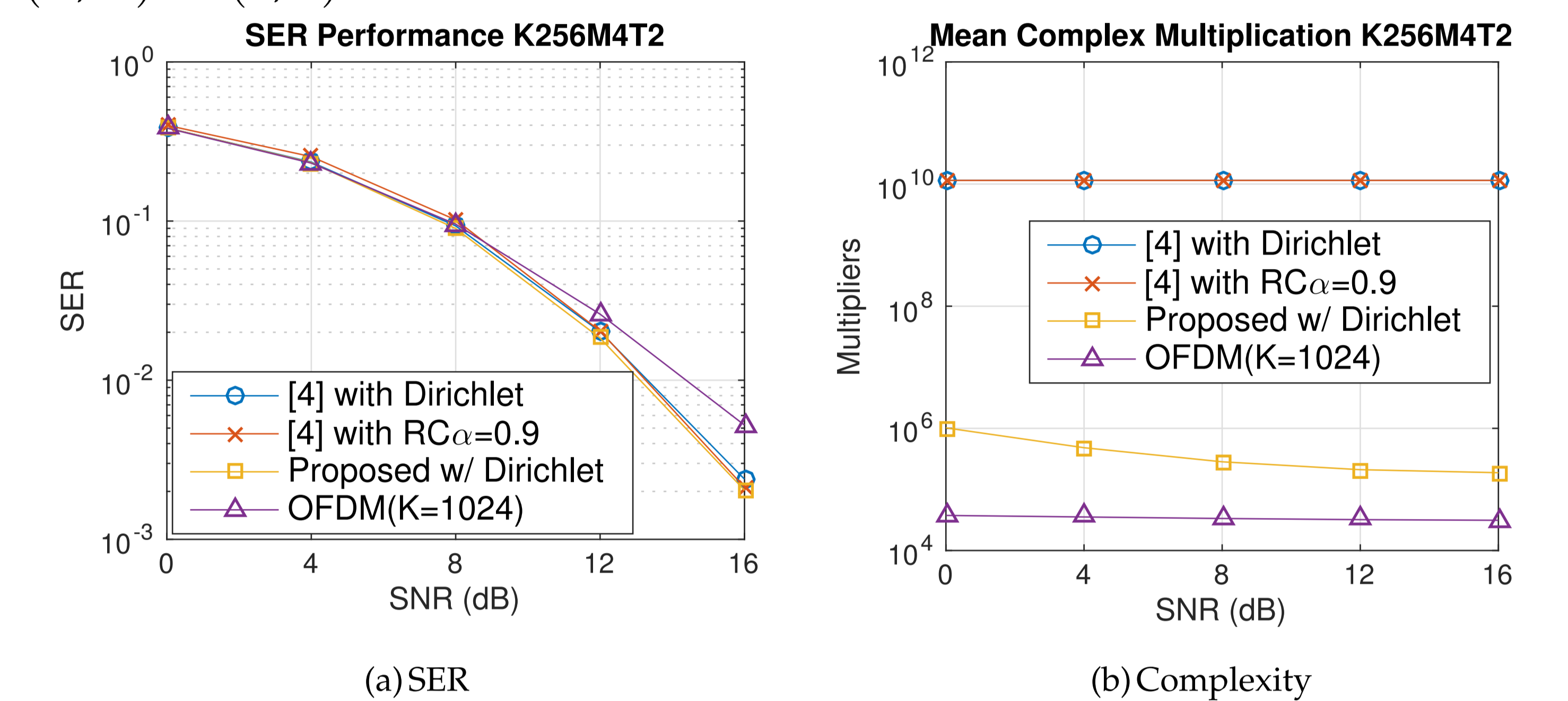


Figure 2: Performances for  $K = 256, M = 4, T = 2$

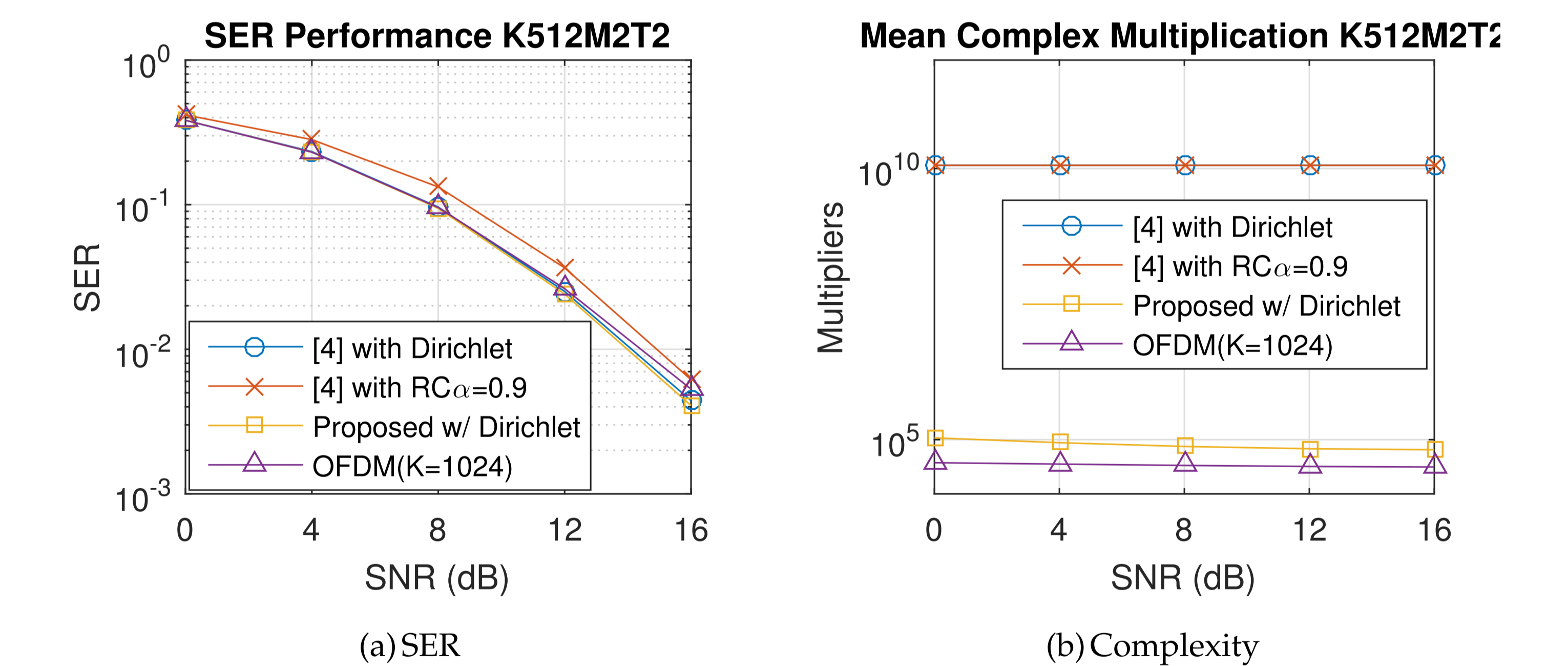


Figure 3: Performances for  $K = 512, M = 2, T = 2$

- Symbol error rate (SER): MIMO-GFDM (FD decoupling) > MIMO-GFDM ([4]) > MIMO-OFDM (subcarrier spacing  $\leftrightarrow$  frequency diversity)
- Complexity: MIMO-GFDM significantly improves via FD decoupling.

## Conclusion

- MIMO-GFDM achieves FD decoupling w/ proposed prototype filters.
- Dirichlet filter w/ proposed scheme outperforms widely-applied RC filter in terms of SER and complexity performances for MIMO-GFDM.

## References

- [4] M. Matthé, I. Gaspar, D. Zhang, and G. Fettweis, "Near-ML Detection for MIMO-GFDM," in *Veh. Technol. Conf. (VTC Fall), 2015 IEEE 82nd*, Sept. 2015, pp. 1–2.