Introducing the Orthogonal Periodic Sequences for the Identification of Functional Link Polynomial Filters

Alberto Carini
DIA – Università di Trieste
acarini@units.it

Simone Orcioni and Stefania Cecchi
DII – Università Politecnica delle Marche
s.orcioni@univpm.it, s.cecchi@univpm.it

Abstract

We introduce the **orthogonal periodic sequences** (OPSs), a family of deterministic signals, for the identification of **functional link polynomial** (FLiP) filters. The OPSs share many characteristics of the **perfect periodic sequences** (PPSs). As the PPSs, they allow the perfect identification of a FLiP filter on a finite time interval with the **cross-correlation method**. In contrast to PPSs, OPSs can identify also **non-orthogonal** FLiP filters, as the Volterra filters. With OPSs, the input sequence can be any **persistently exciting** sequence and can also be **quantized**. OPSs can often identify FLiP filters with a sequence period and a computational complexity much smaller than that of PPSs.

FLiP filters

FLiP filters are a class of **linear-in-the-parameters** (LIP) nonlinear filters. They are a linear combination of basis functions, product of nonlinear expansions of delayed input samples. In **diagonal form**:

$$y(n) = \sum_{p=0}^{R-1} \sum_{m=0}^{N_p-1} h_p(m) f_p(n-m)$$

where $f_p(n)$ are the zero lag basis functions, with $f_p(n) \in \{1,\ g_1[x(n)],\ g_2[x(n)],\ g_1[x(n)]g_1[x(n-1)], \ldots, g_1[x(n)]g_1[x(n-D)], g_3[x(n)], \ldots\}$.

When $g_i(\xi)$, $\forall i$, satisfy the Stone-Weierstrass theorem, the FLiP filters are **universal approximators**.

They include many families of polynomial filters, as the **Volterra** for $g_i(\xi) = \xi^i$, the **Legendre nonlinear** (LN), $g_i(\xi) \in \{1, \xi, (3\xi^2 - 1)/2, \xi(5\xi^2 - 3)/2, \ldots\}$, the **Wiener nonlinear** (WN), $g_i(\xi) \in \{1, \xi, \xi^2 - \sigma_x^2, \xi^3 - 3\sigma_x^2\xi, \ldots\}$.

Some FLiP filters have orthogonal basis functions for some input distribution, e.g., LN and WN, thus allowing the identification of the coefficients using the **cross-correlation method**.

Orthogonal FLiP filters also admit **PPSs**, i.e., periodic sequences that guarantee the perfect orthogonality of the basis functions over a period.

Using a PPS input, an orthogonal FLiP filter can still be identified with the cross-correlation method,

$$h_i(j) = < y(n) f_i(n-j) >_L \ / < f_i^2(n) >_L \ .$$

OPSs

Each OPS allows the estimation of **a diagonal** $h_i(j)$ of the FLiP filter with the cross-correlation method.

We consider a **periodic input** sequence $x(n)$ of period L, **persistently exciting** the FLiP filter.

We want to find $z_i(n)$ of period L such that for any j, with $0 \le j \le N_i - 1$,

$$h_i(j) = < y(n) z_i(n-j) >_L .$$

For $i > 0$, it can be proved that $z_i(n)$ **must satisfy**

$$\begin{aligned} < z_i(n) >_L &= 0, \\ < f_i(n) z_i(n) >_L &= 1, \\ < f_i(n-m_i) z_i(n) >_L &= 0, \\ < f_p(n-m_p) z_i(n) >_L &= 0, \end{aligned}$$

for all $-(N_i - 1) < m_i \le N_i - 1$ and $m_i \ne 0$, $-(N_i - 1) \le m_p \le N_p - 1$ and $0 < p \le R-1$ with $p \ne i$.

The system has Q_i equations and L variables. For $L \ge Q_i$ it **always** admits **a solution**.

Let us write the system in matrix form,

$$\mathbf{S}\mathbf{z} = \mathbf{d}$$

The minimum norm solution of the system is

$$\mathbf{z} = \mathbf{S}(\mathbf{S}\mathbf{S}^T)^{-1}\mathbf{d}.$$

The elements of $\mathbf{S}\mathbf{S}^T$ are **cross-correlations** between basis functions with different time delays. By properly sorting the rows of $\mathbf{S}$, $\mathbf{S}\mathbf{S}^T$ is **block Toeplitz** and admits efficient algorithms for its inversion [1].

When $L \ge Q = \max_i Q_i$, it is possible to develop a **set of OPSs** and estimate with the same input $x(n)$ **all diagonals** of the FLiP filter. The same $x(n)$ could be used for estimating **different** types of **FLiPs filters**.

Output noise effect

We study the effect of an additive output noise $\nu(n)$.

The **mean square deviation** (MSD) of $f_i(n-j)$ is

$$\begin{aligned} \text{MSD}_{i,j} &= E[(h_i(j) - \tilde{h}_i(j))^2] \\ &= E[(< \nu(n) z_i(n-j) >_L)^2]. \end{aligned}$$

$\text{MSD}_{i,j}$ is proportional to the noise power σ_ν^2 and inversely proportional to $< f_i^2(n) >_L$.

To compare the OPSs we define the **noise gain**,

$$G_{\nu,i,j} = \text{MSD}_{i,j} < f_i^2(n-j) >_L / E[\nu^2(n)].$$

For **PPSs**, it can be proved $G_{\nu,i,j}$ is **always 1**.

On the contrary, for **OPSs** it is:

$$G_{\nu,i,j} = < z_i^2(n) >_L \cdot < f_i^2(n) >_L .$$

Experimental results

We have considered the identification of a **real device**, a Behringer Mic 100 Vacuum Tube Preamplifier.

Working at 8 kHz sampling frequency, the device has a memory lower than 20 samples.

Different signals have been applied for identification:

- **two PPSs** for LN and WN filters (with order 3, memory 20, and period $L = 357\,956$);
- **eight periodic sequences** with uniform and Gaussian distributions, quantized with 10 bits, and with periods

$$[6140, 2^{13}, 2^{14}, 2^{15}, 2^{16}, 2^{17}, 2^{18}]$$

OPSs for LN, WN and Volterra filters of order 3, memory 20, have been derived and used for identification.

Thirteen different settings have been considered for the preamplifier. The SNR was around 50 dB.

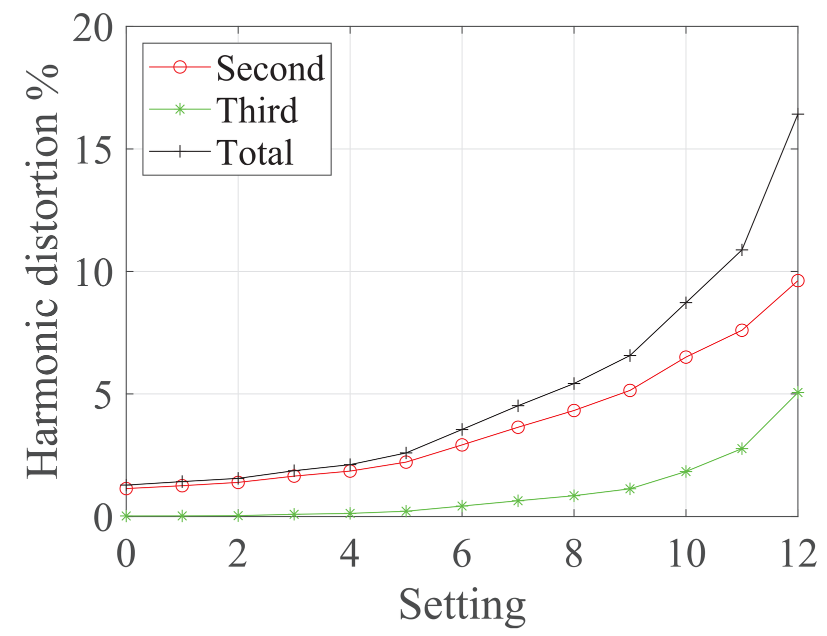

Fig. 1. Second, third, and total harmonic distortion.

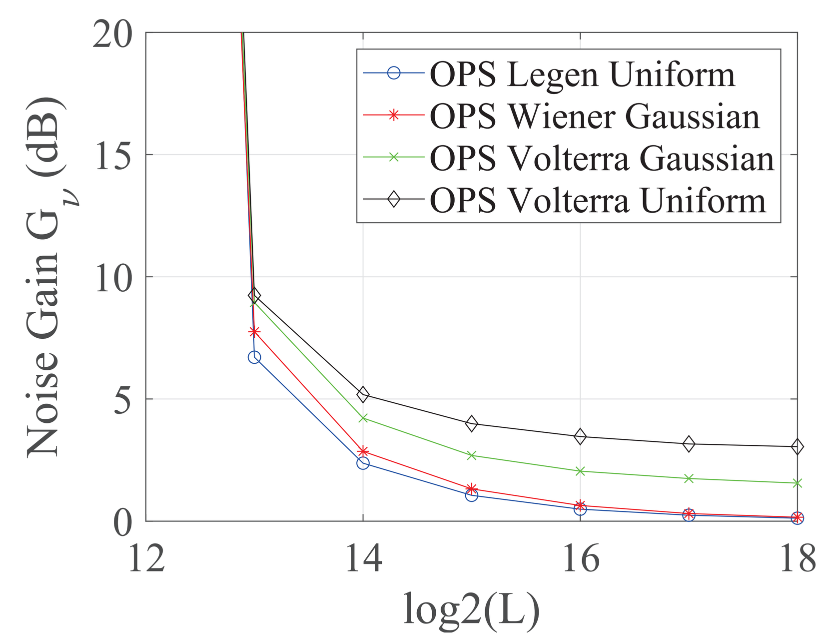

Fig. 2. Noise Gain of OPSs for LN, WN and Volterra filters.

References

[1] G.-O. Glentis and N. Kalouptsidis, "Efficient algorithms for the solution of block linear systems with Toeplitz entries," *Linear Algebra and its Applications*, vol. 179, 1993

[2] A. Carini, S. Orcioni, and S. Cecchi, *Orthogonal periodic sequences*, http://www2.units.it/ipl/res_OPSeqs.htm

Normalized MSEs

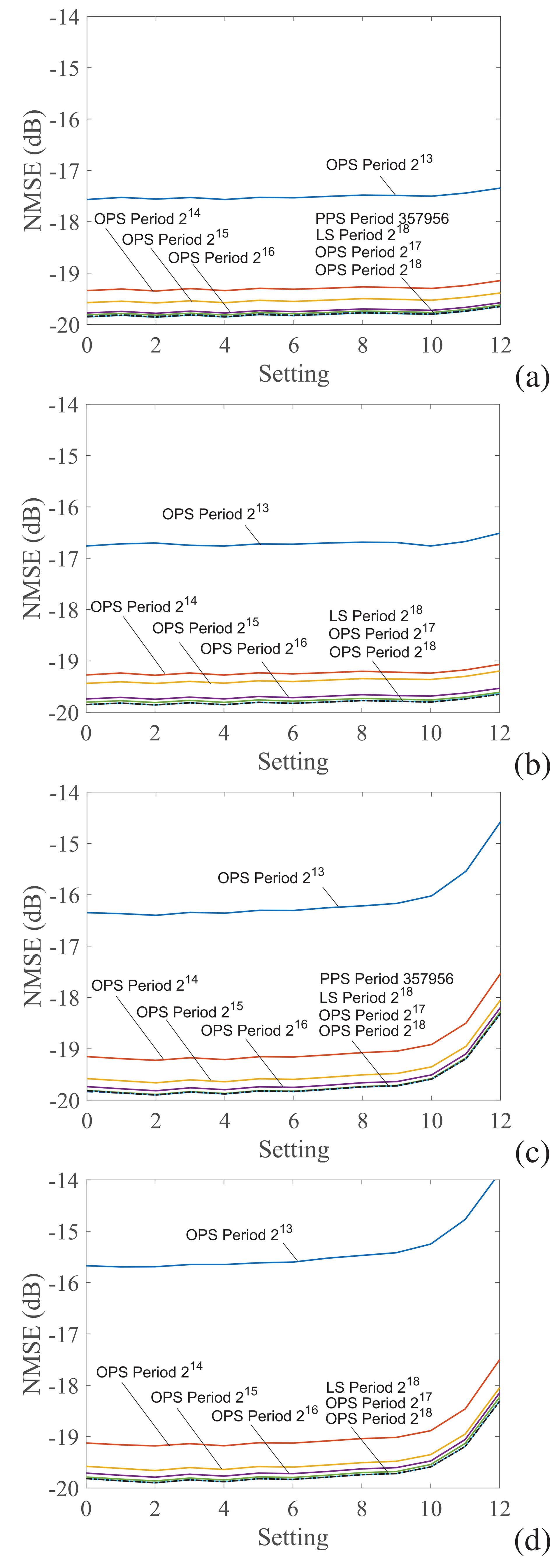

Fig. 3. NMSEs for (a) LN filter and (b) Volterra filter on uniform distribution input, and for (c) WN filter and (d) Volterra filter on Gaussian distribution input.