Gridless DOA Estimation via Alternating Projections

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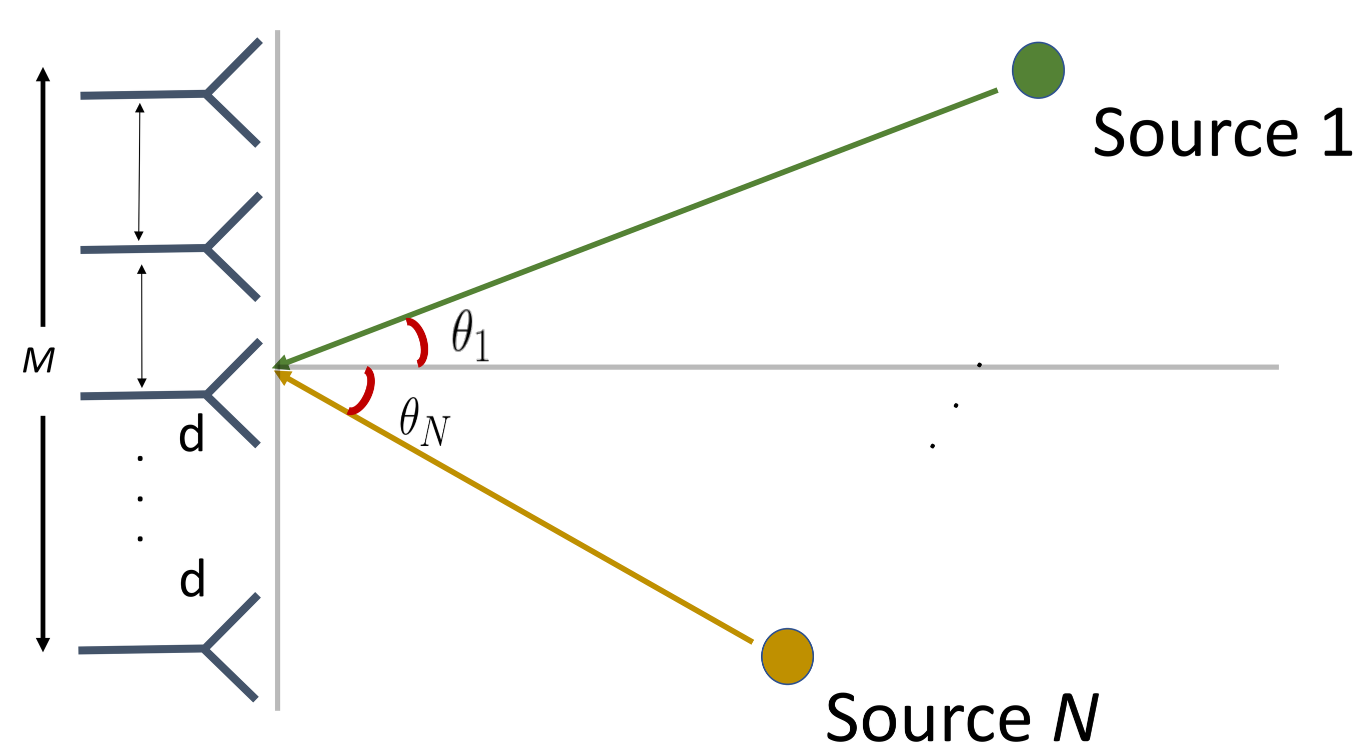
Overview: Direction of Arrival (DOA) Estimation has recently been combined with compressive sensing to produce sparsity based algorithms which achieve better accuracy than traditional methods. The most promising of these algorithms is the family of *gridless* algorithms.

Objective: Estimate the DOA of all incoming signals



Signal Model:

$$\mathbf{Y} = \mathbf{A}\mathbf{X}$$

$$\begin{bmatrix} \uparrow & & \uparrow \\ \mathbf{y}_1, & \ldots, & \mathbf{y}_L \\ \downarrow & & \downarrow \end{bmatrix} = \begin{bmatrix} \uparrow & & \uparrow \\ \mathbf{a}(\theta_1), & \ldots, & \mathbf{a}(\theta_N) \\ \downarrow & & \downarrow \end{bmatrix} \begin{bmatrix} \leftarrow \mathbf{x}_1 \rightarrow \\ \leftarrow \mathbf{x}_2 \rightarrow \\ \vdots \\ \leftarrow \mathbf{x}_N \rightarrow \end{bmatrix}$$

$\mathbf{x}_i \in \mathbb{C}^L$ is the length L signal sent from source i,
$\mathbf{y}_i \in \mathbb{C}^M$ is the received signal across the array at snapshot i
$\mathbf{a}(\theta_i) \in \mathbb{C}^M$ is the 'array steering vector' from angle θ_i

$$\mathbf{a}(\theta) = \begin{bmatrix} e^{-j\frac{2\pi}{\lambda}0\sin\theta} \\ e^{-j\frac{2\pi}{\lambda}1\sin\theta} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}(M-1)\sin\theta} \end{bmatrix}$$ is the *array steering vector* for angle θ

Gridless DOA Estimation:

$$\begin{aligned} \underset{\mathbf{u},\mathbf{W}}{\text{minimize}} \quad & \text{rank}(\mathbf{T}(\mathbf{u})) \\ \text{subject to} \quad & \begin{bmatrix} \mathbf{T}(\mathbf{u}) & \mathbf{Y} \\ \mathbf{Y}^{\mathsf{H}} & \mathbf{W} \end{bmatrix} \succeq 0. \end{aligned}$$

where $\mathbf{T}(\mathbf{u})$ is an $M \times M$ Toeplitz matrix

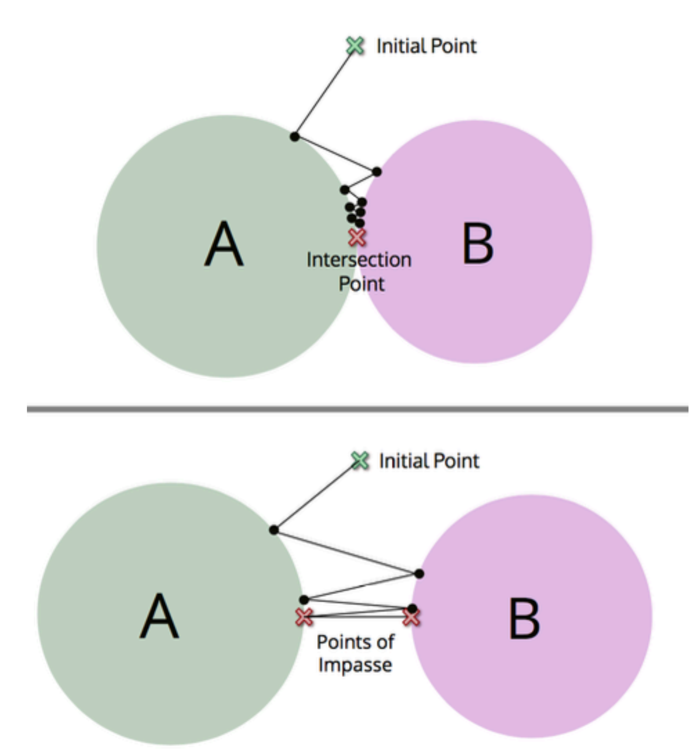
Once $\mathbf{T}(\mathbf{u})$ is estimated the DOAs are recovered via *Vandermonde Decomposition*

$$\mathbf{T}(\mathbf{u}) = \mathbf{A}\mathbf{D}\mathbf{A}^{\mathsf{H}}$$

The convex relaxation of the above optimization is a Semi-Definite Program (SDP), which is solvable but not computationally efficient.

ADMM: ADMM, or *Alternating Directions Method of Multipliers* is the current state of the art algorithm for solving SDP optimizations.

Alternating Projections: an optimization algorithm which iteratively projects an initial estimate between two or more sets until a point of intersection is reached

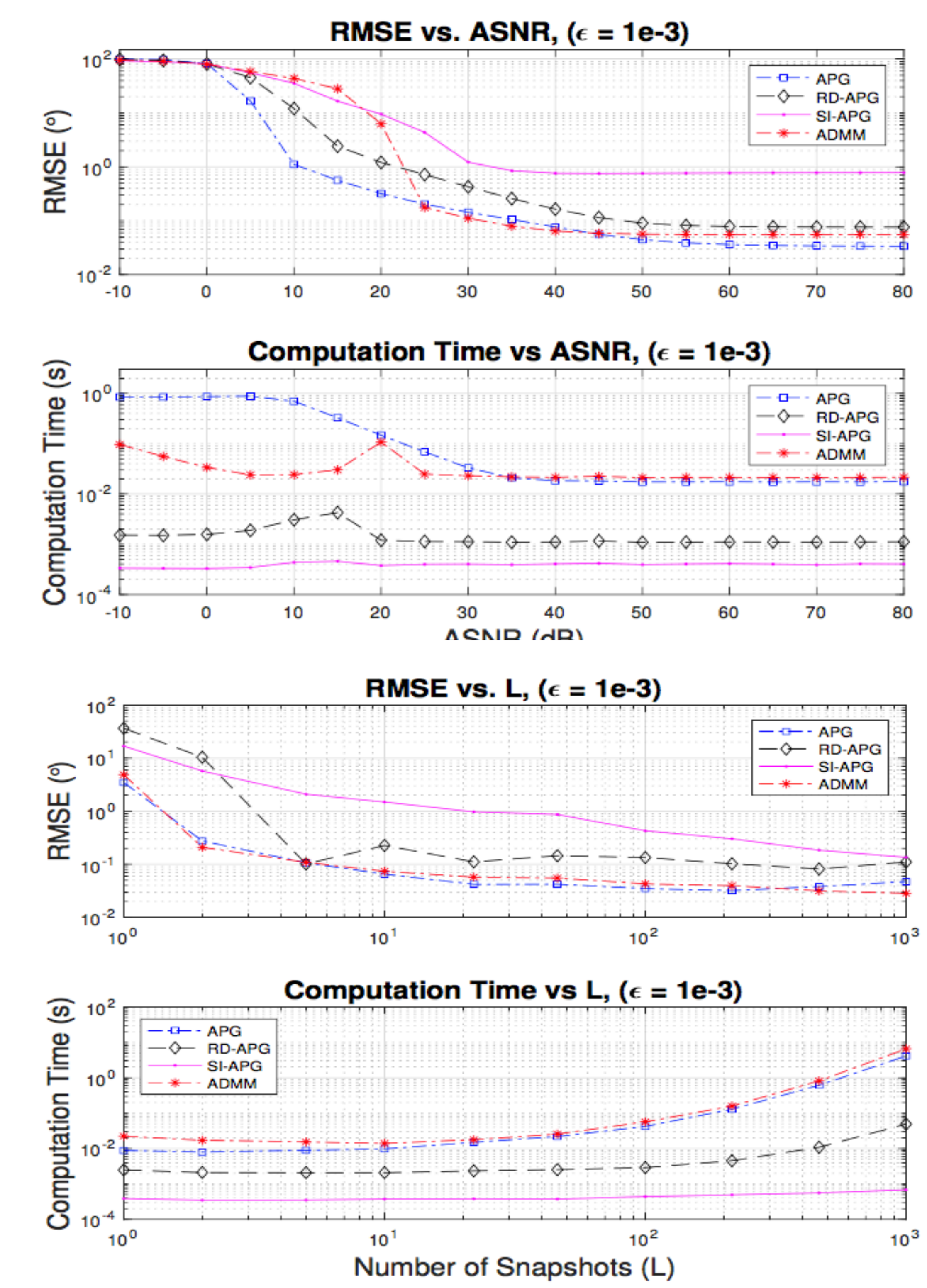


Claim: The gridless DOA estimation optimization (1) can be solved directly and efficiently using alternating projections

Alternating Projections based Gridless DOA estimation (APG):

1.) Generate random initialization of $\mathbf{T}(\mathbf{u})$
2.) Project $\mathbf{T}(\mathbf{u})$ to the Toeplitz Set
3.) Project $\mathbf{T}(\mathbf{u})$ to be Rank N
4.) Construct $\mathbf{S} = \begin{bmatrix} \mathbf{T}(\mathbf{u}) & \mathbf{Y} \\ \mathbf{Y}^{\mathsf{H}} & \mathbf{W} \end{bmatrix}$
5.) Project $\mathbf{S}$ to the PSD cone, recover $\mathbf{T}(\mathbf{u})$
6.) Repeat steps 2-5 until convergence

Related Algorithms: In addition to APG, we have introduced a reduced dimension variant dubbed **RD-APG**, and a single iteration variant dubbed **SI-APG**



M = 20,
N = 3,
L = 50,
250 trials

M = 20,
N = 3,
ASNR = 20dB,
250 trials