Time domain spherical harmonic analysis for adaptive noise cancellation over a spatial region

Australian National University

Huiyuan Sun, Thushara D. Abhayapala, Prasanga N. Samarasinghe

Contact:{huiyuan.sun, thushara.abhayapala, prasanga.samarasinghe }@anu.edu.au

Summary

- Propose a time domain method for spherical harmonic analysis.
- The proposed method efficiently reduces the latency of the system, benefits real-time signal processing.
- Develop two time-space domain feed-forward adaptive algorithms for spatial ANC based on the time domain spherical harmonic analysis and evaluate the performance.

Problem Formulation

◇ Consider a spatial ANC system with Q onmi-directional error microphones and L secondary loudspeakers, with a source-free region of interest of radius r.

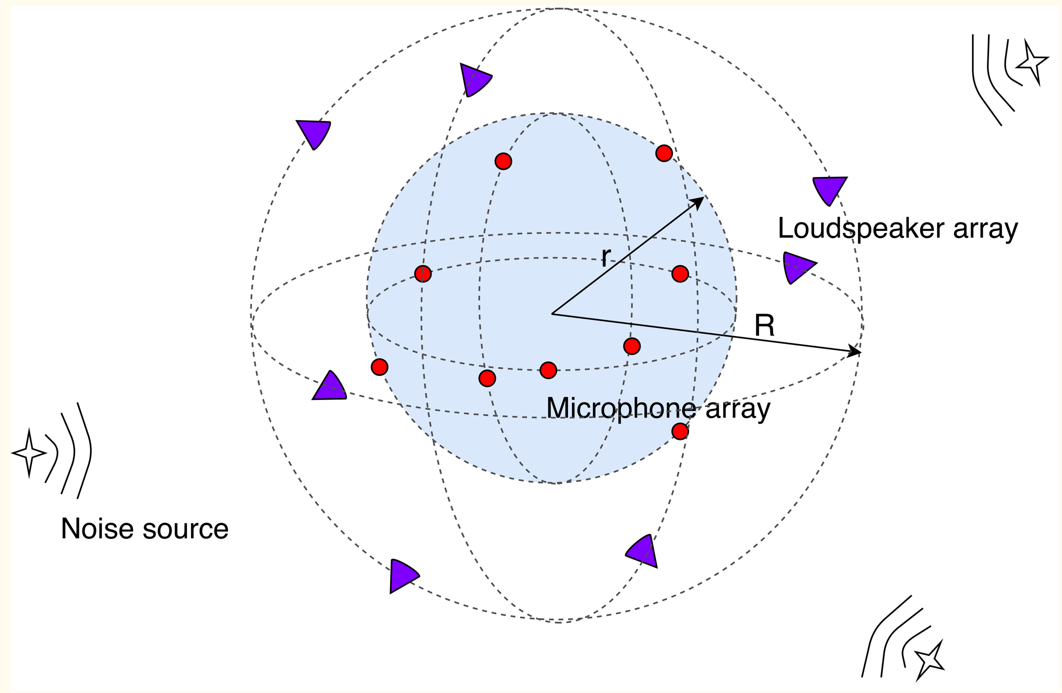


Figure 1: ANC system setup with a spatial region of interest

◇ Total residual sound field at any point $\boldsymbol{x} = (r, \theta, \phi)$ in the region

$$e(\boldsymbol{x}, t) = p(\boldsymbol{x}, t) + y(\boldsymbol{x}, t)$$

$$= \mathcal{N}(t) * g(\boldsymbol{x}|\boldsymbol{y_n}, t) + \sum_{\ell=1}^{L} d_\ell(t) * g(\boldsymbol{x}|\boldsymbol{y_\ell}, t).$$

◇ Driving signal is generated by weighted noises with feed-forward system

$$d_\ell(t) = \mathcal{N}(t) * w_\ell(t).$$

Spherical harmonic analysis

◇ Spherical harmonics decomposition of an N^{th} order residual sound field in frequency domain [1]

$$E(\boldsymbol{x}, f) = \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} \Gamma_\nu^\mu(f) j_\nu\left(\frac{2\pi f r_x}{c}\right) Y_\nu^\mu(\theta_x, \phi_x),$$

◇ $\Gamma_\nu^\mu(k)$: **Frequency domain** spherical harmonic coefficients.

◇ For any $r_x < r$, the infinite summation can be truncated to $V = \lceil kr \rceil$ [1].

◇ Using the Fourier transform relationship between $e(\boldsymbol{x}, t)$ and $E(\boldsymbol{x}, f)$:

$$e(\boldsymbol{x}, t) = \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \gamma_\nu^\mu(t) * p_\nu(t) Y_\nu^\mu(\theta_x, \phi_x),$$

◇ $\gamma_\nu^\mu(t)$: **Time domain** spherical harmonic coefficients.

◇ $p_\nu(t)$ is the inverse Fourier transform of $j_\nu(2\pi f r_x / c)$:

$$p_\nu(t) = \frac{i^\nu c}{2 r_x} P_\nu\left(\frac{tc}{r_x}\right).$$

◇ $a_\nu(t) * p_\nu(t) \approx \delta(t)$, a filter designed according to the inverse Fourier transform of $1/j_\nu(2\pi f r/c)$.

◇ With Q error microphone regularly distributed on a sphere of r [1] and the orthogonality of $Y_\nu^\mu(\theta_q, \phi_q)$:

$$\eta_\nu^\mu(t) = p_\nu(t) * \gamma_\nu^\mu(t) \approx \sum_{q=1}^{Q} e(\boldsymbol{x_q}, t) Y_\nu^\mu(\theta_q, \phi_q),$$

$$\gamma_\nu^\mu(t) \approx a_\nu(t) * \sum_{q=1}^{Q} e(\boldsymbol{x_q}, t) Y_\nu^\mu(\theta_q, \phi_q).$$

Time-space domain adaptive algorithm

◇ Sample the signal with sampling period of T where $t = nT$.

◇ The driving signal of the ℓ^{th} loudspeaker is expressed as a vector $\boldsymbol{w}_\ell$ with the length of L_w, matrix $\boldsymbol{W} = [\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \cdots, \boldsymbol{w}_L]$.

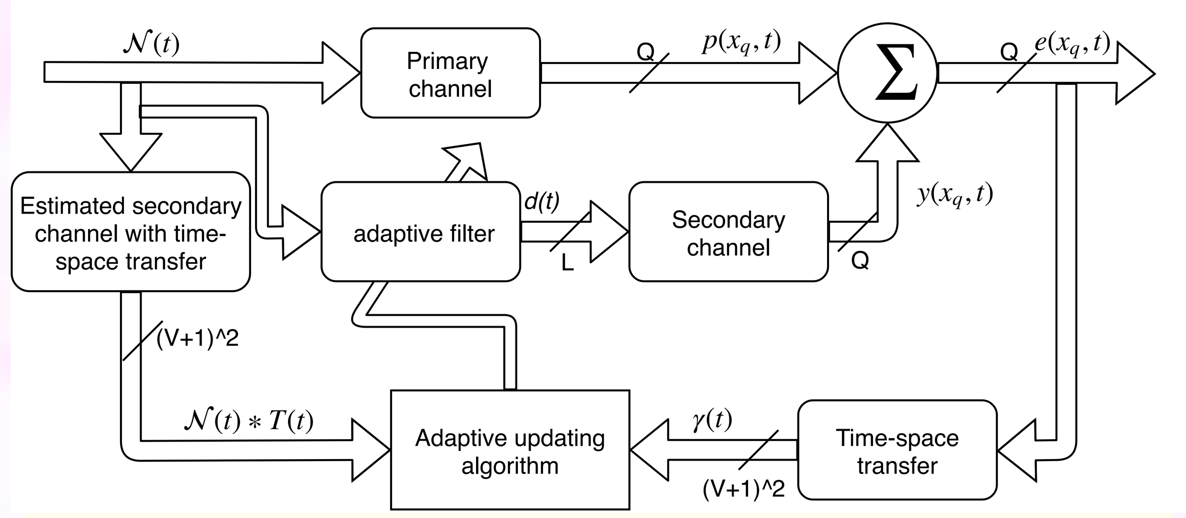


Figure 2: Time-space domain adaptive system

• Minimizing squared residual sound field coefficient error over region (MSE-R)

◇ Define the adaptive algorithm cost function

$$\xi(n) = \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \|\gamma_\nu^\mu(n)\|^2.$$

◇

$$\nabla \xi(n) = \frac{\partial \xi(n)}{\partial \boldsymbol{W}(n)} = 2 \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \gamma_\nu^\mu(n)\, \boldsymbol{S}_\nu^\mu(n),$$

where $\boldsymbol{S}_\nu^\mu(n)$ is a matrix with its ℓ^{th} column and τ^{th} row element at time-index n given by $s_\nu^\mu(\ell, \tau) = \mathcal{N}(n - \tau) * T_{\nu,\ell}^\mu(n)$.

◇ Hence, for ℓ^{th} secondary loudspeaker, the τ^{th} element of the update equation is [2]

$$\boldsymbol{w}_{\ell,\tau}(n+1) = \boldsymbol{w}_{\ell,\tau}(n) - \lambda \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \gamma_\nu^\mu(n) \times [\boldsymbol{N}(n - \tau) * T_{\nu,\ell}^\mu(n)].$$

• Minimizing squared residual sound field coefficient error over region (MSE-B)

◇ Define an alternative cost function

$$\xi(n) = \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \|\eta_\nu^\mu(n)\|^2.$$

◇

$$\nabla \xi(n) = 2 \eta_\nu^\mu(n) [\boldsymbol{S}_\nu^\mu(n) * p_\nu(n)].$$

◇ Hence, for ℓ^{th} secondary loudspeaker, the τ^{th} element of the update equation is

$$\boldsymbol{w}_{\ell,\tau}(n+1) = \boldsymbol{w}_{\ell,\tau}(n) - \lambda \sum_{\nu=0}^{V} \sum_{\mu=-\nu}^{\nu} \eta_\nu^\mu(n) \times \{[\boldsymbol{N}(n - \tau) * T_{\nu,\ell}^\mu(n)] * p_\nu(n)\}$$

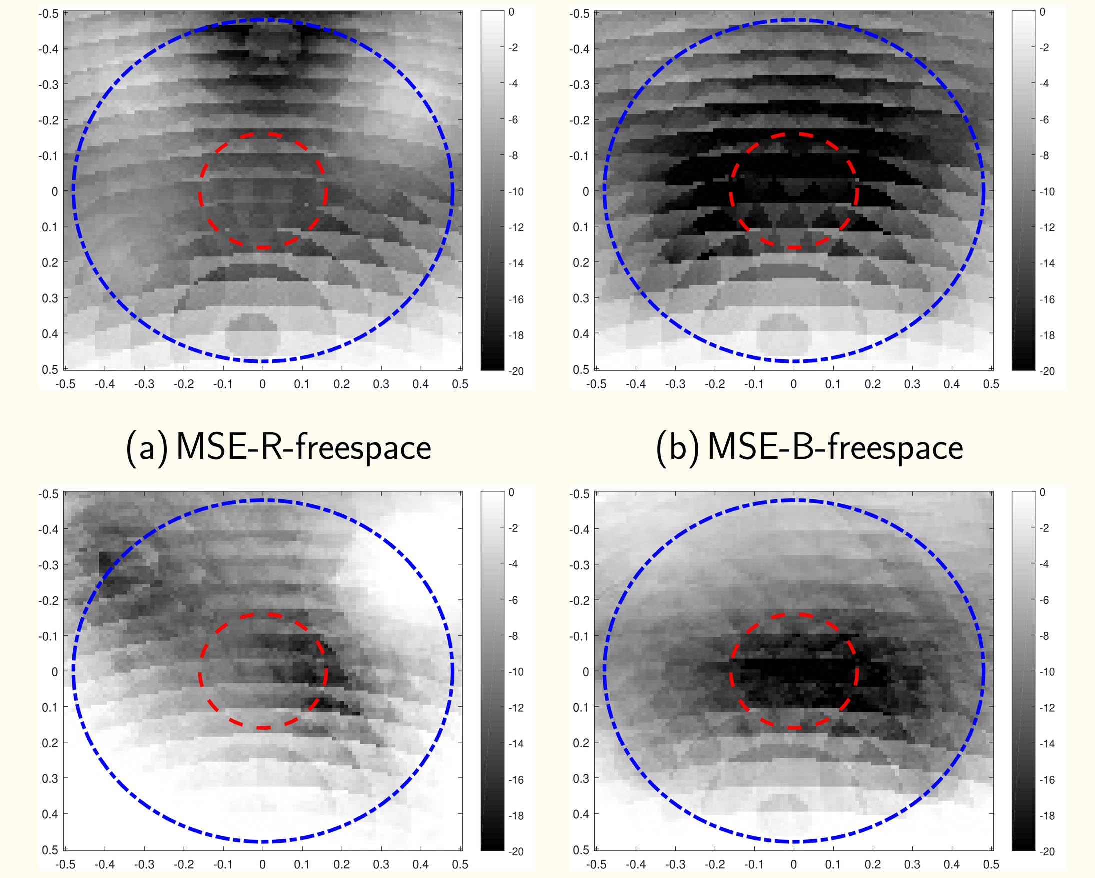
Results

◇ Noise frequency band: 20-600Hz; Radius of region: 0.16 m; $V = \lceil kr \rceil = 2$; $Q = L = (V+1)^2 = 9$;

◇ Radius of microphone array: 0.16 m; Radius of Loudspeakers array: 0.48 m;

◇ Noise source: $(2\text{m}, \pi/2, \pi/2)$ with reference microphone nearby;

◇ Room Size: 4 m × 5 m × 3 m; Reflection coefficients: [0.9, 0.7, 0.8, 0.6, 0.5 0.8].



(a) MSE-R-freespace (b) MSE-B-freespace

(c) MSE-R-room (d) MSE-B-room

Figure 3: Noise reduction performance with MSE-R, MSE-B with multi tones in freespace.

Table 1: Average performance inside the region of interest.

	Freespace			Room		
	MP	MSE-R	MSE-B	MP	MSE-R	MSE-B
Single sine wave	5.55	18.76	21.27	8.60	20.09	23.67
Multi sine wave	6.52	14.06	20.92	8.85	13.43	17.51
Random noise	16.11	16.85	21.85	5.56	10.83	9.56
Real noise	9.00	10.03	13.50	4.40	5.72	4.41

References

[1] T. D. Abhayapala and D. B. Ward, "Theory and design of high order sound field microphones using spherical microphone array," in *Proc. IEEE ICASSP*, vol. 2, pp. 1949–1952, 2002.

[2] S. M. Kuo and D. R. Morgan, "Active noise control systems: algorithms and dsp implementations," *John Wiley & Sons, Inc.*, 1995.