FRI Sensing: Sampling images along unknown curves

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Motivation

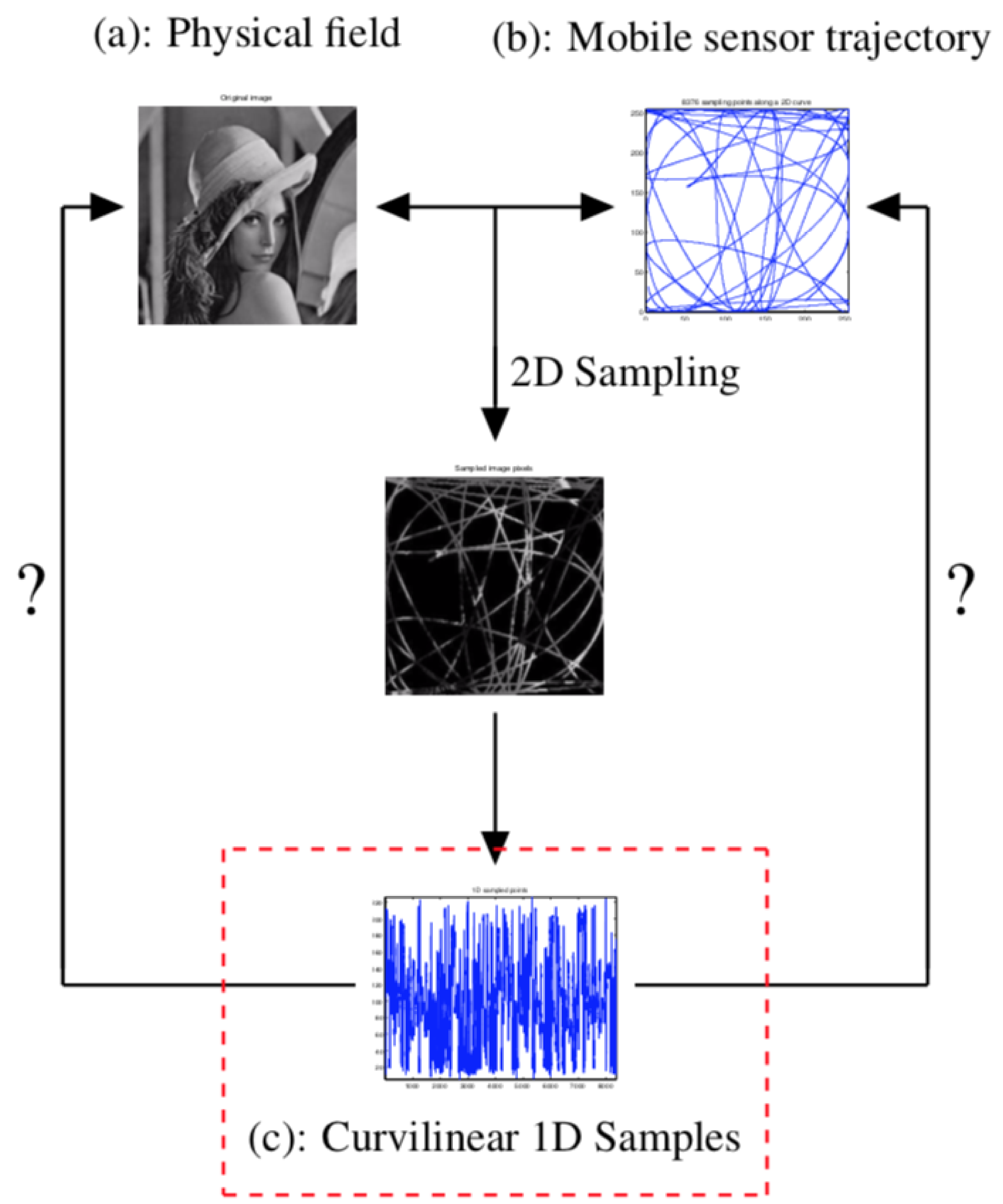
Combining the sensor data with its position information is essential in many applications. Currently, people obtain the location mainly through Global Positioning System devices.

GPS is infeasible in many scenarios due to three aspects:

1. Complex terrain environments
2. Energy limitations
3. Signal power attenuation

Problem Statement

Our goal is to extract and reveal the multi-D information hidden within a series of 1D temporal samples.



We explore the possibility of reconstructing the image and curve from a sequence of 1D signals obtained from a unique mobile sensor without any positioning devices.

Contributions

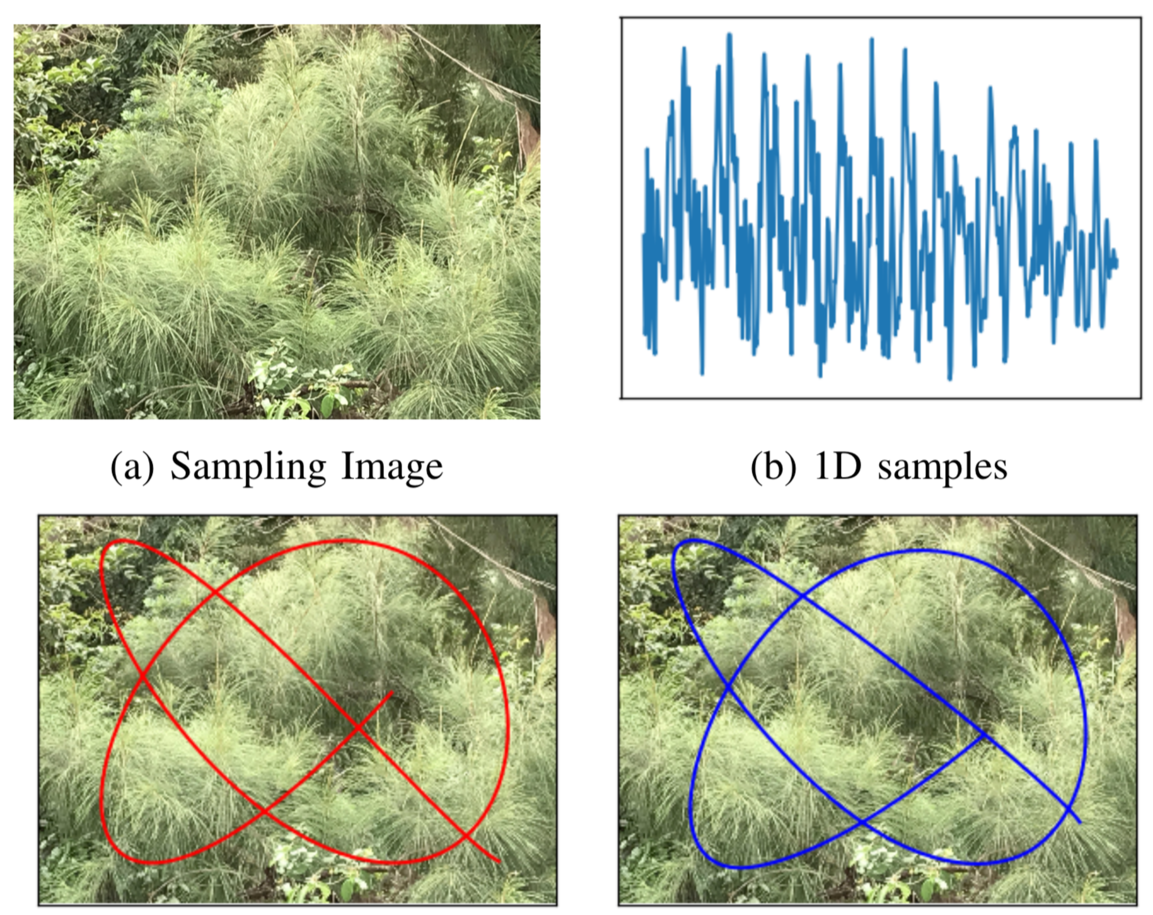
We explore a new task called FRI Sensing and propose a novel algorithm to solve it efficiently.

Our main contributions are

1. Exploration of a new task
2. Investigation of the feasible reconstruction hypothesis
3. Formulation as a 1D frequency estimation problem
4. Proposing a novel algorithm to solve the problem accurately

Results

Our algorithm makes FRI sensing possible: we reconstruct the mobile trajectory (d) from the 1D temporal signal (b) sampled from a natural image (a) along the ground truth curve (c).

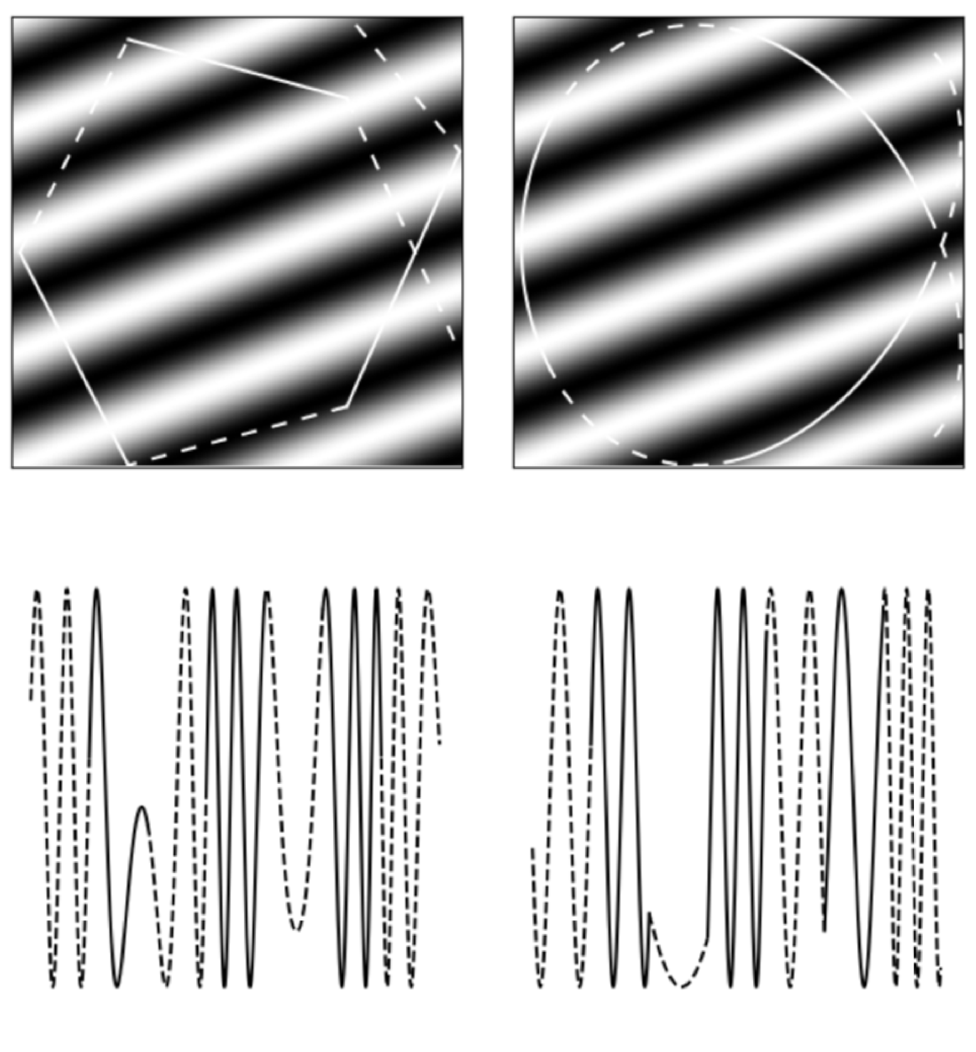


(a) Sampling Image (b) 1D samples
(c) Ground truth curve (d) Reconstructed curve

The final reconstruction accuracy is strongly influenced by

1. Curve curvature
2. Image resolution
3. Local richness of frequency contents

Method



Assume the image is

$$I(\mathbf{r}) = \sum_{k=1}^{K} C_k e^{j\mathbf{u}_k^T \mathbf{r}},\ K \geqslant 2 \tag{1}$$

And the sampling curve is

$$r(t) = \mathbf{a}_l t + \mathbf{b}_l,\ l = 1, 2, \cdots L,\ L \geqslant 2 \tag{2}$$

The key idea is that sampling a sum of 2D sinusoids uniformly along a straight line results in a sum of 1D sinusoids as shown in (a).

$$s_l(t) = \sum_{k=1}^{K} C_{l,k} e^{j\omega_{l,k} t},\ l = 1, 2, \cdots L \tag{3}$$

By appropriately arrange the sinusoids of each segment, we can have

$$\boldsymbol{\Omega} = \begin{bmatrix} \omega_{1,1} & \omega_{2,1} & \cdots & \omega_{L,1} \\ \omega_{1,2} & \omega_{2,2} & \cdots & \omega_{L,2} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{1,K} & \omega_{2,K} & \cdots & \omega_{L,K} \end{bmatrix} \tag{4}$$

$$= \underbrace{[\mathbf{u}_1, \mathbf{u}_2 \cdots \mathbf{u}_K]^T}_{K \times 2 \text{ matrix } \mathbf{U}} \cdot \underbrace{[\mathbf{a}_1, \mathbf{a}_2 \cdots \mathbf{a}_L]}_{2 \times L \text{ matrix } \mathbf{A}} \tag{5}$$

Based on the above analysis, there is at least two significant sinusoids ($K \geqslant 2$) and two straight line segments ($L \geqslant 2$) in (a) to make FRI Sensing possible.

Conclusion and Future Work

Our algorithm makes accurate and robust inverse reconstruction possible. Through this paper, we show that:

1. There is valuable and adequate 2D information hidden within the 1D samples
2. This 2D information can be recovered under the hypotheses

Future works mainly focus on:

1. The quantitative reconstruction hypothesis of the image and curve
2. The extension of the framework to non-stationary images, e.g. natural images

References

[1] Ruiming Guo and Thierry Blu. FRI Sensing: Sampling Images Along Unknown Curves In *ICASSP '19*