TOEPLITZ MATRIX COMPLETION FOR DIRECTION FINDING USING A MODIFIED NESTED LINEAR ARRAY

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Motivation

Background

- Recently, a modified nested linear array (MNLA) has been reported for a potential in increasing the degree-of-freedom.
- However, there exist some "holes" in its difference co-array, which results in missing "lags" and limited performance of direction-of-arrival (DOA) estimation.
- The interest of this paper is to tackle the problem caused by the missing lags in MNLA.

Our Approach

- Construct a covariance matrix with Toeplitz Hermitian structure.
- Derive and solve a semidefinite program.
- Perform DOA estimation with the obtained Toeplitz covariance matrix.

Contribution

- The problem caused by the missing lags in MNLA is solved.
- The performance of DOA estimation is improved.

Signal Model

The MNLA with total sensor number $L = M + N$, is depicted in Figure 1. The observation data using MNLA can be formed as

$$\mathbf{x}(t) = \mathbf{B}\mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{B}$ is the steering matrix, $\mathbf{s}(t)$ and $\mathbf{n}(t)$ denote the signal and noise, respectively. The data covariance matrix is defined as follows

$$\mathbf{R}_x \triangleq E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{B}\mathbf{R}_s\mathbf{B}^H + \sigma_n\mathbf{I}.$$

- **Remark 1:** In the case of uniform linear array (ULA), the resulting covariance matrix is a Toeplitz Hermitian matrix.
- **Remark 2:** Here, the array configuration under consideration is MNLA rather than ULA. Thus, $\mathbf{R}_x$ is not a Toeplitz matrix, and the DOA estimation performance would be limited whenever it is directly used base on traditional direction finding techniques.
- **Remark 3:** To this end, a Toeplitz matrix completion procedure is applied to transform the MNLA covariance matrix to a ULA counterpart before performing DOA estimation.

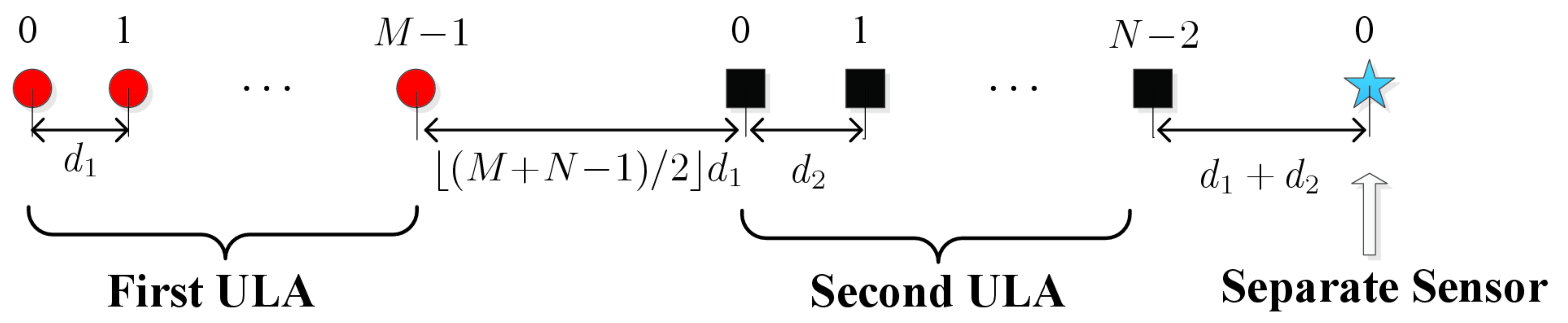

Figure 1: Illustration of configuration of MNLA.

Methodology

Progress of Toeplitz Matrix Completion

- A four-step scheme:

Step 1:

Construct a Toeplitz matrix $\mathbf{T}_0$. — Using the existing lags in $\mathbf{R}_x$, while the missing lags being filled with zeros

⇓

Step 2:

Formulate a low-rank matrix recovery problem. —

$$\min_{\mathbf{w}} \ \mathrm{rank}(\mathbf{T}) \quad \text{s.t. } \mathbf{T} = \mathbf{T}_0 + \sum_{l \in \mathscr{H}} \left(w_l \mathbf{I}^l_{(+)} + w_l^* \mathbf{I}^l_{(-)} \right)$$

⇓

Step 3:

Reform the problem into a semidefinite program. —

$$\min_{\mathbf{w}} \ \mathrm{trace}(\mathbf{T}) \quad \text{s.t.} \begin{cases} \mathbf{T} = \mathbf{T}_0 + \sum_{l \in \mathscr{H}} \left(w_l \mathbf{I}^l_{(+)} + w_l^* \mathbf{I}^l_{(-)} \right), \\ \mathbf{T} \succeq \mathbf{0} \end{cases}$$

⇓

Step 4:

Solve the problem. — Using CVX

- An example: $K = 1$ signal and $L = 5$ sensors. The sensor locations are $\{0, 1, 3, 7, 12\}$. The corresponding covariance matrix is

$$\begin{aligned} \mathbf{R}_x^{(\mathrm{Ex})} &= \mathbf{b}(\theta_1)\sigma_s\mathbf{b}^H(\theta_1) + \sigma_n\mathbf{I} \\ &= \sigma_s \begin{bmatrix} \beta^0(\theta_1) & \beta^{-1}(\theta_1) & \beta^{-3}(\theta_1) & \beta^{-7}(\theta_1) & \beta^{-12}(\theta_1) \\ \beta^1(\theta_1) & \beta^0(\theta_1) & \beta^{-2}(\theta_1) & \beta^{-6}(\theta_1) & \beta^{-11}(\theta_1) \\ \beta^3(\theta_1) & \beta^2(\theta_1) & \beta^0(\theta_1) & \beta^{-4}(\theta_1) & \beta^{-9}(\theta_1) \\ \beta^7(\theta_1) & \beta^6(\theta_1) & \beta^4(\theta_1) & \beta^0(\theta_1) & \beta^{-5}(\theta_1) \\ \beta^{12}(\theta_1) & \beta^{11}(\theta_1) & \beta^9(\theta_1) & \beta^5(\theta_1) & \beta^0(\theta_1) \end{bmatrix} + \sigma_n\mathbf{I}. \end{aligned}$$

It can be seen that some lags in $\mathbf{R}_x^{(\mathrm{Ex})}$, including $\beta^8(\theta_1)$, $\beta^{10}(\theta_1)$, $\beta^{-8}(\theta_1)$, and $\beta^{-10}(\theta_1)$, are missing. The Toeplitz covariance matrix is constructed as $\mathbf{T}_0^{(\mathrm{Ex})} = \mathrm{toep}(\mathbf{c}, \mathbf{r})$, where

$$\begin{aligned} \mathbf{c} = &[\beta^0(\theta_1), \beta^1(\theta_1), \beta^2(\theta_1), \beta^3(\theta_1), \beta^4(\theta_1), \beta^5(\theta_1), \\ &\beta^6(\theta_1), \beta^7(\theta_1), 0, \beta^9(\theta_1), 0, \beta^{11}(\theta_1), \beta^{12}(\theta_1)]^T \\ \mathbf{r} = &[\beta^0(\theta_1), \beta^{-1}(\theta_1), \beta^{-2}(\theta_1), \beta^{-3}(\theta_1), \beta^{-4}(\theta_1), \beta^{-5}(\theta_1), \\ &\beta^{-6}(\theta_1), \beta^{-7}(\theta_1), 0, \beta^{-9}(\theta_1), 0, \beta^{-11}(\theta_1), \beta^{-12}(\theta_1)]. \end{aligned}$$

DOA Estimation with Toeplitz Covariance Matrix

Once the Toeplitz covariance matrix is obtained, classical approaches like multiple signal classification (MUSIC) algorithm can be adopted for DOA estimation.

Results

Angle Resolution Comparison

- $K = 2$ signals, $L = 5$ sensors.
- Angle resolution comparison: SNR is set as −5 dB, 0 dB, and 5 dB (from row 1 to row 3), and the angle separation Δ_θ between two sources is set as 2°, 3°, 6°, and 9° (from column 1 to column 4).
- It is seen that, the proposed method outperforms the others in any experimental situations.

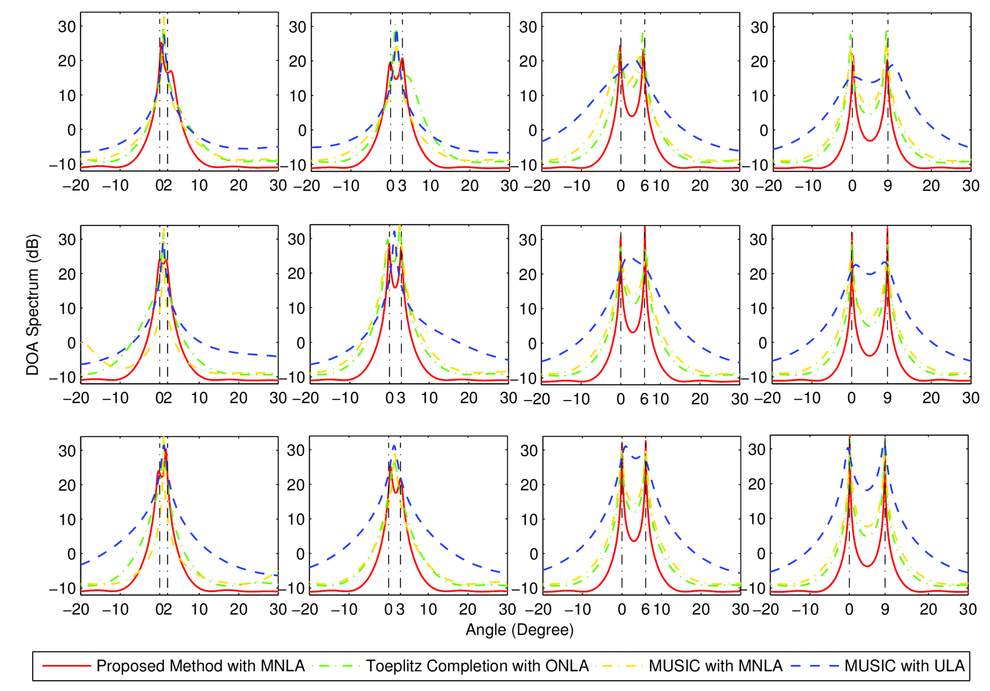

Figure 2: Angle resolution comparison.

DOA RMSE Comparison

- $K = 2$ signals $\{0°, 10°\}$, $L = 5$ sensors, 500 snapshots, SNR ranges from −4 dB to 12 dB.
- $K = 2$ signals, $L = 5$ sensors, SNR is 10 dB, the number of snapshots varies from 10 to 500.

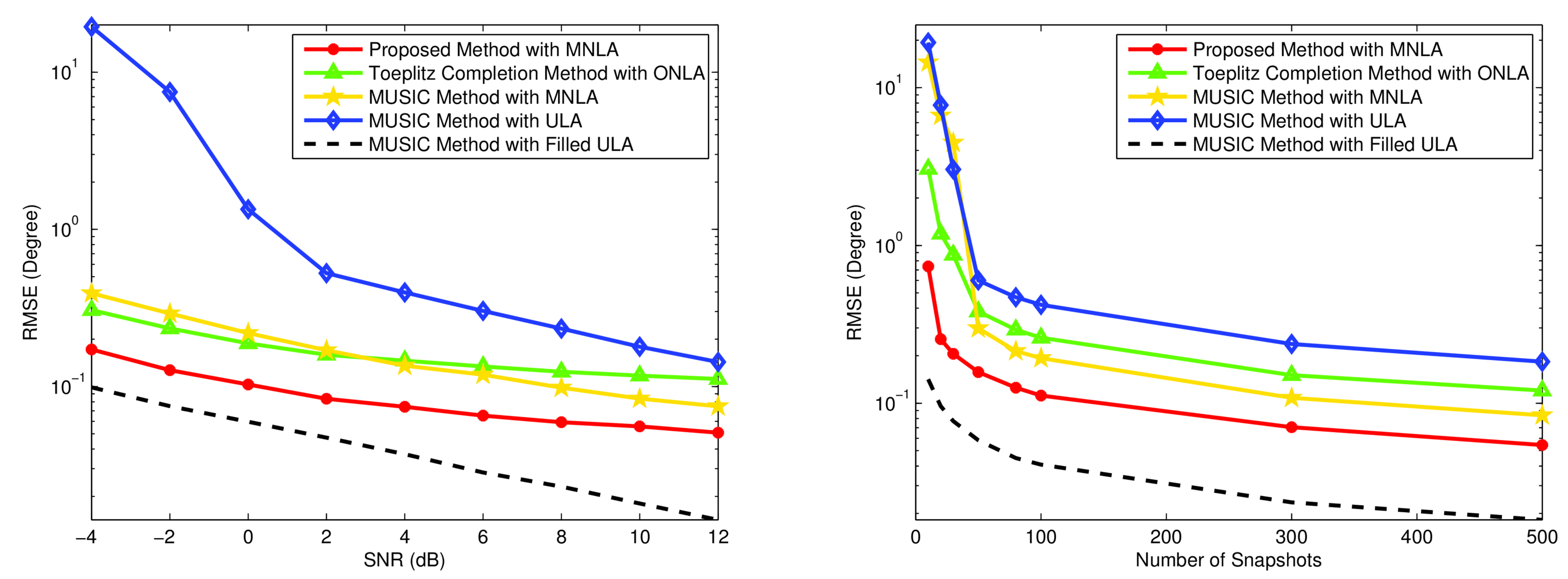

Figure 3: DOA RMSE comparison.

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If you have any further questions, please do not hesitate to contact us via emailing to Huiping Huang (h.huang@spg.tu-darmstadt.de).