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GRIDLESS SUPER-RESOLUTION DOA ESTIMATION WITH UNKNOWN MUTUAL COUPLING

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Introduction

Motivation

The resolution of two or more closely spaced sources with high precision from very few snapshots and the mutual coupling effect between the antenna elements are two significant issues involved in the DOA estimation method.

Main Goal

Tackling the super-resolution and the unknown mutual coupling problems simultaneously in fewer snapshots and lower SNR condition.

Implement

System Model

Consider a ULA shown in Fig. 1, the received model for multiple measurement vectors (MMVs) under T subsequent observation is

$$\mathbf{X}=\mathbf{CAS}+\mathbf{N}\in\mathbb{C}^{M\times T}$$

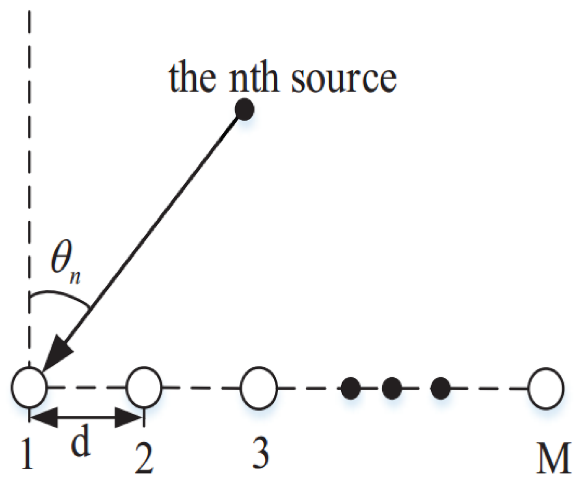

Fig. 1 Uniform linear array

where

$$\mathbf{C}=\begin{bmatrix} 1 & c_1 & \cdots & c_{P-1} & & & & & \\ c_1 & 1 & c_1 & \cdots & c_{P-1} & & \mathbf{0} & & \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & & & \\ c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} & & \\ & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} \\ & \mathbf{0} & & \ddots & \vdots & \ddots & \ddots & \ddots & \vdots \\ & & & & c_{P-1} & \cdots & c_1 & 1 & c_1 \\ & & & & & c_{P-1} & \cdots & c_1 & 1 \end{bmatrix}_{M\times M}$$

(2) → mutual coupling matrix (MCM)

$\mathbf{A}=[\mathbf{a}(f_1),\mathbf{a}(f_2),\cdots,\mathbf{a}(f_N)]$ (3) → array manifold

$\mathbf{a}(f_n)=[1,\beta(f_n),\cdots,\beta(f_n)^{M-1}]^T$ (4) → steering vector

$$\beta(f_n)=\exp(j2\pi f_n)\ \text{when}\ d=\lambda/2,\ f_n=\frac{1+\sin(\theta_n)}{2}\in[0,1],\quad \theta_n\in[-90^o,90^o]$$

Solution

By analyzing the structure of mutual coupling matrix $\mathbf{C}$ in equation (2), we note that the center part of $\mathbf{C}$ is cyclic. Thus we construct a selection matrix $\mathbf{F}=[\mathbf{0}_{[M\text{-}2(P\text{-}1)]\times(P\text{-}1)}\mathbf{I}_{M\text{-}2(P\text{-}1)}\mathbf{0}_{[M\text{-}2(P\text{-}1)]\times(P\text{-}1)}]$ such that

$$\bar{\mathbf{X}}=\mathbf{FX}=\mathbf{FCAS}+\mathbf{FN}=\bar{\mathbf{C}}\mathbf{AS}+\mathbf{FN}=\bar{\mathbf{A}}\bar{\mathbf{S}}+\mathbf{FN}\in\mathbb{C}^{[M-2(P-1)]\times T}\quad(6)$$

$$\bar{\mathbf{C}}\mathbf{a}(f)=\mathrm{H}(f)\bar{\mathbf{a}}(f)$$

$$\mathrm{H}(f)=\sum_{l=1-P}^{P-1}c_{|l|}\beta(f)^{l+P-1}$$

where

$$\bar{\mathbf{A}}=[\bar{\mathbf{a}}(f_1),\bar{\mathbf{a}}(f_2),\cdots,\bar{\mathbf{a}}(f_N)]\quad(6)$$

$$\bar{\mathbf{a}}(f)=[1,\beta(f),\beta(f)^2,\cdots,\beta(f)^{M-2P+1}]^T\quad(7)$$

$$\bar{\mathbf{C}}=\begin{bmatrix} c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} & 0 & 0 \\ 0 & \ddots & \vdots & \ddots & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & c_{P-1} & \cdots & c_1 & 1 & c_1 & \cdots & c_{P-1} \end{bmatrix}_{[M-2(P-1)]\times M}\quad(8)$$

The noise-free measurement matrix of $\bar{\mathbf{X}}$ is

$$\bar{\mathbf{X}}_0=\bar{\mathbf{A}}\bar{\mathbf{S}}=\sum_{i=1}^{N}\bar{\mathbf{a}}(f_i)\bar{\mathbf{s}}_i^T=\sum_{i=1}^{N}r_i\bar{\mathbf{a}}(f_i)\mathbf{\Phi}_i\quad(9)$$

where $\bar{\mathbf{s}}_i^T$ is the ith row of $\bar{\mathbf{S}}$, $r_i=\|\bar{\mathbf{s}}_i^T\|_2>0$ and $\mathbf{\Phi}_i=r_i^{-1}\bar{\mathbf{s}}_i^T$ with $\|\mathbf{\Phi}_i\|_2=1$.

Then the set of atoms is

$$\mathcal{A}=\{\hat{\mathbf{a}}_M(f,\mathbf{\Phi})=\bar{\mathbf{a}}(f)\mathbf{\Phi}\mid f\in[0,1],\mathbf{\Phi}\in\mathbb{C}^{1\times T},\|\mathbf{\Phi}\|_2=1\}\quad(10)$$

and the atomic norm of $\bar{\mathbf{X}}_0$ is

$$\|\bar{\mathbf{X}}_0\|_{\mathcal{A}}=\inf\{\sum_k r_k\mid\sum_{i=1}^{K}r_k\hat{\mathbf{a}}_M(f_i,\mathbf{\Phi}_i),r_k\geq 0\}\quad(11)$$

The corresponding semidefinite programming (SDP) formulation of (11) is

$$\min_{\mathbf{W},\mathbf{u}}\ \frac{1}{2\sqrt{M}}(tr(\mathbf{W})+tr(\mathbf{T}(\mathbf{u})))\quad \text{s.t.}\begin{bmatrix}\mathbf{W} & \bar{\mathbf{X}}_0^H\\ \bar{\mathbf{X}}_0 & \mathbf{T}(\mathbf{u})\end{bmatrix}\succeq 0\quad(12)$$

where $\mathbf{W}\in\mathbb{C}^{T\times T}$, $\mathbf{T}(\mathbf{u})$ is a Hermitian Toeplitz matrix with $\mathbf{u}\in\mathbb{C}^{M-2(P-1)}$. According to the Caratheodory theorem, $\mathbf{T}(\mathbf{u})$ is decomposed as

$$\mathbf{T}(\mathbf{u})=\bar{\mathbf{A}}\mathbf{Z}\bar{\mathbf{A}}^H=\sum_{i=1}^{N}z_i\bar{\mathbf{a}}(f_i)\bar{\mathbf{a}}^H(f_i)\quad(13)$$

where $\mathbf{Z}=diag(\mathbf{z}),\mathbf{z}=[z_1,z_2,\cdots,z_N]^T$.

Based on (5) and (12), the final SDP problem for MMVs model is

$$\min_{\mathbf{X},\mathbf{W},\mathbf{u}}\ \frac{1}{2}\|\mathbf{X}-\bar{\mathbf{X}}\|_F^2+\frac{\tau}{2\sqrt{M}}(tr(\mathbf{W})+tr(\mathbf{T}(\mathbf{u})))\quad \text{s.t.}\begin{bmatrix}\mathbf{W} & \mathbf{X}^H\\ \mathbf{X} & \mathbf{T}(\mathbf{u})\end{bmatrix}\succeq 0\ .\quad(14)$$

After obtaining the optimal solution $\mathbf{T}(\mathbf{u})$, the standard ESPRIT method is utilized to estimate the DOAs.

Simulation

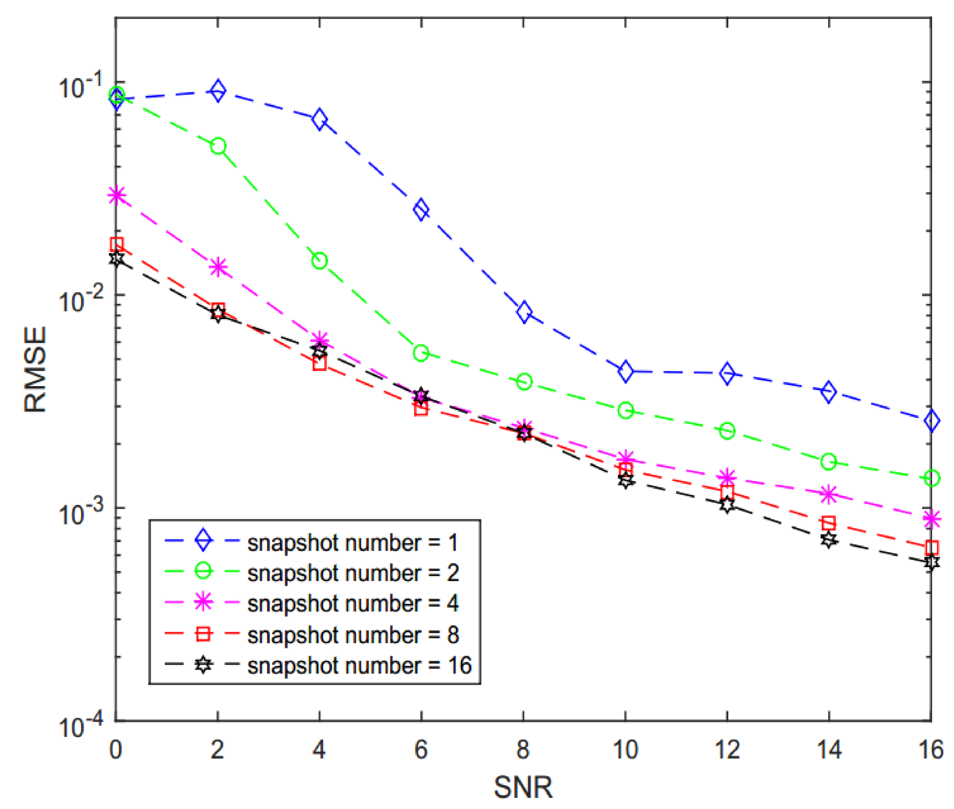

Fig. 2 The RMSEs of SMV and MMVs.

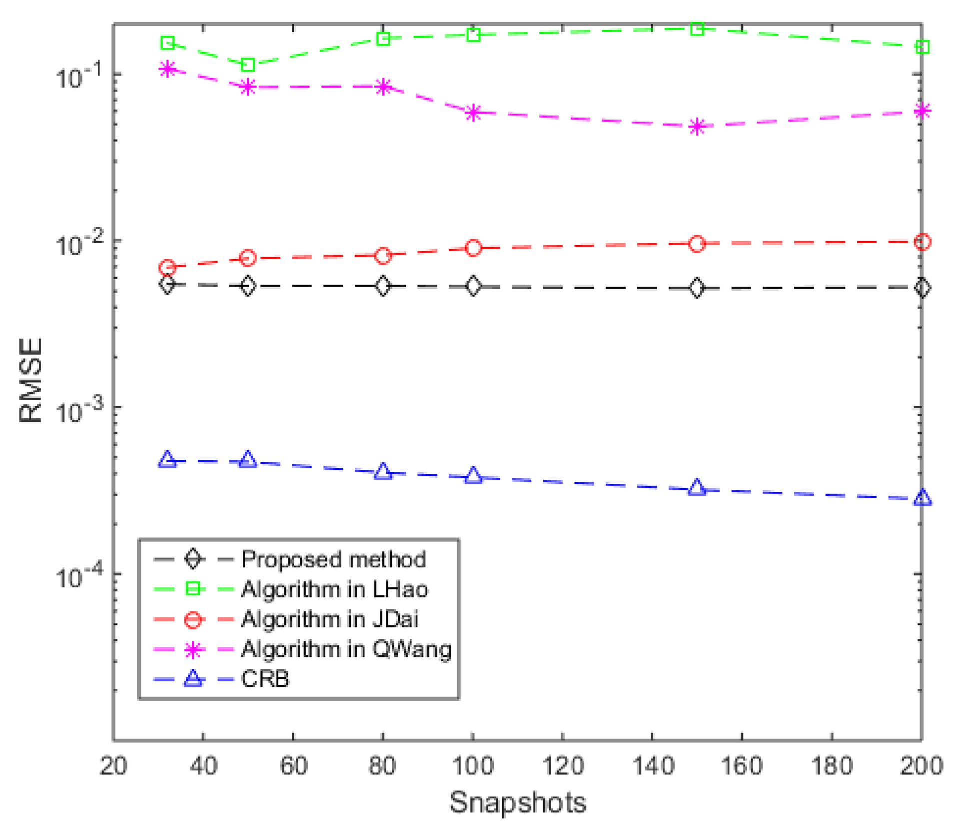

Fig. 3 The RMSEs versus snapshots.

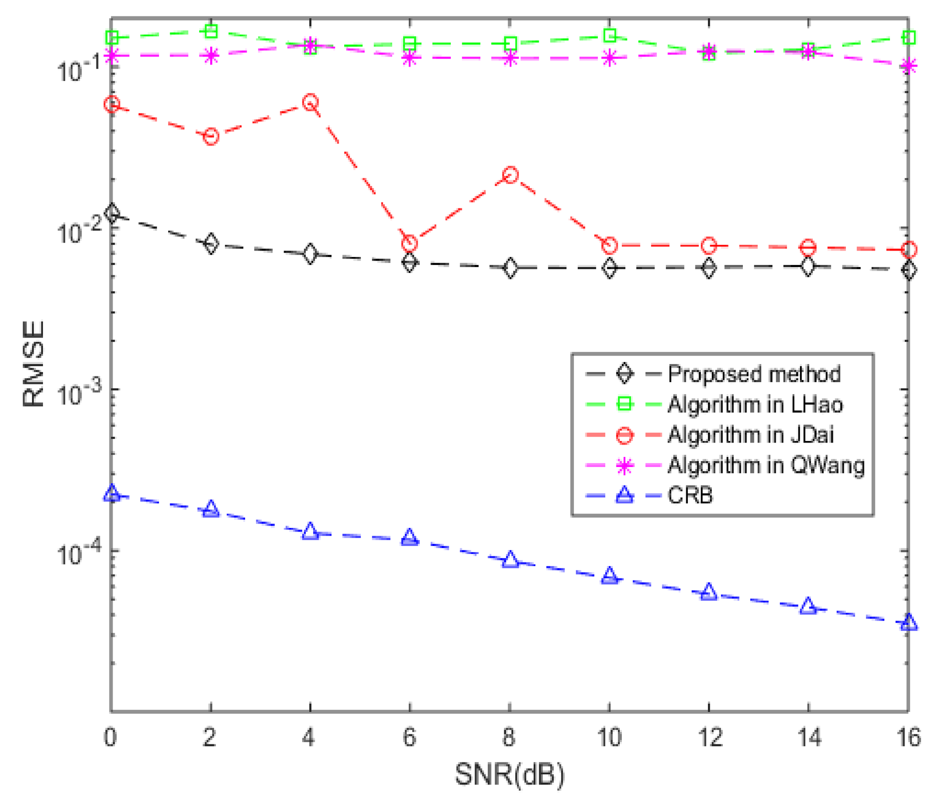

Fig. 4 The RMSEs versus SNR.

Summary

Algorithm

Algorithm
Step1: A new clean steering vector is obtained based on the banded symmetric Toeplitz structure of MCM; Step2: Atomic norms associated with the array structure are generated, which achieves super-resolution estimation problem by directly working on the continuous parameter domain; Step3: A semidefinite programming (SDP) method is derived to solve this atomic norm minimization problem; Step4: The standard ESPRIT method is applied to T(u) to estimate the DOAs.

Results

1) An gridless DOA estimation method simultaneously considering about the super-resolution problem and the mutual coupling effect is proposed;
2) The proposed method achieves the super-resolution and demonstrates a superior performance over existing methods in the case of low SNR and fewer snapshots.

Future Study

In the future study, the property (such as the noncircularity) of the signal can be exploited to enhance the performance of the proposed method.

References

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