Low-complexity Detection and Performance Analysis For Decode-and-Forward Relay Networks

Yuxin Lu and Wai Ho Mow

SENG School of Engineering

香港科技大學 THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Abstract

- We propose two new detectors with near-optimal error performance. One has linear complexity for general modulations and the other has constant complexity for PAM signals
- We design an algorithm for computing the upper bound on the symbol error rate (SER) of the new detectors

Motivation

The state-of-the-art detectors, such as the piece-wise linear detector (PLD), achieve near-optimal SER performance with linear complexity. **However, the signal structures are not fully exploited to further reduce the detection complexity**

System Model

- 3-node DF relay network with one source (S), one relay (R) and one destination (D)
- **D is assumed to have the instantaneous CSI of the S-D and R-D links and only the statistical CSI of the S-R link**
- In the first time slot, S broadcasts its signals. In the next slot, R detects and transmits to D
- Received signals are

$$y_{\mathrm{X,Y}} = h_{\mathrm{X,Y}} x_{\mathrm{X}} + n_{\mathrm{X,Y}},$$

where $\mathrm{X,Y} \in \{\mathrm{s,d,r}\}$ denote the transmitting and receiving nodes, respectively

Relay Detection

- **Closed-form solution**

$$x_r = 2\left\lfloor \frac{\Re\{y_{s,r}^* h_{s,r}\}}{2|h_{s,r}|^2} + \frac{M+1}{2} \right\rceil_M - (M+1),$$

where $\lfloor n \rceil_M \triangleq \arg\min_{m\in\mathbb{Z}, m\in[1,M]} |n-m|$ with $\mathbb{Z}$ as the integer set

- The average SER is denoted as ε

Proposed Detectors at the Destination

- Almost MLD (AMLD) at the destination: $\max_{x_s\in\mathcal{X}} \Pr(y_{s,d}|x_s) \sum_{x_r\in\mathcal{X}} \Pr(x_r|x_s)\Pr(y_{r,d}|x_r)$, where $\Pr(x_r|x_s) = 1-\varepsilon$ if $x_r = x_s$; $\Pr(x_r|x_s) = \varepsilon/(M-1)$ otherwise
- Apply **max-sum approximation** to AMLD:

$$\max_{x_s\in\mathcal{X}} \left\{ \Pr(y_{s,d}|x_s) \max\left\{ (1-\varepsilon)\Pr(y_{r,d}|x_s), \max_{x_r\in\mathcal{X}, x_r\neq x_s} \varepsilon/(M-1)\Pr(y_{r,d}|x_r) \right\}\right\}$$

- Further obtain the **proposed linear-complexity MAMLD and constant-complexity NMLD** as

$$\hat{x}_s = \arg\min_{x_s\in\mathcal{X}} \left\{ \frac{|y_{s,d} - h_{s,d}x_s|^2}{N_{s,d}} + \min\left\{ \frac{|y_{r,d} - h_{r,d}x_s|^2}{N_{r,d}}, \min_{x_r\in\mathcal{X}} \frac{|y_{r,d} - h_{r,d}x_r|^2}{N_{r,d}} + \eta \right\}\right\} \quad \textbf{MAMLD}$$

$$= \arg\min \left\{ \min_{\mathbf{x}\in\mathcal{X}^2, x_s = x_r} f(\mathbf{x}), \min_{\mathbf{x}\in\mathcal{X}^2} f(\mathbf{x}) + \eta \right\} \quad \textbf{NMLD}$$

where $f(\mathbf{x}) \triangleq \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$, $\mathbf{y} = \left[\frac{\mathrm{Re}\{y_{s,d}^* h_{s,d}\}}{|h_{s,d}|}, \frac{\mathrm{Re}\{y_{r,d}^* h_{r,d}\}}{|h_{r,d}|}\right]^T$, $\mathbf{H} = [|h_{s,d}|, 0; 0, |h_{r,d}|]$, $\mathbf{x} = [x_s, x_r]^T$, $\mathbf{y} \sim \mathcal{N}(\mathbf{Hx}, \mathbf{\Sigma})$, and $\eta \triangleq \log\frac{(1-\varepsilon)(M-1)}{\varepsilon}$

- The detection is essentially a **two-dimensional lattice decoding problem** with $\mathbf{Hx}$ as the lattice points, and $\mathbf{y}$ as the observations

Designed Algorithm

Algorithm 1: A high-level description

Input: $h_{s,d}, h_{r,d}, \eta, M, r_d, r_{nd}, \mathcal{W}_i, i\in[1,M]$, $\mathbf{v}_{k,j}, k\in[1,M-1], j\in[1,M-k]$

Output: $\mathcal{V}_i^{nd}, i\in[1,M]$, $\mathcal{V}_{k,j}^{d}, k\in[1,M-1], j\in[1,M-k]$

```
/* Procedure 1: update V_i^nd based on selection condition 1 */
for i = 1 : M do
    for (m, n) ∈ W_i do
        if d_d(m, n) < r_d then
            V_i^nd ← V_i^nd ∪ (m, n) ;
/* Procedure 2: update S_k and I_k */
for k = 1 : M − 1 do
    for m = (1 − M) : (M − k − 1) do
        update S_k and I_k;
/* Procedure 3: update V_{k,j}^d based on selection condition 2 */
for k = 1 : M − 1 do
    for j = 1 : M − k do
        for m ∈ v_{k,j} do
            update V_{k,j}^d;
```

SER Upper Bound

◇ **We design an algorithm to find the Voronoi-relevant cover set** (should cover all the Voronoi-relevant neighbors) for each point, and the union bound of the SER can be calculated

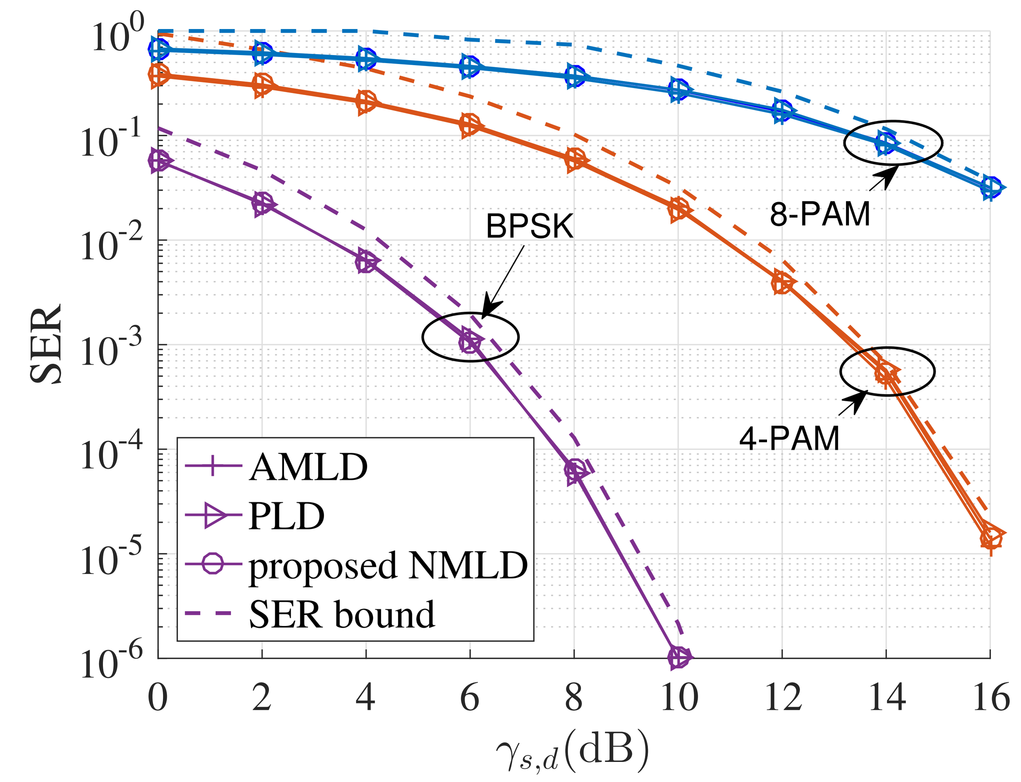
◇ General ideas of the algorithm are:

- Construct an **original decision region** whose minimal circumcircle has radius r
- Calculate the distance d from the interest point to the decision boundary between this point and its neighbor
- **Compare** d **and** r and update the cover set based on the defined **selection condition**

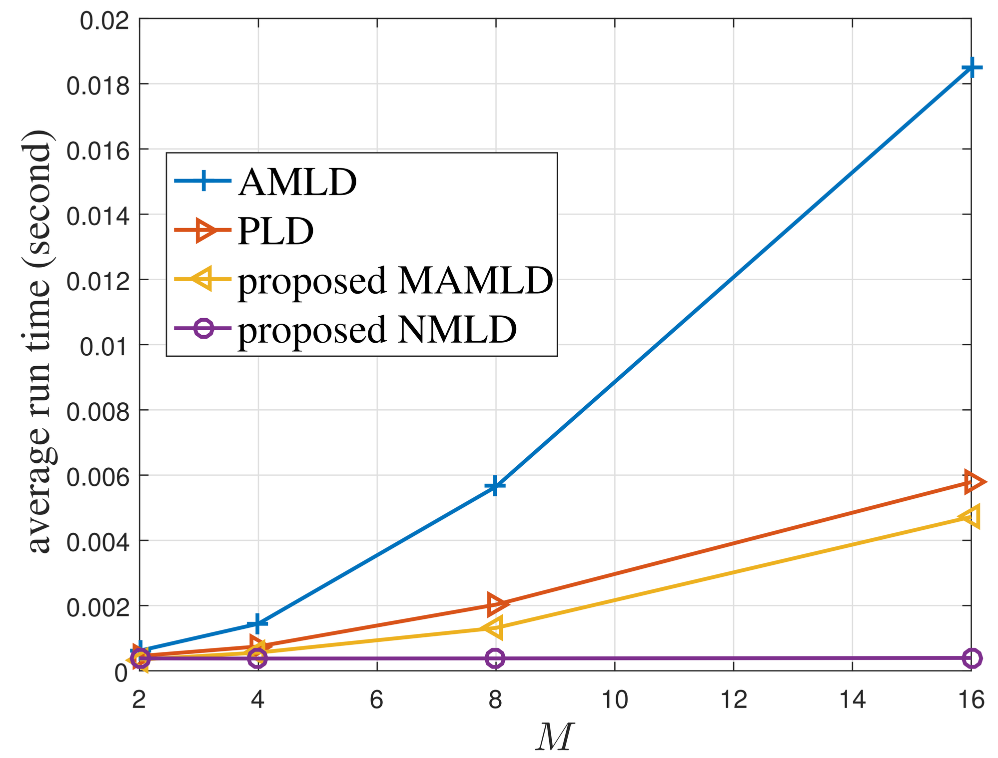
Conclusion

- By exploiting the **signal structure** of the PAM signals, the near-optimal detection complexity can be reduced to **constant**
- **We should also exploit the signal structure of the PAM signals to reduce the complexity of the designed algorithm further**

Simulation Results



- The SER performance of NMLD approaches that of the AMLD
- The proposed bound is quite tight in the high SNR regime



- The runtimes of the proposed MAMLD and NMLD are linear and invariant with M, respectively
- Significantly lower complexity

Reference

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[2] Ankur Bansal and Manav R Bhatnagar, "Performance analysis of PL decoder for M-QAM constellation in DF cooperative system," in *Proc. SPCOM*, 2012.

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