ALGEBRAICALLY-INITIALIZED EXPECTATION MAXIMIZATION FOR HEADER-FREE COMMUNICATION

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Introduction

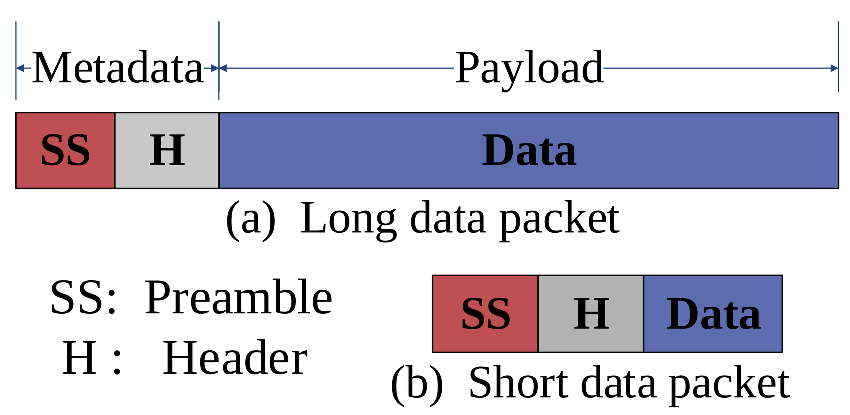

Figure 1:Example of the long and short data packet.

- It has been a trend to have low latency in wireless communication systems.
  - **Example.** URLLC, MM2M [1].
- Transmission latency may be highly suboptimal when applying traditional methods to transmit short packets [2].
- One simple solution is to exclude the header, i.e.,
  - **H** in Figure 1.
- Such an exclusion leads to the so-called *header-free communication*.

Application Context

- Figure 2 depicts an application of header-free communication in massive IoT networks.

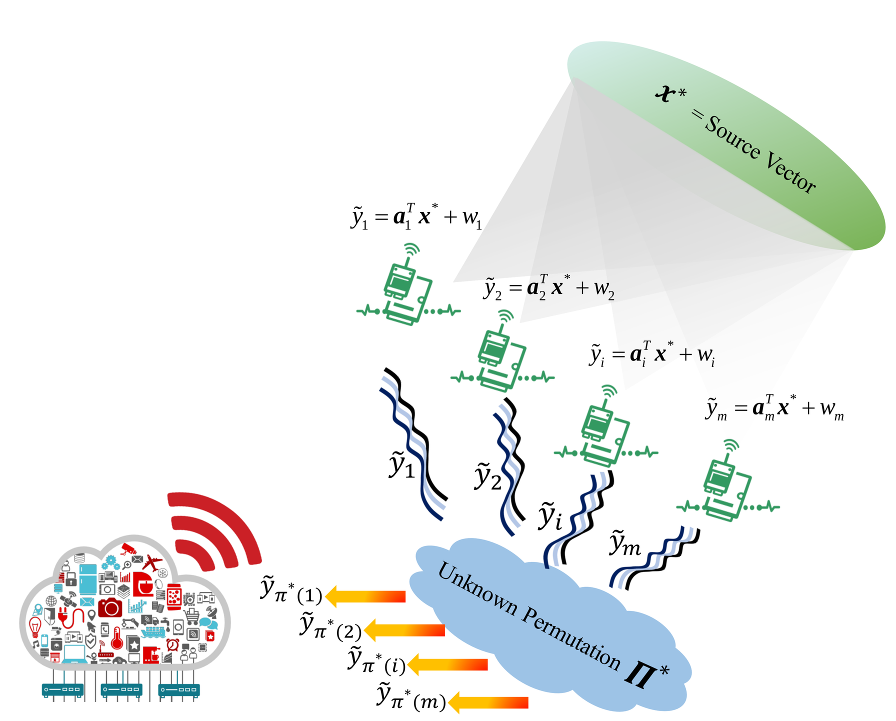

Figure 2:IoT networks with header-free communication.

Modeling

- each sensor $s_i, i \in \{1,\dots,m\} =: [m]$ has a corrupted measurement $x_j^* + w_{i,j}$ at time j for $j \in [n]$, where $w_{i,j} \in \mathbb{R}$ is noise.
- sensor s_i sends the weighted average
$$a_{i,1}(x_1 + w_{i,1}) + \cdots + a_{i,n}(x_n + w_{i,n}) \quad (1)$$
to the fusion center, where $a_{i,j}$'s are weights.
- the center receives
$$\tilde{y}_i := \boldsymbol{a}_i^\top \boldsymbol{x}^* + w_i. \quad (2)$$
- packets sent with headers:
$$\tilde{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{x}^* + \boldsymbol{w}. \text{ (Linear Regression)} \quad (3)$$
- header-free communication ($\boldsymbol{y} = \boldsymbol{\Pi}^\top \tilde{\boldsymbol{y}}$ received):
$$\boldsymbol{\Pi}\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}^* + \boldsymbol{w}. \text{ (Shuffled Linear Regression)} \quad (4)$$

Power Sum Polynomial System

- Power Sum Polynomials:
$$p_k(\boldsymbol{z}) = \Sigma_{i=1}^m z_i^k, \forall k \in [n]. \quad (5)$$
- (5) is permutation invariant:
$$p(\boldsymbol{z}) = p(z_{\pi(1)}, \dots, z_{\pi(m)}) =: p(\boldsymbol{\Pi}\boldsymbol{z}), \quad (6)$$
- hence
$$p_k(\boldsymbol{A}\boldsymbol{x}^* + \boldsymbol{w}) = p_k(\boldsymbol{\Pi}^*\boldsymbol{y}) = p_k(\boldsymbol{y}) \quad (7)$$
$$\iff p_k(\boldsymbol{A}\boldsymbol{x}^* + \boldsymbol{w}) - p_k(\boldsymbol{y}) = 0. \quad (8)$$
- Power Sum Polynomial System $\hat{\mathcal{P}}$:
$$\hat{p}_k(\boldsymbol{x}) := p_k(\boldsymbol{A}\boldsymbol{x}) - p_k(\boldsymbol{y}) = 0, k \in [n]. \quad (9)$$
- (9) is solvable by standard algorithms in computational algebraic geometry, yielding finitely many solutions [3].

Contributions

- a working algorithm for the noisy shuffled linear regression problem for small dimensions (n).
  - at a complexity linear in the number m of sensors.
- suitable for massive IoT sensor networks with header-free communication.

The AIEM Algorithm

Algebraic Initialization.

1. $\{\hat{\boldsymbol{x}}_i\}_{i=1}^L \leftarrow$ solutions of $\hat{\mathcal{P}}$ (9);
2. take the real parts of the roots:
$$\{(\hat{\boldsymbol{x}}_i)_{\mathbb{R}}\}_{i=1}^L \leftarrow \{a : a + ib \in \{\hat{\boldsymbol{x}}_i\}_{i=1}^L\}. \quad (10)$$
3. extract the most *suitable* one:
$$\hat{\boldsymbol{x}}_0 \leftarrow \operatorname*{argmin}_{i \in [L]} \{\min_{\boldsymbol{\Pi}} \|\boldsymbol{\Pi}\boldsymbol{y} - \boldsymbol{A}(\hat{\boldsymbol{x}}_i)_{\mathbb{R}}\|_2\}. \quad (11)$$

Expectation Maximization.

- iteratively solve (12) via sorting, and
$$\boldsymbol{\Pi}_t \leftarrow \operatorname*{argmin}_{\boldsymbol{\Pi}} \|\boldsymbol{\Pi}\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}_{t-1}\|_2 \quad (12)$$
- solve (13) via least-squares until convergence.
$$\boldsymbol{x}_t \leftarrow \operatorname*{argmin}_{\boldsymbol{x} \in \mathbb{R}^n} \|\boldsymbol{\Pi}_t\boldsymbol{y} - \boldsymbol{A}\boldsymbol{x}\|_2 \quad (13)$$
- return the final estimate, say $\boldsymbol{x}_T =: \hat{\boldsymbol{x}}$, for $\boldsymbol{x}^*$.

Results

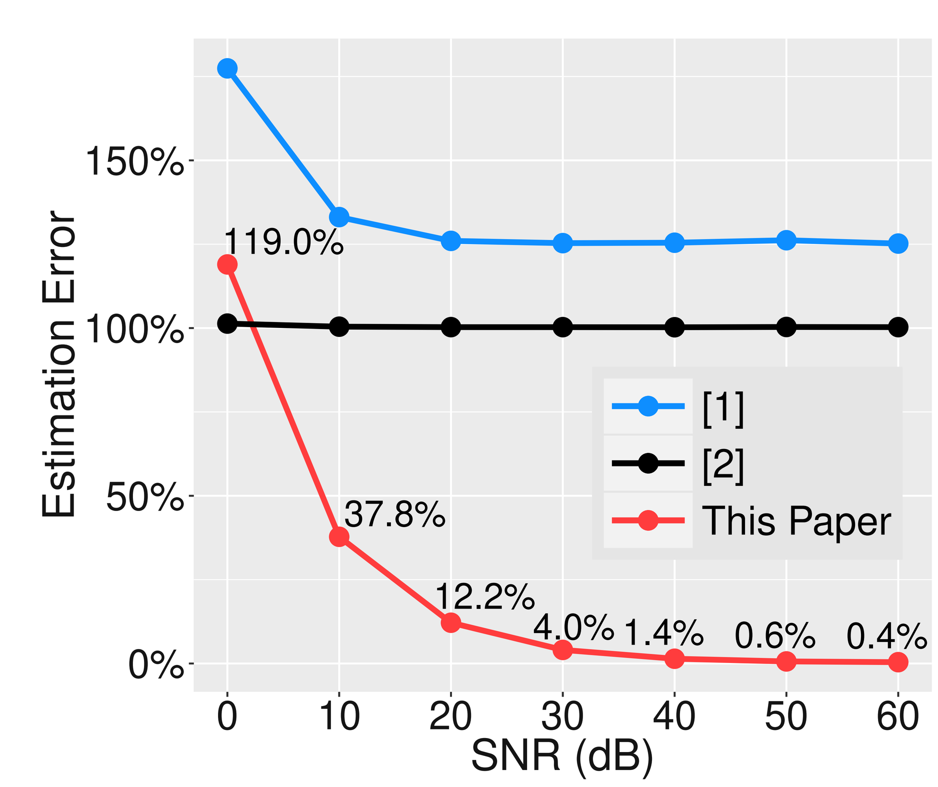

Figure 3:Estimation error $100 \times \frac{\|\boldsymbol{x}^* - \hat{\boldsymbol{x}}\|_2}{\|\boldsymbol{x}^*\|_2}\%$.

Future Work

- Shuffle Linear Regression in high dimension
- Extension to other applications, such as
  - multi-target tracking
  - genome-assembly

References

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