Fast Double-coupled Nonnegative Canonical Polyadic Decomposition

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Introduction

background

- Coupled tensor decomposition has become a popular technique for the simultaneous analysis of multiblock tensors [1].
- It enables the simultaneous extraction of common components and individual components.
- It is reasonable to expect identical elicited information among subjects since ongoing EEG are collected under the same stimulus.
- Time consumption would go extremely heavy due to the high-dimensional and non-negative nature of ongoing EEG.

Objective

To develop an efficient data-driven coupled tensor decomposition algorithm.

Proposed algorithm

Coupled tensor decomposition (or LCPTD [2]) model

- Each factor matrix $\boldsymbol{U}^{(n,s)} = \left[\boldsymbol{U}_C^{(n)}, \boldsymbol{U}_I^{(n,s)}\right]$ consists of two parts: $\boldsymbol{U}_C^{(n)} \in \Re^{I_n \times L_n}, 0 \le L_n \le R$ shared by all tensors with coupling information and $\boldsymbol{U}_I^{(n,s)} \in \Re^{I_n \times (R-L_n)}$ representing individual characteristics of each single tensor block.

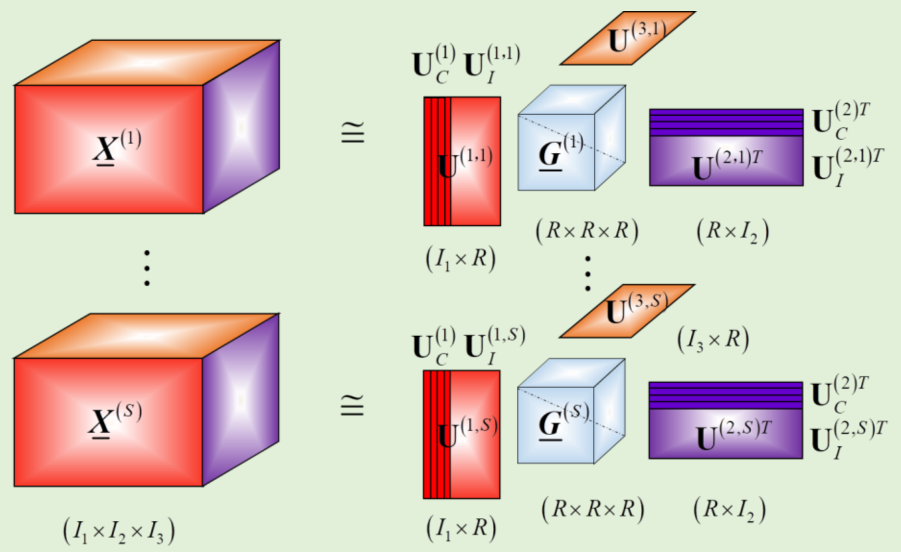

Fig.1 Conceptual illustration of dual-coupled LCPTD model

Realization of FDC-NCPD

- Squared Euclidean Divergence minimization
- Hierarchical Alternating Least Squares (HALS)
- Fast Hierarchical Alternating Least Squares (Fast HALS [3])
- The object function can be expressed as:

$$\text{minimize} \sum_{s=1}^{S} \left\| \underline{\boldsymbol{X}}^{(s)} - \sum_{r=1}^{R} \lambda_r^{(s)} \boldsymbol{u}_r^{(1,s)} \circ \boldsymbol{u}_r^{(2,s)} \circ \cdots \circ \boldsymbol{u}_r^{(N,s)} \right\|_F^2$$

$$s.t.\ \boldsymbol{u}_r^{(n,1)} = \cdots = \boldsymbol{u}_r^{(n,S)}\ for\ r \le L_n,$$

$$\left\| \boldsymbol{u}_r^{(n,s)} \right\| = 1, n = 1 \cdots N, r = 1 \cdots R, s = 1 \cdots S$$

Experiments and Results

Exp1. Validation of synthetic data

- NTF-HALS, NTF-FastHALS, LCPTD-HALS and FDC-NCPD
- Convergence speed: Execution time and iteration number, 30 runs
  SNR = 20 dB, $I_{1,2,3} = \{7n, 8n, 9n\}$, $R = 4n$, $L_{1,2} = 2n$, $S = 10$
- Decomposition quality: Fit and PI, 20 runs
  SNR = -5~20 dB, $I_{1,2,3} = \{40,50,60\}$, $R = 30$, $L_{1,2} = 20$, $S = 10$
- Evaluation index: Execution time, iteration number, Fit and PI

Exp2. Application of multi-subject ongoing EEG data

- Data collection, data preprocessing can be found in [4]
- Tensor (14): 64 channels × 146 frequency bins × 510 samples
- The coupled information exists on the first two modes.
- DIFFIT suggested $R = 36$. $L_{1,2} = 20$

Tab 1. Performance comparison of two algorithms in Exp2.

Ago.	Comp. Number	Running Time	Fit
LCPTD-HALS	59.3	76442.65	0.7360
FDC-NCPD	65.6	350.97	0.7353

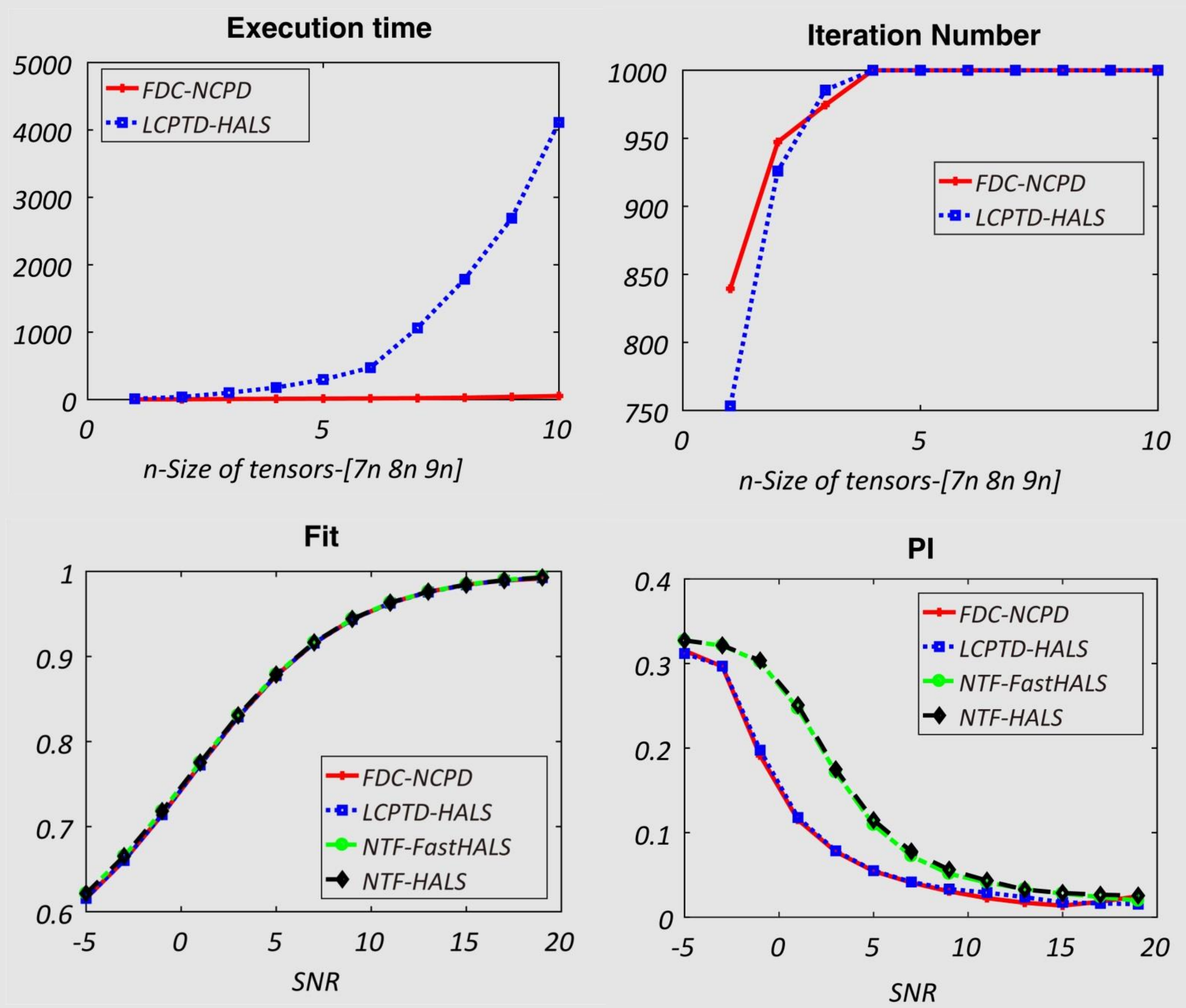

Fig 2. Results of Synthetic data

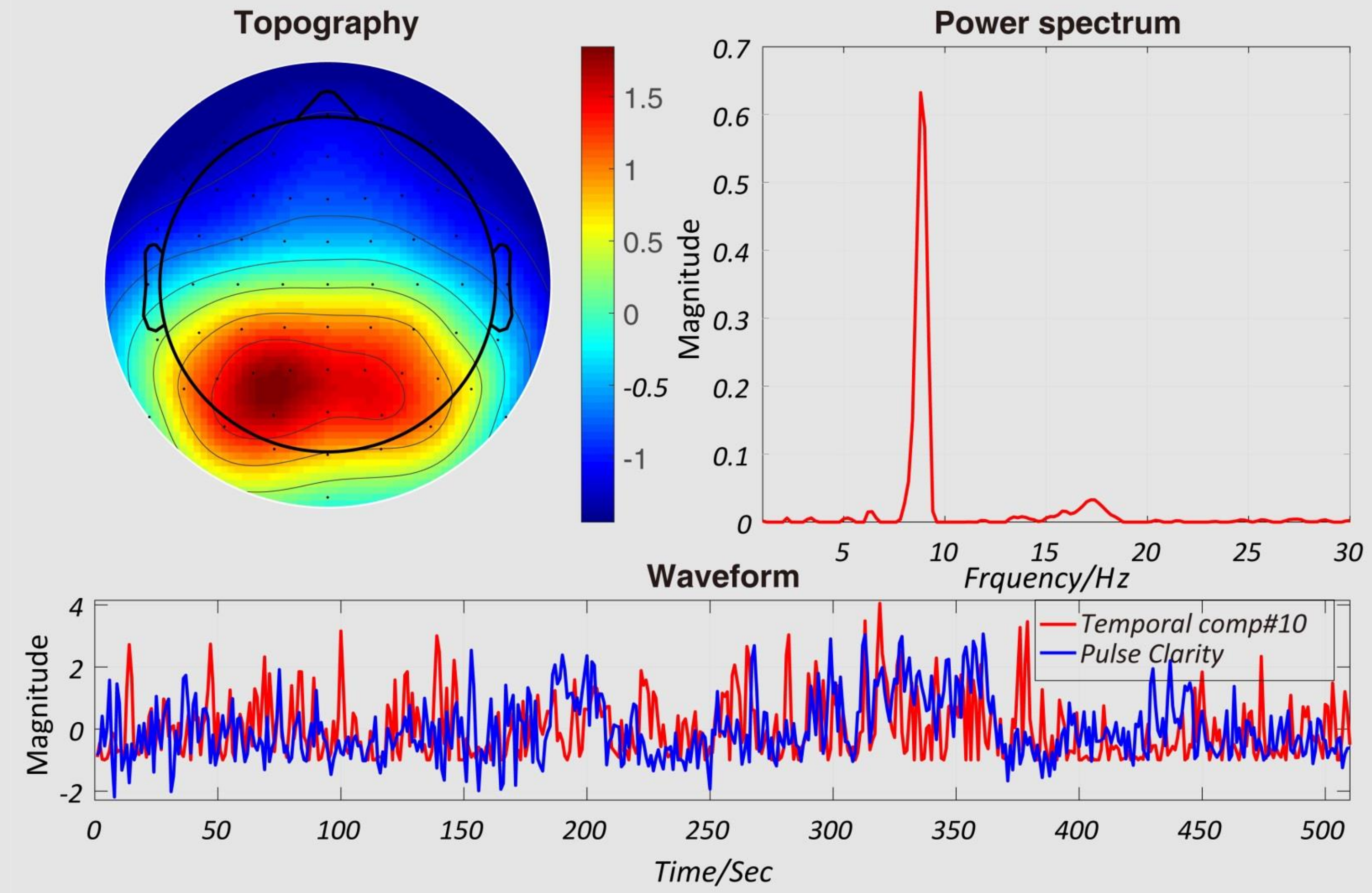

Fig 3. An example of temporal component with its corresponding spatial and spectral components in Exp2.

Conclusion and Future work

- Double coupled tensor-based using LCPTD model and Fast-HALS strategy greatly reduces the computational complexity without compromising the decomposition quality.
- Further analyze brain activation regions and frequency oscillations corresponding to the significantly correlated temporal components

References

[1] G. X. Zhou et al., "Linked component analysis from matrices to high-order tensors: Applications to biomedical data," Proceedings of the IEEE, 2016.

[2] T. Yokota et al., "Linked PARAFAC / CP Tensor Decomposition and Its Fast Implementation for Multi-block,", ICONIP2012.

[3] A. Cichocki et al., "Fast local algorithms for large scale nonnegative matrix and tensor factorizations," IEICE T FUND ELECTR, 2009.

[4] F. Y. Cong et al., "Linking brain responses to naturalistic music through analysis of ongoing eeg and stimulus features," IEEE Transactions on Multimedia, 2013.

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