Analysis of coprime arrays on moving platform

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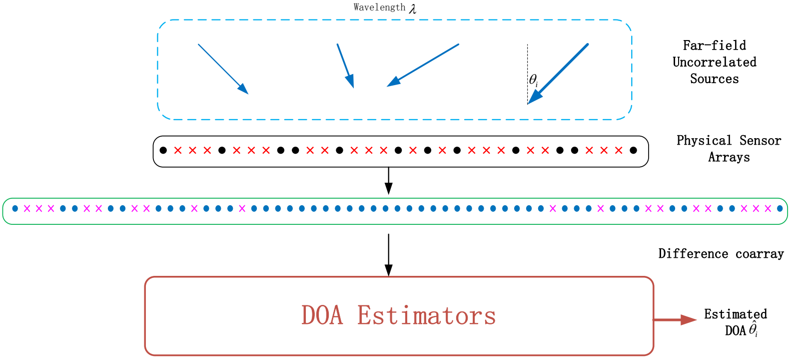
May 16, 2019

Outline

1. Introduction
2. Problem formulation
3. Difference co-array
4. Simulation results
5. Conclusion

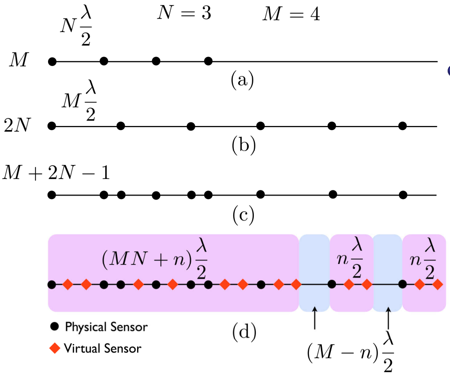
Outline

DOA estiamtion based on a fixed coprime array



[1] *Chun-Lin Liu and P. P. Vaidyanathan, "Robustness of coarrays of sparse arrays to sensor failures," in Proc. IEEE Int. Conf. Acoust., Speech, and Sig. Proc., Calgary, AB, Canada, April, 2017, pp. 3231 – 3235.*

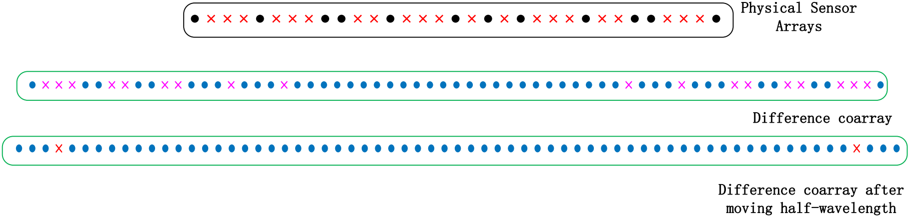
Coprime array on a moving platform



- Requirements:
  (1)The difference co-array of (d) is a hole-free array when
  $$n = \begin{cases} N/2, & if \quad N \quad is \quad even \\ \frac{N-1}{2}, & if \quad N \quad is \quad odd \end{cases}$$
  (2) The signal environment is stationary within a long period for a large aperture array.

[2] *J. Ramirez and J. L. Krolik, "Synthetic aperture processing for passive co-prime linear sensor arrays," Digital Signal Process., vol. 61, pp. 62 – 75, 2017.*

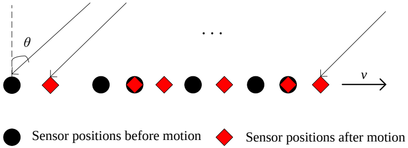
Coprime array moving half-wavelength



- Advantages:
  1. Easy to be applied in practice
  2. Such motion can fill most, if not all of the holes, and as such significantly increases both the contiguous and unique lags

Outline

Passive synthetic aperture



The output of the receive array, at time t, is expressed as

$$\mathbf{x}(t) = \sum_{q=1}^{Q} s_q(t) \exp(-jvt\kappa_q)\mathbf{a}(\theta_q) + \boldsymbol{\varepsilon}(t) = \mathbf{A}\mathbf{s}(t) + \boldsymbol{\varepsilon}(t),$$

where v is the velocity of platform,
$\mathbf{a}(\theta_q) = [1, \exp(-jd_2\kappa_q), \cdots, \exp(-jd_L\kappa_q]^T$, $\kappa_q = 2\pi\sin(\theta_q)/\lambda$,
$\mathbf{s}(t) = [s_1(t)\exp(-jvt\kappa_1), s_2(t)\exp(-jvt\kappa_2), \cdots, s_Q(t)\exp(-jvt\kappa_Q)]^T$,
$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_Q)]$, $\boldsymbol{\varepsilon}(t)$ is zero-mean complex additive white Gaussian noise vector.

Passive synthetic aperture

At time $t+\tau$,

$$\mathbf{x}(t+\tau) = \sum_{q=1}^{Q} s_q(t+\tau)\exp(-jvt\kappa_q)\exp(-jv\tau\kappa_q)\mathbf{a}(\theta_q)+\varepsilon(t+\tau)$$
$$= \mathbf{B}\mathbf{s}(t+\tau)+\varepsilon(t+\tau).$$

where

$$\mathbf{B} =[\mathbf{b}(\theta_1),\mathbf{b}(\theta_1),\cdots,\mathbf{b}(\theta_Q)],$$
$$\mathbf{b}(\theta_q) = \exp(-jv\tau\kappa_q)\mathbf{a}(\theta_q),$$
$$= [\exp(-jv\tau\kappa_q),\exp(-j(v\tau+d_2)\kappa_q),\cdots,\exp(-j(v\tau+d_L)\kappa_q)]^T,$$
$$\mathbf{s}(t+\tau) = [s_1(t+\tau)\exp(-jvt\kappa_1), s_2(t+\tau)\exp(-jvt\kappa_2),\quad\cdots,$$
$$s_Q(t+\tau)\exp(-jvt\kappa_Q)]^T.$$

Passive synthetic aperture

For narrowband signals and $v\tau = d = \lambda/2$,

$$\mathbf{x}(t+\tau) = \exp(j2\pi f\tau)\mathbf{B}\mathbf{s}(t) + \varepsilon(t+\tau)$$

$$\mathbf{b}(\theta_q) = [\exp(-jd\kappa_q), \exp(-j(d+d_2)\kappa_q), \cdots, \exp(-j(d+d_L)\kappa_q)]^T$$

Compensating for the phase correction factor $\exp(j2\pi f\tau)$(See Ref.[3]) ,

$$\tilde{\mathbf{x}}(t+\tau) = \mathbf{x}(t+\tau)\exp(-j2\pi f\tau) = \mathbf{B}\mathbf{s}(t) + \tilde{\varepsilon}(t+\tau),$$

where $\tilde{\varepsilon}(t+\tau) = \exp(-j2\pi f\tau)\varepsilon(t+\tau)$.
The output of the synthetic array is

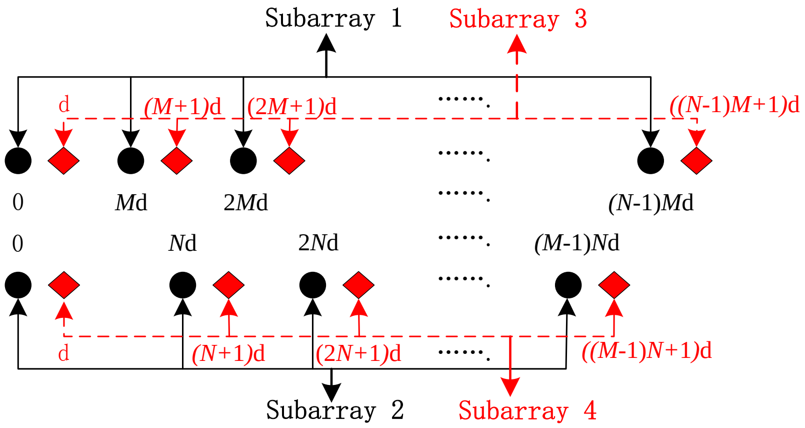
$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \tilde{\mathbf{x}}(t+\tau) \end{bmatrix} = \mathbf{A}_s\mathbf{s}(t) + \begin{bmatrix} \varepsilon(t) \\ \tilde{\varepsilon}(t+\tau) \end{bmatrix},$$

where $\mathbf{A}_s = [\mathbf{a}_s(\theta_1), \mathbf{a}_s(\theta_2), \cdots, \mathbf{a}_s(\theta_Q)]$, $\mathbf{a}_s(\theta_q) = [\mathbf{a}^T(\theta_q), \mathbf{b}^T(\theta_q)]^T$.

[3] *S. Stergios and E. J. Sullivan, "Extended towed array processing by an overlap correlator," J. Acoust. Soc.America, vol. 86, no. 1, pp. 158–171, 1989.*

Outline

Difference co-array



The subarrays of the original and synthetic coprime array($M < N$).

Difference co-array

The set formed from the difference co-array corresponding to the synthetic array is given as

$$\mathbb{S}_\mathrm{c} = \mathbb{S}_{12} \cup \mathbb{S}_{34} \cup \mathbb{S}_{13} \cup \mathbb{S}_{24} \cup \mathbb{S}_{14} \cup \mathbb{S}_{23}.$$

where $\mathbb{S}_{12} = \{Mk_1 - Nk_2\} \cup \{Nk_2 - Mk_1\}$.
Lemma 1: For sets $\mathbb{S}_{13}$, $\mathbb{S}_{24}$, $\mathbb{S}_{14}$ and $\mathbb{S}_{23}$ defined above, $\mathbb{S}_{13} \cup \mathbb{S}_{24} \cup \mathbb{S}_{14} \cup {}_{23} = \mathbb{S}_{14} \cup \mathbb{S}_{23}$.
proof: See Ref. [4].
Utilizing Lemma 1 and $\mathbb{S}_{12} \cup \mathbb{S}_{34} = \mathbb{S}_{12}$, $\mathbb{S}_\mathrm{c}$ is simplified as

$$\mathbb{S}_\mathrm{c} = \mathbb{S}_{12} \cup \mathbb{S}_{14} \cup \mathbb{S}_{23}.$$

Because

$$\mathbb{S}_{14} \cup \mathbb{S}_{23} = \tilde{\mathbb{S}}_{14} \cup \tilde{\mathbb{S}}_{23},$$

[4] *G. Qin, M. G. Amin, and Y. D. Zhang, "DOA estimation exploiting sparse array motions," IEEE Trans. Signal Process., vol. 67, no. 11, pp. 3013 - 3027, June 2019.*

Difference co-array

where

$$\tilde{\mathbb{S}}_{14} = \{Mk_1 - Nk_4 - 1\} \cup \{Nk_2 - Mk_3 - 1\},$$

$$\tilde{\mathbb{S}}_{23} = \{Nk_4 - Mk_1 + 1\} \cup \{Mk_3 - Nk_2 + 1\}.$$

Because $k_1, k_3 \in [0, N-1]$, $k_2, k_4 \in [0, M-1]$, $\tilde{\mathbb{S}}_{14}$ and $\tilde{\mathbb{S}}_{23}$ are equivalent to the following equations.

$$\mathbb{S}_{12}^{\mathrm{L}} = \{Mk_1 - Nk_2 - 1\} \cup \{Nk_2 - Mk_1 - 1\},$$

$$\mathbb{S}_{12}^{\mathrm{R}} = \{Nk_2 - Mk_1 + 1\} \cup \{Mk_1 - Nk_2 + 1\}.$$

Then,

$$\mathbb{S}_{\mathrm{c}} = \mathbb{S}_{12} \cup \mathbb{S}_{12}^{\mathrm{L}} \cup \mathbb{S}_{12}^{\mathrm{R}}.$$

The difference co-array of the synthetic array consists of the difference co-array of the original array and its unit-lag shifted versions in the direction and opposite to direction of motion.

Difference co-array:Example

Assume $M = 4$, $N = 5$, then

$$\mathbb{S}_{12} = \{0, \pm 1, \ \cdots, \pm 8, \pm 10, \pm 11, \pm 12, \pm 15, \pm 16\}$$

The number of unique lags: 27, the number of holes: 6
After array motion,

$$\mathbb{S}_{12}^{\mathrm{L}} = \{0, \pm 1, \cdots, \pm 7, -8, \pm 9, \ 10, \pm 11, -12, -13, 14, 15, -16, -17\}$$

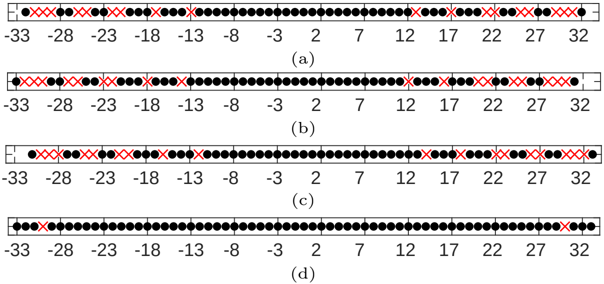
$$\mathbb{S}_{12}^{\mathrm{R}} = \{0, \pm 1, \ \cdots, \pm 7, 8, \pm 9, -10, \pm 11, 12, 13, -14, -15, 16, 17\}$$

$$\mathbb{S}_{\mathrm{c}} = \{0, \pm 1, \cdots, \pm 17\}$$

The number of unique lags: 35, the number of holes: 0

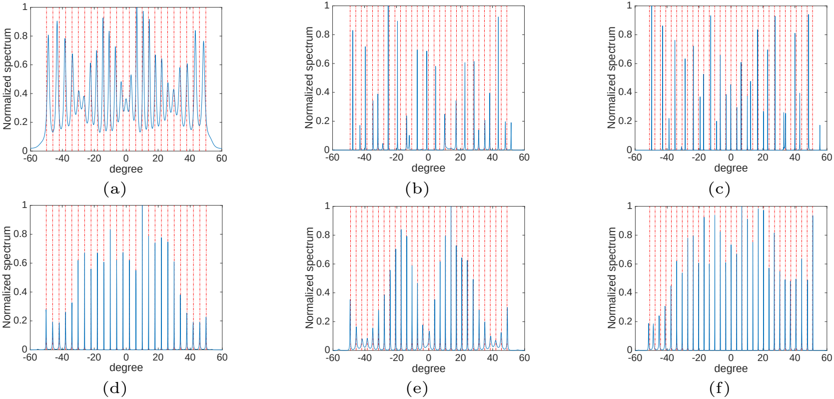
Outline

Simulation results



Difference co-array for M=4, N=9. (a) $\mathbb{S}_{12}$; (b) $\mathbb{S}_{12}^{\mathrm{L}}$; (c) $\mathbb{S}_{12}^{\mathrm{R}}$; (d) $\mathbb{S}_{\mathrm{c}}$.(•: lags; ×: holes.)

Simulation results



Spatial spectra estimation using the LASSO. (a) M=4, N=9, Q=26 for the original array; (b) M=4, N=9, Q=26 for the synthetic array; (c) M=6, N=7, Q=29 for the original array; (d) M=6, N=7, Q=29 for the synthetic array; (e) M=5, N=8, Q=31 for the original array; (f) M=5, N=8, Q=31 for the synthetic array.

Outline

Conclusion

- Analysis of coprime arrays on moving platform
  1. Passive synthetic aperture for moving half-wavelength
  2. Composition of the difference co-array for synthetic array
  3. Camprison of difference co-array corresponding to the oringinal and synthetic array

THANK YOU!