Adaptive Sensing Matrix Design for Greedy Algorithms in MMV Compressive Sensing

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Abstract

Sensing matrix can be designed with low coherence with the measurement matrix to improve the sparse signal recovery performance of greedy algorithms. However, most of the sensing matrix design algorithms are computationally expensive due to large number of iterations. This paper proposes an iteration-free sensing matrix design algorithm for multiple measurement vectors (MMV) compressive sensing. Specifically, sensing matrix is designed in the sense of the local cumulative cross-coherence (LCCC) of the sensing matrix with respect to the measurement matrix when the number of MMV is sufficient and the sparse signals are of full rank. Experiment results verify the effectiveness of the proposed algorithm in terms of improving the sparse signal recovery performance of greedy algorithms.

Introduction

With multiple measurement vectors (MMV) along time instance, the measurement equation of compressive sensing (CS) can be formulated as

$$\boldsymbol{y}_l = \Phi \boldsymbol{x}_l + \boldsymbol{n}_l, \quad l = 1, 2, \cdots, L \tag{1}$$

where $\Phi \in \mathbb{R}^{M\times N}$ $(M < N)$ is the measurement matrix, and $\boldsymbol{x}_l \in \mathbb{R}^{N\times 1}$, $\boldsymbol{y}_l \in \mathbb{R}^{M\times 1}$ and $\boldsymbol{n}_l \in \mathbb{R}^{M\times 1}$ are the vectors of signal, measurement and noise, respectively. Note that (1) can be compactly represented in the form of matrix

$$\boldsymbol{Y} = \Phi \boldsymbol{X} + \boldsymbol{N} \tag{2}$$

where $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_L]$ is the jointly sparse signal, $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_L]$ is the measurement, $\boldsymbol{N} = [\boldsymbol{n}_1, \boldsymbol{n}_2, \cdots, \boldsymbol{n}_L]$ is the measurement noise. In the case of MMV-CS, the vectors $\{\boldsymbol{x}_l\}_{l=1}^{L}$ share the same sparse pattern which means that the matrix $\boldsymbol{X}$ only has a few of rows with non-zero entries.

Usually, the recovery of the sparse signal from its multiple linear measurements can be realized by solving the following optimization problem

$$\min_{\boldsymbol{X}\in\mathbb{R}^{N\times L}} \left\|\boldsymbol{Y} - \Phi\boldsymbol{X}\right\|_F^2 + \lambda \mathcal{R}(\boldsymbol{X}) \tag{3}$$

where $\|\cdot\|_F$ indicates the Frobenius norm, $\mathcal{R}(\cdot)$ is an operator that gives the number of non-zero rows of the input signal $\boldsymbol{X}$. The first term in (3) is the data fidelity term and second one forces the recovered signal to be sparse. The $\lambda > 0$ is a regularization parameter which is the trade-off between data fitting and the sparsity of signal.

Coherence Measurement of Measurement Matrix and Sensing Matrix

A generalized parameter to address the coherence of measurement matrices is the cumulative coherence. The kth cumulative coherence is defined as

$$\mu_c(k, \Phi) = \max_{|J|=k} \sum_{i,j=1,2,\cdots,N, i\notin J, j\in J} \left|\Phi_{.i}^T \Phi_{.j}\right|. \tag{4}$$

It is proved that $\mu_c(k, \Phi) + \mu_c(k-1, \Phi) < 1$ can guarantee the success of both OMP and BP algorithms.

The concept of sensing matrix $\Psi \in \mathbb{R}^{M\times N}$ is proposed for support recovery in OMP and hard thresholding algorithms. The purpose of designing sensing matrix in CS is to reduce the coherence of sensing and measurement matrices, which can improve the recovery performance. The process of sparse recovery by using sensing matrix can be expressed as

$$\hat{X} = \mathcal{R}(Y, \Psi, \Phi, \cdots). \tag{5}$$

The parameter termed as the cumulative cross-coherence (CCC) is proposed to measure the coherence between Φ and Ψ, defined as

$$\tilde{\mu}_c(k, \Psi, \Phi) = \max_{i} \max_{|J|=k, i\notin J} \sum_{j\in J} \left|\Psi_{.i}^T \Phi_{.j}\right|. \tag{6}$$

It has been proved that the smaller CCC between Ψ and Φ results in higher accuracy of support recovery. Furthermore, the local cumulative cross-coherence (LCCC) is defined

$$\hat{\mu}_c(k, \Psi, \Phi_\Gamma) = \max_{|J|=k, J\subseteq\Gamma} \max_{i\notin J} \sum_{j\in J} \left|\Psi_{.i}^T \Phi_{.j}\right| \tag{7}$$

where Γ is the support of sparse signal.

It can be seen from (7) that $\hat{\mu}_c(k, \Psi, \Phi_\Gamma)$ represents the worst case coherence between the columns of the sensing matrix and measurement columns indexed by the support Γ. In other words, $\hat{\mu}_c(k, \Psi, \Phi_\Gamma)$ describes the local coherence between the sensing and the measurement matrix, while $\tilde{\mu}_c(k, \Psi, \Phi)$ describes the global coherence of the sensing matrix with the measurement matrix.

Adaptive Sensing Matrix Design

For sufficient multiple measurement vectors, i.e., $L \geq M$, the recovered signal is expressed as

$$\begin{aligned}
\hat{X}_{i.} &= \Psi_{.i}^T Y = \Psi_{.i}^T(\Phi X + N) = \Psi_{.i}^T \Phi X + \Psi_{.i}^T N \\
&= \Psi_{.i}^T \left[\sum_{j=1}^{N} \Phi_{.j} X(j,1), \sum_{j=1}^{N} \Phi_{.j} X(j,2), \cdots,\right. \\
&\quad \left.\sum_{j=1}^{N} \Phi_{.j} X(j,L)\right] + \Psi_{.i}^T N \\
&= \left[\Psi_{.i}^T \Phi_{.i} X(i,1) + \Psi_{.i}^T \sum_{i\neq j, j=1}^{N} \Phi_{.j} X(j,1), \cdots,\right. \\
&\quad \left.\Psi_{.i}^T \Phi_{.i} X(i,L) + \Psi_{.i}^T \sum_{i\neq j, j=1}^{N} \Phi_{.j} X(j,L)\right] \\
&\quad + \Psi_{.i}^T N.
\end{aligned} \tag{8}$$

It follows from (8) that in order to exactly recover the jointly sparse signal $\boldsymbol{X}$, the terms $\Psi_{.i}^T \Phi_{.i} X(i,1), \cdots, \Psi_{.i}^T \Phi_{.i} X(i,L)$ for $i = 1, 2, \cdots, N$ should be kept distortionless for $\Psi_{.i}^T \Phi_{.i} = 1$, while other terms should be minimized. Given the measurements $\boldsymbol{Y}$, the sensing matrix can be designed as follows

$$\begin{aligned}
&\min_{\Psi_{.i}\in\mathbb{R}^{M\times 1}} \left\|\Psi_{.i}^T Y\right\|_2^2 \\
&\text{s.t.} \quad \Psi_{.i}^T \Phi_{.i} = 1.
\end{aligned} \tag{9}$$

The optimization problem in (9) is a quadratic programming problem with a linear constraint. Its closed form solution is

$$\Psi_{.i} = \frac{R^{-1}\Phi_{.i}}{\Phi_{.i}^T R^{-1} \Phi_{.i}} \tag{10}$$

where $\boldsymbol{R} = \frac{1}{L} Y Y^T$.

Proposition 1 For the SOMP algorithm, the sensing matrix Ψ designed by (10) provides a decreased LCCC and the bound is

$$0 \leq \hat{\mu}_c(K, \Psi, \Phi_\Gamma) \leq \tilde{\mu}_c(K, \Psi, \Phi). \tag{11}$$

Numerical Simulations

The simulation settings are provided as follows. The sparse signal is generated by Gaussian distribution with mean one and variance 0.1. The sizes of Φ and Ψ are both 128×256. The entries of Φ are drawn from Gaussian distribution with zero mean and 1/128. In order to evaluate the performances of the proposed approach, 500 independent trials are carried out at each specific case. The percentage of successful support recovery and the root mean square error (RMSE) of recovered signal are both calculated. For the purpose of comparison, the Alternating Projection (AP) algorithm and Re-weighted (RW) algorithm for sensing matrix design are performed. The conventional approach $\Psi = \Phi$ is also conducted.

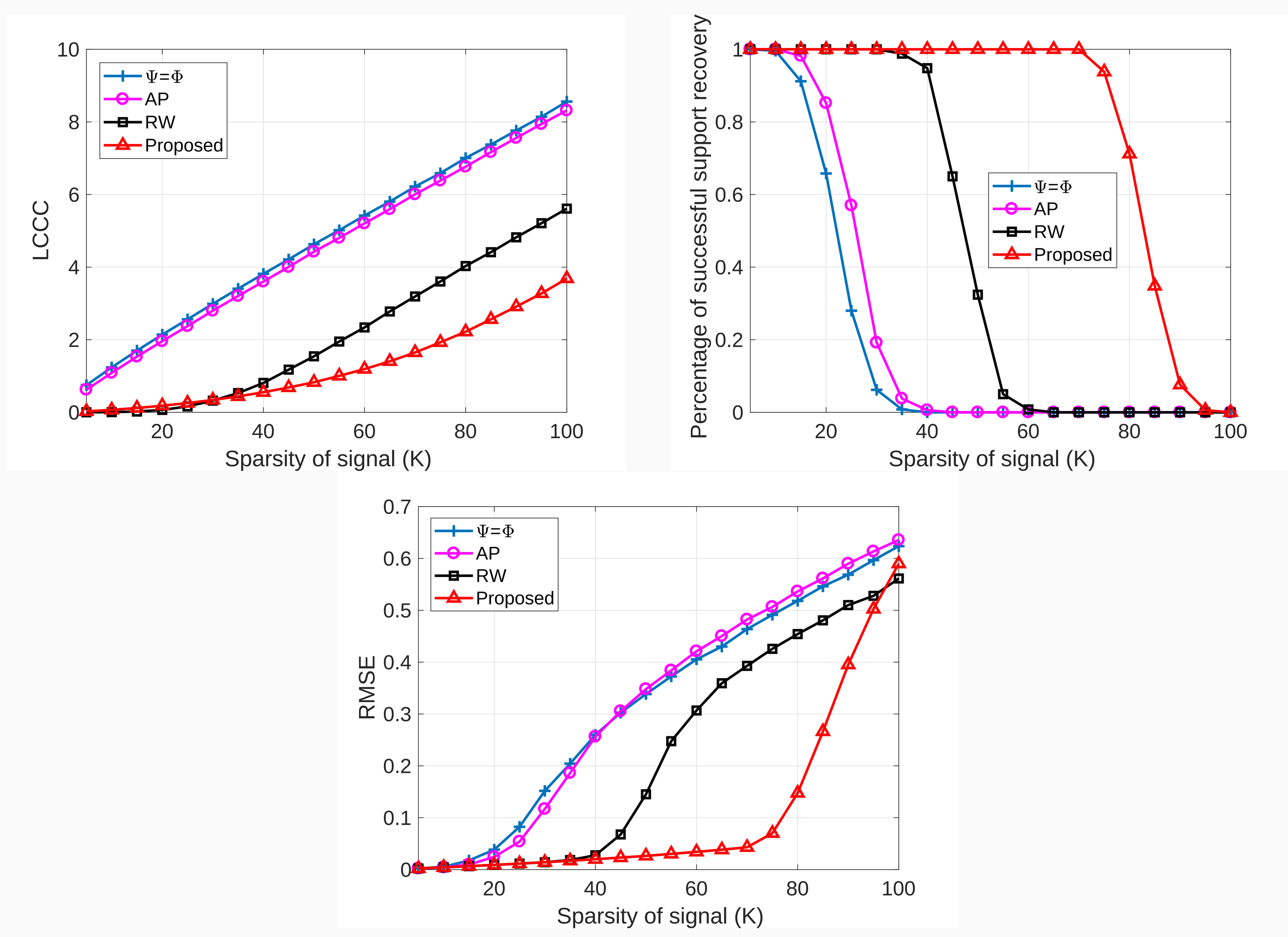

Fig. 1: Simulation results: (1) LCCC versus sparsity of signal with SNR = 20dB and $L = 500$; (2) Percentage of successful recovery versus sparsity of signal with SNR = 20dB and $L = 500$; (3) RMSE versus sparsity of signal with SNR = 20dB and $L = 500$.

Conclusion In this paper, an iteration-free sensing matrix design algorithm for MMV-CS is proposed. To improve the performance of sparse signal recovery, the coherency of Ψ and Φ as well as that of Ψ and Y are exploited. Comparing with the existing methods of sensing matrix design, the proposed algorithm is iteration-free and is able to further enhance the recovery performance. Simulation results confirm the superiority of the proposed approach.