# TOA SOURCE NODE SELF-POSITIONING WITH UNKNOWN CLOCK SKEW IN WIRELESS SENSOR NETWORKS


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## PROBLEMS

- Time-of-arrival (TOA) source node self-positioning with unknown clock skew in wireless sensor networks.
- Localization in the presence of sensor position errors.

## CONTRIBUTIONS

- Asynchronous TOA localization [3] is extended to a more practical case: the clock skew of source node is unknown.
- An SDP based algorithm is developed to solve this problem, which has better performance than the TDOA scheme described in [1].
- The SDP algorithm is tuned to the case of anchor nodes uncertainties, witch are not considered in the TDOA scheme in [1].

## MEASUREMENTS MODEL & PROPOSED METHODS

### 1 Measurements Model

At a common time instant $t_0$, the anchor nodes transmit their signals to the source node. The local time of the source node, $\tau$, is related to the reference time (i.e., synchronized anchor nodes' time), $t$, as

$$\tau = \omega t + \theta \tag{1}$$

where $\omega$ is the clock skew of source node and $\theta$ is the clock offset of source node.
In the line-of-sight propagation conditions, the received TOA measurements in the source node can be expressed as [1, 2]

$$\tau_i = \omega t_0 + \theta + w(\frac{d_i}{c} + n_i),\ i = 1, 2, \cdots, M. \tag{2}$$

where $\tau_i$ is the TOA measurement from the $i$th anchor node, $t_0$ is the unknown transmission time at the anchor nodes, $d_i$ is the distance between the source node and the $i$th anchor node, and $n_i$ is the measurement noise.
The time measurements are converted to range measurements as

$$r_i = \omega z + \omega(d_i + e_i),\ i = 1, 2, \cdots, M. \tag{3}$$

where $r_i = \tau_i c$, $z = (t_0 + \frac{\theta}{\omega})c$, and $e_i = n_i c$.
The MLE problem:

$$\min_{\mathbf{u},\omega,z} \sum_{i=1}^{M} \frac{(\frac{r_i}{\omega} - \|\mathbf{u} - \mathbf{s}_i\| - z)^2}{\sigma_i^2} \tag{4}$$

### 2 Localization Algorithm

Next, an SDP based method is proposed to jointly estimate the position and clock skew of the source node.
First, (4) can be expressed as

$$\min_{\mathbf{u},a,z,\mathbf{d}} (a\mathbf{r} - \mathbf{d} - z\mathbf{1}_M)^T \mathbf{Q}^{-1} (a\mathbf{r} - \mathbf{d} - z\mathbf{1}_M) \tag{5a}$$
$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\|,\ i = 1, 2, \cdots, M. \tag{5b}$$

where $a = \frac{1}{\omega}$, $\mathbf{r} = [r_1, r_2, \cdots, r_M]^T$, $\mathbf{d} = [d_1, d_2, \cdots, d_M]^T$. Let $\mathbf{A} = [\mathbf{r}, -\mathbf{1}_M]$, and $\mathbf{y} = [a, z]^T$. Then (5) can be recast as

$$\min_{\mathbf{u},\mathbf{y},\mathbf{d}} (\mathbf{A}\mathbf{y} - \mathbf{d})^T \mathbf{Q}^{-1} (\mathbf{A}\mathbf{y} - \mathbf{d}) \tag{6a}$$
$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\| \tag{6b}$$

Let the gradient of the objective function in (6) with respect to $\mathbf{y}$ equal zero:

$$-2\mathbf{A}^T \mathbf{Q}^{-1}(\mathbf{A}\mathbf{y} - \mathbf{d}) = 0, \tag{7}$$

which results in

$$\mathbf{y} = (\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{d}. \tag{8}$$

Substituting (8) into (6) yields

$$\min_{\mathbf{u},\mathbf{d}} (\mathbf{H}\mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{H}\mathbf{d}) \tag{9a}$$
$$\text{s.t. } d_i = \|\mathbf{u} - \mathbf{s}_i\| \tag{9b}$$

where $\mathbf{H} = \mathbf{A}(\mathbf{A}^T\mathbf{Q}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Q}^{-1} - \mathbf{I}_M$.
By letting $\mathbf{D} = \mathbf{d}\mathbf{d}^T$, (9a) can be rewritten as

$$(\mathbf{H}\mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{H}\mathbf{d}) = \text{tr}(\mathbf{d}\mathbf{d}^T\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) = \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}). \tag{10}$$

Further, by letting $y_s = \mathbf{u}^T\mathbf{u}$, from (9b), we have

$$\mathbf{D}_{i,i} = d_i^2 = \|\mathbf{u} - \mathbf{s}_i\|^2 = \mathbf{y}_s - 2\mathbf{u}^T\mathbf{s}_i + \mathbf{s}_i^T\mathbf{s}_i,\ i = 1, 2, \ldots, M \tag{11}$$

and

$$\begin{aligned}\mathbf{D}_{i,j} &= d_i d_j = \|\mathbf{u} - \mathbf{s}_i\|\,\|\mathbf{u} - \mathbf{s}_j\| \geq |(\mathbf{u} - \mathbf{s}_i)^T(\mathbf{u} - \mathbf{s}_j)| \\ &= |\mathbf{y}_s - \mathbf{u}^T(\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T\mathbf{s}_j|,\ 1 \leq i < j \leq M.\end{aligned} \tag{12}$$

An SDP-based localization algorithm is finally expressed as

$$\min_{\mathbf{u},y_s,\mathbf{d},\mathbf{D}} \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) + \eta\text{tr}(\mathbf{D}) \tag{13a}$$
$$\text{s.t. } \mathbf{D}_{i,i} = \mathbf{y}_s - 2\mathbf{u}^T\mathbf{s}_i + \mathbf{s}_i^T\mathbf{s}_i, \tag{13b}$$
$$\|\mathbf{u} - \mathbf{s}_i\| \leq d_i, \tag{13c}$$
$$\mathbf{D}_{i,j} \geq |\mathbf{y}_s - \mathbf{u}^T(\mathbf{s}_i + \mathbf{s}_j) + \mathbf{s}_i^T\mathbf{s}_j|, \tag{13d}$$
$$\begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0} \tag{13e}$$
$$\begin{bmatrix} \mathbf{I}_m & \mathbf{u} \\ \mathbf{u}^T & \mathbf{y}_s \end{bmatrix} \succeq 0 \tag{13f}$$

The next step would be to calculate $\mathbf{y}$, i.e., (8) using the value of $\hat{\mathbf{u}}$ obtained. The clock skew is estimated from $\mathbf{y}$ as

$$\hat{\omega} = \frac{1}{\mathbf{y}(1)}. \tag{14}$$

### 3 Localization Algorithm With Position Uncertainties

When the locations of the anchor nodes are not precise, which is mostly the case in practice, the sensor positions with errors can be expressed as [3]

$$\mathbf{s}_i = \mathbf{s}_i^0 + \boldsymbol{\beta}_i,\ i = 1, 2, \cdots, M. \tag{15}$$

where $\mathbf{s}_i^0$ is the actual but unknown sensor position and $\boldsymbol{\beta}_i$ represents the position error, which is assumed to be a zero-mean Gaussian vector with a known covariance $\delta_i^2\mathbf{I}_2$. The MLE problem is expressed as

$$\min_{\mathbf{u},\omega,z,\mathbf{s}_i^0} \sum_{i=1}^{M} \frac{(\frac{r_i}{w} - \|\mathbf{u} - \mathbf{s}_i^0\| - z)^2}{\sigma_i^2} + \sum_{i=1}^{M} \frac{\|\mathbf{s}_i - \mathbf{s}_i^0\|^2}{\delta_i^2} \tag{16}$$

where $z$ and $\mathbf{s}_i^0$ are the nuisance parameters.
Eq. (16) can be expressed as

$$\min_{\mathbf{X},\mathbf{y},\mathbf{d}} (\mathbf{A}\mathbf{y} - \mathbf{d})^T\mathbf{Q}^{-1}(\mathbf{A}\mathbf{y} - \mathbf{d}) + \left\|(\mathbf{B} - \mathbf{X}(:, 2: M+1))\mathbf{W}^{\frac{1}{2}}\right\|_F^2 \tag{17a}$$
$$\text{s.t. } d_i = \|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\| \tag{17b}$$

where $\mathbf{X} = [\mathbf{u}, \mathbf{s}_1^0, \mathbf{s}_2^0, \ldots, \mathbf{s}_M^0]$, $\mathbf{B} = [\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M]$, $\mathbf{W} = \text{diag}\left([\delta_1^{-2}, \delta_2^{-2}, \cdots, \delta_M^{-2}]\right)$.
The SDP-based localization algorithm with anchor node position errors are expressed as

$$\begin{aligned}&\min_{\mathbf{d},\mathbf{D},\mathbf{X},\mathbf{Y}} \text{tr}(\mathbf{D}\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H}) - 2tr(\mathbf{W}\mathbf{B}^T\mathbf{X}(:, 2: M+1)) \\ &+ \text{tr}(\mathbf{W}\mathbf{Y}(2: M+1, 2: M+1)) + \eta tr(\mathbf{D})\end{aligned} \tag{18a}$$
$$\text{s.t. } \mathbf{D}_{i,i} = \mathbf{Y}(1,1) - 2\mathbf{Y}(1, i+1) + \mathbf{Y}(i+1, i+1), \tag{18b}$$
$$\|\mathbf{X}(:, 1) - \mathbf{X}(:, i+1)\| \leq d_i, \tag{18c}$$
$$\begin{aligned}&\mathbf{D}_{i,j} \geq |\mathbf{Y}(1,1) - \mathbf{Y}(1, i+1) - \mathbf{Y}(1, j+1) \\ &+ \mathbf{Y}(i+1, j+1)|,\ 1 \leq i < j \leq M.\end{aligned} \tag{18d}$$
$$\begin{bmatrix} 1 & \mathbf{d}^T \\ \mathbf{d} & \mathbf{D} \end{bmatrix} \succeq \mathbf{0} \tag{18e}$$
$$\begin{bmatrix} \mathbf{I}_m & \mathbf{X} \\ \mathbf{X}^T & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0}. \tag{18f}$$

## SIMULATIONS

There are six anchor nodes, and their true positions are $[0, 0]^T m$, $[400, 0]^T m$, $[800, 0]^T m$, $[800, 800]^T m$, $[400, 800]^T m$, $[0, 800]^T m$. $t_0$, $\omega$, and $\theta$ are uniformly distributed within $[10, 40]ns$, $[0.995, 1.005]$, and $[1, 10]ns$, respectively. Both TOA measurement errors and anchor node position errors are assumed to be independent and identically distributed, i.e., $\sigma_i^2 = \sigma^2, \delta_i^2 = \delta^2$, and $\eta$ is set to $10^{-7}$, $10^{-6}$, $10^{-5}$, $10^{-4}$, $10^{-3}$ for the proposed algorithms and the algorithms in [3].

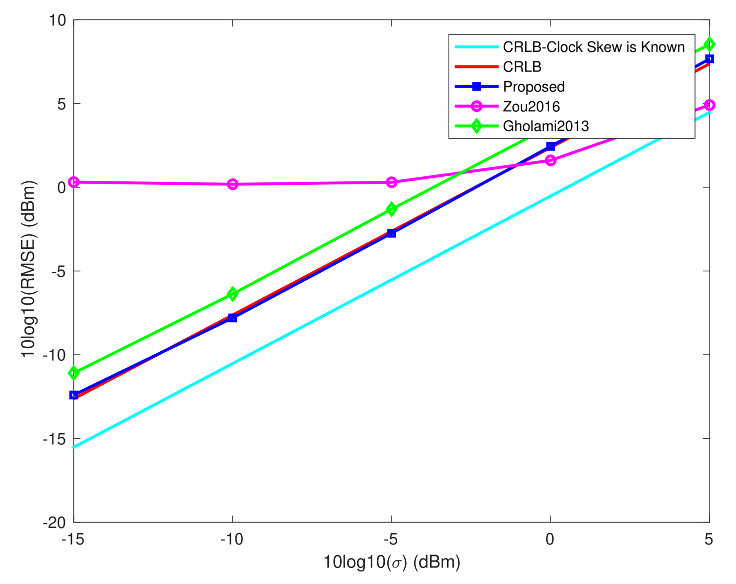


**Figure 1:** No sensor position errors, RMSE of position vs. $\sigma$, $\mathbf{u} = [200, 100]^T m$.

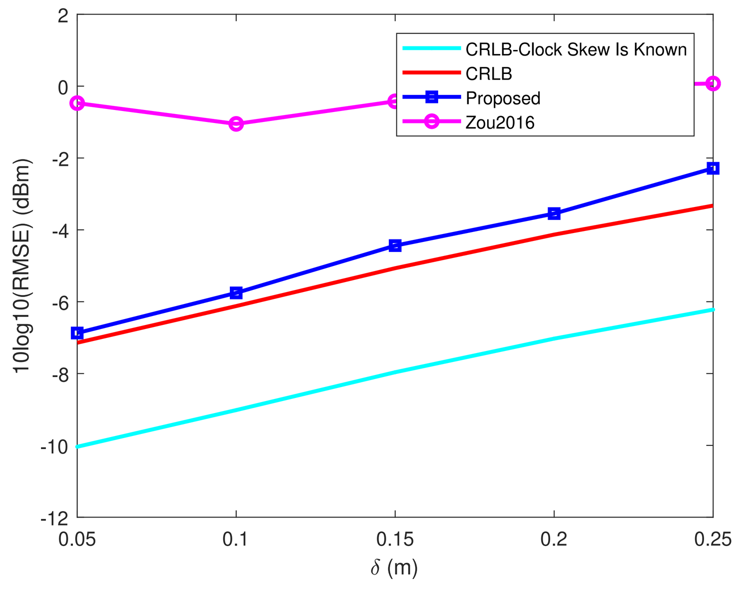


**Figure 2:** Exist sensor position errors, RMSE of position vs. $\delta$, $\mathbf{u} = [200, 100]^T m$, $\sigma = 0.1m$.

## REFERENCES


[1] M.R. Gholami, S. Gezici, and E.G. Strom, "TDOA based positioning in the presence of unknown clock skew," *IEEE Transaction on Communications*, vol. 61, no. 6, pp. 2522-2534, June 2013.

[2] Y, Zou, H, Liu and Q. Wan, "Joint synchronization and localization in wireless sensor networks using semidefinite programming," *IEEE Internet of Things Journal*, vol. 5, no. 1, pp. 199–205, Feb. 2018.

[3] Y. Zou, and Q. Wan, "Asynchronous Time-of-Arrival-Based Source Localization with Sensor Position Uncertainties," *IEEE Communications Letters*, vol. 20, no. 9, pp. 1860-1863, Sep. 2016.


## CONTACT INFORMATION


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