Analysis of Broadband GEVD-Based Blind Source Separation

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Signal Processing in the Information Age

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GEVD-Based Blind Source Separation: Overview

Broadband Blind Source Separation

- Consider a broadband MIMO system:

$$x_i[n] = \sum_{j=1}^{M} \sum_{m=-\infty}^{\infty} H_{i,j}[n-m] s_j[m] \quad 1 \leq i \leq N$$

$$\boldsymbol{x}(z) = \boldsymbol{s}(z)\boldsymbol{H}(z)$$

- We are interested in estimating the source signals $\boldsymbol{s}(z)$ without knowledge of the channel transfer functions $\boldsymbol{H}(z)$
- Can we use a narrowband solution as a guide?
  - Stationary problem requires higher-order statistics
  - Non-stationary problem can be solved using generalised eigenvalue decomposition (GEVD)

GEVD-based Blind Source Separation

- Narrowband: Parra and Sajda, Yeredor, Tomé
- Approach:
  1. Assume sources have non-stationary statistics
  2. Assume channel mixing matrix is stationary
  3. Collect two datasets at different times
  4. Form two space-time covariance matrices
  5. Calculate GEVD: $AU = BU\Lambda$
  6. Use matrix of generalised eigenvectors to unmix signals
- Broadband algorithms:
  - Corr, Pestana, Weiss, Redif, & Moonen
  - Redif
- No analysis showing validity of step 6 or conditions under which it works

Analysis of GEVD-based Blind Source Separation

1. Existence of pGEVD
   - GEVD not well defined if both matrices are rank deficient (Bai) – Without loss of generality, we assume that $\boldsymbol{B}(z)$ is invertible
   - Weiss et. al. show that a parahermitian pEVD exists but with caveats
   - GEVD can be cast as a parahermitian EVD

$$\boldsymbol{A}(z)\boldsymbol{U}(z) = \boldsymbol{B}(z)\boldsymbol{U}(z)\boldsymbol{\Lambda}(z)$$

becomes:

$$\boldsymbol{C}(z)\boldsymbol{V}(z) = \boldsymbol{V}(z)\boldsymbol{\Lambda}(z)$$

where

$$\boldsymbol{V}(z) = \boldsymbol{L}(z)\boldsymbol{U}(z)$$

– $\boldsymbol{L}(z)$ is derived from $\boldsymbol{B}(z)$ and is full rank

Analysis of GEVD-based Blind Source Separation

1. Existence of pGEVD ...
   - Weiss et. al. show that:
     - The generalised eigenvalues $\mathbf{\Lambda}(z)$ exist as unique, convergent but likely infinite-length Laurent series provided they are Hölder continuous on the unit circle
     - If the eigenvalues are analytic functions on the unit circle, and $\boldsymbol{V}(z)$ is Hölder continuous with $\alpha > 1$ on the unit circle, then the generalised eigenvectors $\boldsymbol{U}(z)$ exist as a convergent Laurent series.
     - The generalised eigenvectors $\boldsymbol{U}(z)$ are unique up to an arbitrary phase response
     - Analytic eigenvalues do not exist if $\boldsymbol{C}(z)$ can be made block pseudo-circulant via a paraunitary similarity transformation
     - If the generalised eigenvalues or eigenvalues are not analytic, they can generally be approximated by Laurent polynomials.

Analysis of GEVD-based Blind Source Separation

2. Relationship between pGEVD and BB-BBS
   - We have two space-time covariance matrices:

     $$\boldsymbol{R}_i(z) = \boldsymbol{H}^P(z)\boldsymbol{S}_i(z)\boldsymbol{H}(z) \quad 1 \leq i \leq 2$$

     where $\boldsymbol{S}_i(z)$ is the cross spectral density (CSD) matrix for the i-th dataset and is diagonal
   - pGEVD requires that $\boldsymbol{R}_2(z)$ must be full rank
     Hence $\boldsymbol{H}(z)$ and $\boldsymbol{S}_2(z)$ need to be full rank i.e. $M \leq N$
     If $M < N$ we have an under-determined BSS problem: assume $M = N$.

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2. Relationship between pGEVD and BB-BBS ...
   - Consider two pGEVD Problems:

$$\boldsymbol{R}_1(z)\boldsymbol{U}(z) = \boldsymbol{R}_2(z)\boldsymbol{U}(z)\Lambda_R(z)$$

$$\boldsymbol{S}_1(z)\boldsymbol{V}(z) = \boldsymbol{S}_2(z)\boldsymbol{V}(z)\Lambda_S(z)$$

   - After some algebra we find:

$$\Lambda_R(z) = \Lambda_S(z)\,(\equiv \Lambda(z))$$

$$\Lambda(z) = \boldsymbol{S}_1(z)/\boldsymbol{S}_2(z)$$

$$\boldsymbol{U}(z) = \boldsymbol{H}^{-1}(z)\boldsymbol{V}(z)$$

   - Hence $\boldsymbol{U}(z)$ is nearly the unmixing matrix
     – Need to look at $\boldsymbol{V}(z)$

Analysis of GEVD-based Blind Source Separation

2. Relationship between pGEVD and BB-BBS . . .
   - Consider $\boldsymbol{V}(z)$ in:

$$\boldsymbol{S}_1(z)\boldsymbol{V}(z) = \boldsymbol{S}_2(z)\boldsymbol{V}(z)\Lambda(z)$$

   - Note $\boldsymbol{S}_1(z)$, $\boldsymbol{S}_2(z)$ and $\Lambda(z)$ are diagonal
   - After some maths we find:

$$\boldsymbol{V}(z) = \boldsymbol{\Delta}(z)\Pi$$

     where $\boldsymbol{\Delta}(z)$ is a block-diagonal matrix and Π is a permutation
   - The size of the blocks in $\boldsymbol{\Delta}(z)$ is determined by the algebraic multiplicity of the eigenvalues

Analysis of GEVD-based Blind Source Separation

1. Signal Recovery
   - Recall that

     $$\boldsymbol{x}(z) = \boldsymbol{s}(z)\boldsymbol{H}(z)$$

     and

     $$\boldsymbol{U}(z) = \boldsymbol{H}^{-1}(z)\boldsymbol{\Delta}(z)\Pi$$

   - Hence

     $$\hat{\boldsymbol{s}}(z) = \boldsymbol{x}(z)\boldsymbol{U}(z) = \boldsymbol{s}(z)\boldsymbol{\Delta}(z)\Pi$$

   - Thus source signals with distinct PSD ratios can be recovered up to a scaling and permutation
   - Source signals with identical PSD ratios could be subject to parunitary mixing as well

Computer Simulations

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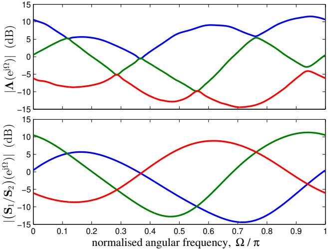
- The analysis assumes the existence of an analytic pGEVD
- Existing algorithms are based on the use of pEVD algorithms which produce majorised solutions



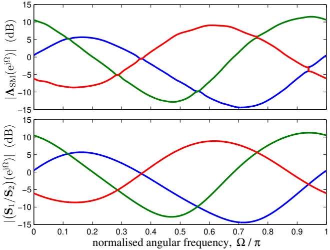
Top: Majorised Eigenvalues $\mathbf{\Lambda}(z)$
– Redif algorithm (not analytic)

Bottom: True Eigenvalues $\mathbf{\Lambda}_0(z)$

- In the absence of an analytic pGEVD, the simulations were based on a pseudo-algorithm which requires human intervention
- The permutation that makes the eigenvalues a smooth function of frequency is determined manually

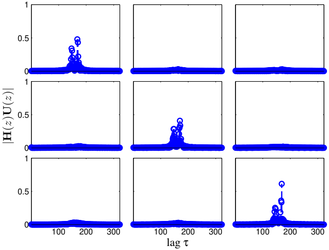
Computer Simulations

- Expt. 1: Distinct Spectrally Unmajorised Sources



Top: Analytic Eigenvalues $\Lambda(z)$
– Pseudo-analytic algorithm

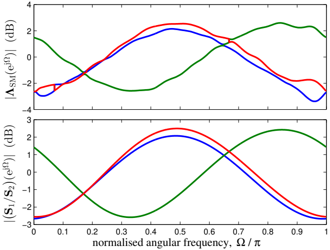
Bottom: PSD Ratios $S_1(z)/S_2(z)$

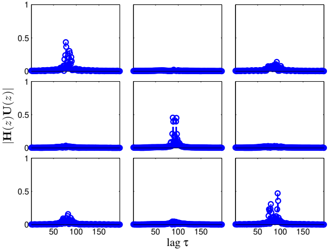


Signal Separation Matrix
$H(z)U(z)$
– Good separation

Computer Simulations

- Expt. 2: Indistinct Sources





Top: Analytic Eigenvalues $\mathbf{\Lambda}(z)$
– Pseudo-analytic algorithm

Bottom: PSD Ratios $\boldsymbol{S}_1(z)/\boldsymbol{S}_2(z)$

Signal Separation Matrix
$\boldsymbol{H}(z)\boldsymbol{U}(z)$
– Poor separation

Conclusions

- Broadband blind signal separation problem with non-stationary signals
- Potentially solution using a polynomial GEVD (generalisation of narrowband case)
- pGEVD can be shown to exist with similar caveats to the pEVD (Weiss et. al.)
- Need PSD ratios (eigenvalues) to be distinct
- Signals are recovered up to scaling and permutation
- Need an analytic pGEVD algorithm