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GLOBAL ENERGY EFFICIENCY MAXIMIZATION IN NON-ORTHOGONAL INTERFERENCE NETWORKS

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Summary

- Resource allocation (RA) over rate region: Non-convex (NC) due to powers
- Computational complexity grows exponentially in the number of variables
- SoA (e.g. canonical monotonic optimization (MO)): treat all variables as NC
- Energy efficient RA: Fractional objective → SoA combines Dinkelbach's Algorithm with MO
- Novel resource allocation framework:
  - Fractional objectives
  - Differentiate between convex and non-convex variables
  - Numerically stable and guaranteed convergence
  - Feasible solution even if terminated prematurely
  - ∼10,000× faster than SoA & (additional) ∼800× faster than Dinkelbach's Algorithm
  - C++ implementation on GitHub: `https://github.com/bmatthiesen`

Motivation

- Global Energy Efficiency:
  Key performance indicator in 5G+ networks
- Non-orthogonal interference networks:
  Beyond treating interference as noise
- Problem (R) Non-convex in $\boldsymbol{p}$, linear in $\boldsymbol{R}$
- SoA solution:
  - Decompose: inner linear & outer monotonic program
  - Fractional Objective: Dinkelbach's Algorithm
  - → 3 layer algorithm
- **Goals:** Keep polynomial complexity in $\boldsymbol{R}$, fast solution, easily applicable framework

Problem (RA)

$$\max_{\boldsymbol{p},\boldsymbol{R}} \frac{\sum_k R_k}{\boldsymbol{\phi}^T\boldsymbol{p} + P_c} \quad \text{s.t.} \quad \boldsymbol{a}_i^T\boldsymbol{R} \leq \log\left(1 + \frac{\boldsymbol{b}_i^T\boldsymbol{p}}{\boldsymbol{c}_i^T\boldsymbol{p} + \sigma_i}\right), \forall i, \quad \boldsymbol{R} \geq 0, \quad \boldsymbol{p} \in [\boldsymbol{0}, \boldsymbol{P}]$$

Robust Global Optimization [1]

Problem (P)

$$\max_{\boldsymbol{x}\in[\boldsymbol{a},\boldsymbol{b}]} f(\boldsymbol{x}) \quad \text{s.t.} \quad g_i(\boldsymbol{x}) \leq 0, \quad i = 1, 2, \dots, m$$

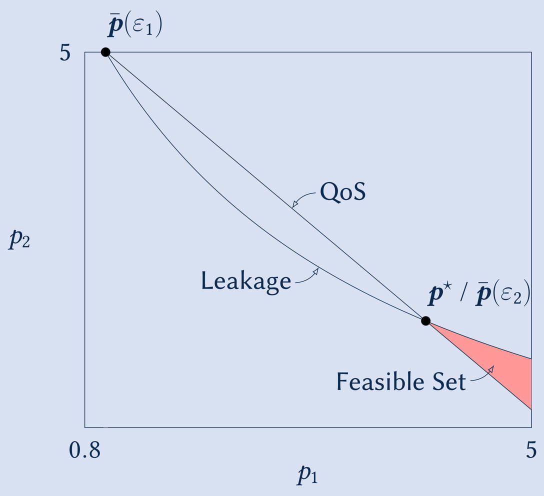
- g_i, $i = 1, 2, \dots, m$: Non-convex functions
- SoA: Branch & Bound or Outer Approximation
- Convergence in finite iterations not guaranteed
- Usual approach: Solve ε-relaxed problem
- Numerical problems:
  - Might give incorrect solution far away from optimum
  - Isolated optimal solutions hard to compute

Example (Numerical Problems of ε-Approximate Solutions)

Consider a MAC with QoS and individual eavesdropper information leakage constraints:

$$\begin{aligned} \min_{p_1,p_2}\ & p_1 \\ \text{s.t.}\ & \log(1 + |h_1|^2 p_1 + |h_2|^2 p_2) \geq Q + \varepsilon && \text{(QoS)} \\ & \log(1 + |g_1|^2 p_1) + \log(1 + |g_2|^2 p_2) \leq L + \varepsilon && \text{(Leakage)} \\ & p_1 \leq P_1, \quad p_2 \leq P_2 \end{aligned}$$

Numerical example:
- $|h_i|^2 = 10$, $|g_1|^2 = \frac{1}{2}$, $|g_2|^2 = 1$, $Q = \log(61)$, $L = \log(8.99)$
- True optimal solution: $\boldsymbol{p}^\star = (4.00665, 1.99335)$
- ε-approximate solution $\bar{\boldsymbol{p}}(\varepsilon)$:
  - $\varepsilon_1 = 10^{-3}$: $\bar{\boldsymbol{p}}(\varepsilon_1) = (0.995843, 5)$
  - $\varepsilon_2 = 10^{-4}$: $\bar{\boldsymbol{p}}(\varepsilon_2) = (4.00541, 1.99417)$



Solution: ε-essential feasibility

A solution of (P) is said to be *essential* (ε, η)*-optimal* if it satisfies

$$f(\boldsymbol{x}^*) + \eta \geq \sup\{f(\boldsymbol{x}) | \boldsymbol{x} \in [\boldsymbol{a}, \boldsymbol{b}], \forall i : g_i(\boldsymbol{x}) \leq -\varepsilon\}, \qquad \text{for some } \eta > 0.$$

- $\varepsilon, \eta \to 0$: essential (ε, η)-optimal solution is a nonisolated feasible point which is optimal
- SIT: Solve sequence of feasibility problems

$$\min_{\boldsymbol{x}\in[\boldsymbol{a},\boldsymbol{b}]} \max_{i=1,\dots,m} g_i(\boldsymbol{x}) \quad \text{s.t.} \quad f(\boldsymbol{x}) \geq \gamma \tag{Q_γ}$$

- Efficient solution with Branch & Bound if $f(\boldsymbol{x})$ is *nice* (i.e., easy feasibility test)
- For all $\varepsilon > 0$: $\min(Q_\gamma) > -\varepsilon \Rightarrow \max(P_\varepsilon) < \gamma$

Algorithm (Successive Incumbent Transcending (SIT) Scheme)

Step 0 Initialize $\bar{\boldsymbol{x}}$ with the best known nonisolated feasible solution and set $\gamma = f(\bar{\boldsymbol{x}}) + \eta$; otherwise do not set $\bar{\boldsymbol{x}}$ and choose $\gamma \leq f(\boldsymbol{x})\ \forall \boldsymbol{x} : g_i(\boldsymbol{x}) \leq 0$, $i = 1, \dots, m$.

Step 1 Check if (P) has a nonisolated feasible solution $\boldsymbol{x}$ satisfying $f(\boldsymbol{x}) \geq \gamma$; otherwise, establish that no such ε-essential feasible $\boldsymbol{x}$ exists and go to Step 3. →Solve ($Q\gamma$)

Step 2 Update $\bar{\boldsymbol{x}} \leftarrow \boldsymbol{x}$ and $\gamma \leftarrow f(\bar{\boldsymbol{x}}) + \eta$. Go to Step 1.

Step 3 Terminate: If $\bar{\boldsymbol{x}}$ is set, it is an essential (ε, η)-optimal solution; else Problem (P) is ε-essential infeasible.

References

[1] H. Tuy, "$\mathcal{D}(\mathcal{C})$-optimization and robust global optimization", *J. Global Optim.*, vol. 47, no. 3, pp. 485–501, Oct. 2009.

[2] B. Matthiesen and E. A. Jorswieck, "Efficient global optimal resource allocation in non-orthogonal interference networks", submitted to *IEEE Trans. Signal Process.*, Oct. 2018.

[3] ——,"Weighted sum rate maximization for non-regenerative multi-way relay channels with multi-user decoding", in *Proc. IEEE Int. Workshop Comput. Adv. Multi-Sensor Adapt. Process. (CAMSAP)*, IEEE, Curaçao, Dutch Antilles, Dec. 2017.

Problem Statement & Solution

Problem (R)

$$\max_{(\boldsymbol{x},\boldsymbol{\xi})\in\mathcal{X}\times\Xi} \frac{f^+(\boldsymbol{x},\boldsymbol{\xi})}{f^-(\boldsymbol{x},\boldsymbol{\xi})} \quad \text{s.t.} \quad g_i^+(\boldsymbol{x},\boldsymbol{\xi}) - g_i^-(\boldsymbol{x}) \leq 0, \ i = 1, \dots, m$$

Dual Problem (Q)

$$\min_{(\boldsymbol{x},\boldsymbol{\xi})\in\mathcal{X}\times\Xi} \max_{i=1,\dots,m} \left(g_i^+(\boldsymbol{x},\boldsymbol{\xi}) - g_i^-(\boldsymbol{x})\right) \quad \text{s.t.} \quad \frac{f^+(\boldsymbol{x},\boldsymbol{\xi})}{f^-(\boldsymbol{x},\boldsymbol{\xi})} \geq \gamma$$

- Non-convex variables $\boldsymbol{x}$, convex variables $\boldsymbol{\xi}$
- Dual Problem has robust feasible set → no isolated feasible points! (cf. Lemma 1)

Technical Requirements:
- Ξ closed convex
- $f^-(\boldsymbol{x},\boldsymbol{\xi}) > 0$
- $\{g_i^-(\boldsymbol{x})\}$: common maximizer over every box $\mathcal{M} \subseteq \mathcal{M}_0 \supseteq \mathcal{X}$
- Separable functions $(\boldsymbol{x},\boldsymbol{\xi})$: $h(\boldsymbol{x},\boldsymbol{\xi}) = h_x(\boldsymbol{x}) + h_\xi(\boldsymbol{\xi})$ and for all γ ($\gamma \geq 0$ if $f^+(\boldsymbol{x},\boldsymbol{\xi}) \geq 0$)
  - $\gamma f_x^-(\boldsymbol{x}) - f_x^+(\boldsymbol{x})$, $g_{1,x}^+(\boldsymbol{x}), \dots, g_{m,x}^+(\boldsymbol{x})$: common minimizer over every $\mathcal{X} \cap \mathcal{M}$ (⋆)
  - $\gamma f_\xi^-(\boldsymbol{\xi}) - f_\xi^+(\boldsymbol{\xi})$, $g_{1,\xi}^+(\boldsymbol{\xi}), \dots, g_{m,\xi}^+(\boldsymbol{\xi})$: convex
  - $\gamma f_x^-(\boldsymbol{x}) - f_x^+(\boldsymbol{x})$ increasing if $\mathcal{X}$ normal, or decreasing if $\mathcal{X}$ conormal

Core Problem: Compute lower bound for (Q) over box $\mathcal{M}$
- With $\bar{\boldsymbol{x}}_{\mathcal{M}}^* \in \arg\max_{\boldsymbol{x}\in\mathcal{M}} g_i^-(\boldsymbol{x})$ for all i:

$$\begin{aligned} \min_{\boldsymbol{x},\boldsymbol{\xi}} \max_{i=1,2,\dots,m} & \left\{g_{i,\xi}^+(\boldsymbol{\xi}) + g_{i,x}^+(\boldsymbol{x}) - g_i^-(\bar{\boldsymbol{x}}_{\mathcal{M}}^*)\right\} \\ \text{s.t.}\ & \gamma f_\xi^-(\boldsymbol{\xi}) - f_\xi^+(\boldsymbol{\xi}) + \gamma f_x^-(\boldsymbol{x}) - f_x^+(\boldsymbol{x}) \leq 0 \\ & \boldsymbol{\xi} \in \Xi, \ \boldsymbol{x} \in \mathcal{X} \cap \mathcal{M}. \end{aligned}$$

- With $\underline{\boldsymbol{x}}_{\mathcal{M}}^*$ being the common minimizer in (⋆):

$$\begin{aligned} \min_{\boldsymbol{\xi}\in\Xi} \max_{i=1,2,\dots,m} & \left\{g_{i,\xi}^+(\boldsymbol{\xi}) + g_{i,x}^+(\underline{\boldsymbol{x}}_{\mathcal{M}_i}^*) - g_i^-(\bar{\boldsymbol{x}}_{\mathcal{M}_i}^*)\right\} \\ \text{s.t.}\ & \gamma f_\xi^-(\boldsymbol{\xi}) - f_\xi^+(\boldsymbol{\xi}) + \gamma f_x^-(\underline{\boldsymbol{x}}_{\mathcal{M}_i}^*) - f_x^+(\underline{\boldsymbol{x}}_{\mathcal{M}_i}^*) \leq 0. \end{aligned} \tag{B}$$

  → Convex optimization problem
- Solve with Branch & Bound and combine with SIT Algorithm

Algorithm (Solution of Problem (R))

Step 0. Initialize $\varepsilon, \eta > 0$ and $\mathcal{M}_0 = [\boldsymbol{p}^0, \boldsymbol{q}^0]$, $\mathscr{P}_1 = \{\mathcal{M}_0\}$, $\mathscr{R} = \emptyset$, $k = 1$, and γ such that $\gamma \leq \frac{f^+(\boldsymbol{x},\boldsymbol{\xi})}{f^-(\boldsymbol{x},\boldsymbol{\xi})}$ for all feasible $(\boldsymbol{x},\boldsymbol{\xi})$.

Step 1. For each box $\mathcal{M} \in \mathscr{P}_k$:
- Set $\beta(\mathcal{M})$ as the solution of (B) or $\beta(\mathcal{M}) = \infty$ if (B) is infeasible.
- Add $\mathcal{M}$ to $\mathscr{R}$ if $\beta(\mathcal{M}) \leq -\varepsilon$.

Step 2. Terminate if $\mathscr{R} = \emptyset$: If $\bar{\boldsymbol{x}}$ is not set, then (R) is ε-essential infeasible; else $\bar{\boldsymbol{x}}$ is an essential (ε, η)-optimal solution.

Step 3. Let $\mathcal{M}^k = \arg\min\{\beta(\mathcal{M}) \,|\, \mathcal{M} \in \mathscr{R}\}$ and solve the feasibility problem

$$\text{find } \boldsymbol{\xi} \in \Xi \quad \text{s.t.} \quad g_i^+(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*, \boldsymbol{\xi}) - g_i^-(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*) \leq 0, \ i = 1, \dots, m.$$

If feasible go to Step 4; otherwise go to Step 5.

Step 4. $\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*$ is a nonisolated feasible solution satisfying $\frac{f^+(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*,\boldsymbol{\xi})}{f^-(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*,\boldsymbol{\xi})} \geq \gamma$ for some $\boldsymbol{\xi} \in \Xi$. Solve

$$\min_{\boldsymbol{\xi}\in\Xi} \frac{f^+(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*, \boldsymbol{\xi})}{f^-(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*, \boldsymbol{\xi})} \quad \text{s.t.} \quad g_i^+(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*, \boldsymbol{\xi}) - g_i^-(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^*) \leq 0, \ i = 1, \dots, m. \tag{1}$$

If $\bar{\boldsymbol{x}}$ is not set or $v(1) > \gamma - \eta$, set $\bar{\boldsymbol{x}} = \underline{\boldsymbol{x}}_{\mathcal{M}^k}^*$ and $\gamma = v(1) + \eta$.

Step 5. Bisect $\mathcal{M}^k$ via $(\boldsymbol{v}^k, j_k)$ where $j_k \in \arg\max_j\{|\bar{x}_{\mathcal{M}^k,j}^* - \underline{x}_{\mathcal{M}^k,j}^*\}$ and $\boldsymbol{v}^k = \frac{1}{2}(\underline{\boldsymbol{x}}_{\mathcal{M}^k}^* + \bar{\boldsymbol{x}}_{\mathcal{M}^k}^*)$. Remove $\mathcal{M}^k$ from $\mathscr{R}$. Let $\mathscr{P}_{k+1} = \{\mathcal{M}_-^k, \mathcal{M}_+^k\}$. Increment k and go to Step 1.

Application to Problem (RA)

Identify

$$\boldsymbol{\xi} \mathrel{\widehat{=}} \boldsymbol{R} \qquad \boldsymbol{x} \mathrel{\widehat{=}} \boldsymbol{p} \qquad \Xi = [\boldsymbol{0}, \infty) \qquad \mathcal{X} = [\boldsymbol{0}, \boldsymbol{P}] \qquad f^+(\boldsymbol{p}, \boldsymbol{R}) = \sum_k R_k$$

$$f^-(\boldsymbol{p}, \boldsymbol{R}) = \boldsymbol{\phi}^T\boldsymbol{p} + P_c \qquad g_i^+(\boldsymbol{p}, \boldsymbol{R}) = \boldsymbol{a}_i^T\boldsymbol{R} + \log\left(\boldsymbol{c}_i^T\boldsymbol{p} + \sigma_i\right) \qquad g_i^-(\boldsymbol{p}) = \log\left(\left(\boldsymbol{b}_i^T + \boldsymbol{c}_i^T\right)\boldsymbol{p} + \sigma_i\right)$$

- For a box $\mathcal{M} = [\underline{\boldsymbol{x}}, \bar{\boldsymbol{x}}]$:

$$\arg\max_{\boldsymbol{p}\in\mathcal{M}} g_i^-(\boldsymbol{p}) = \bar{\boldsymbol{x}} \qquad \arg\min_{\boldsymbol{p}\in\mathcal{M}} \gamma f_x^-(\boldsymbol{p}) = \arg\min_{\boldsymbol{p}\in\mathcal{M}} g_{i,x}^+(\boldsymbol{p}) = \underline{\boldsymbol{x}}$$

- Problem (B) is a LP:

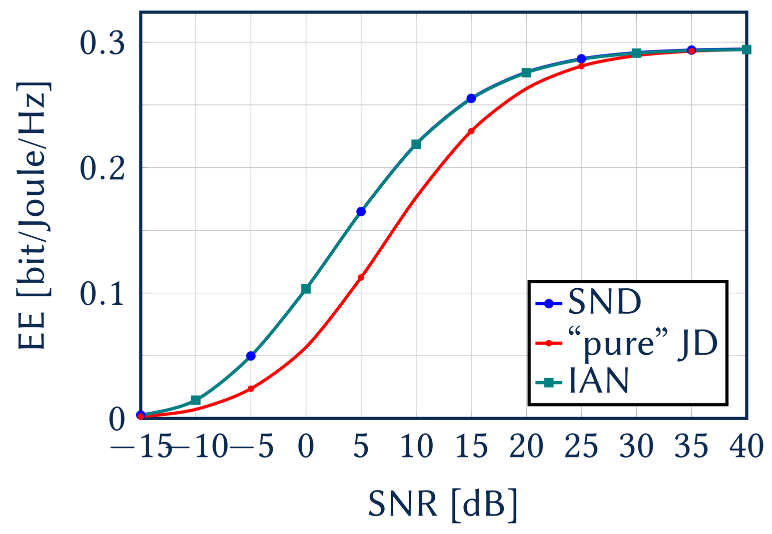
$$\min_{\boldsymbol{R}\geq 0} \max_{i=1,2,\dots,m} \left\{\boldsymbol{a}_i^T\boldsymbol{R} + \log\left(\boldsymbol{c}_i^T\underline{\boldsymbol{x}} + \sigma_i\right) - \log\left(\left(\boldsymbol{b}_i^T + \boldsymbol{c}_i^T\right)\bar{\boldsymbol{x}} + \sigma_i\right)\right\} \quad \text{s.t.} \quad \sum_k R_k + \gamma\left(\boldsymbol{\phi}^T\underline{\boldsymbol{x}} + P_c\right) \leq 0$$

Numerical Evaluation

- Example: Gaussian MWRC with AF relaying and multiple unicast transmissions.
- Treating interference as noise vs. purely joint decoding vs. simultaneous non-unique decoding
- Algorithm vs. Dinkelbach (auxiliary problem solved with our algorithm)

SNR		0 dB	20 dB	40 dB
Algorithm	Mean	5.1438 s	0.1771 s	0.155 s
	Median	3.2781 s	0.0762 s	0.06 s
Dinkelbach	Mean	377.1501 s	145.4181 s	36.969 s
	Median	162.811 s	23.027 s	16.9229 s

Mean and median run times of EE computation for "pure" JD and different solvers, all with precision $\eta = 0.01$



EE of MWRC with AF relaying. Averaged over 1000 i.i.d. channels.

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