Robust Gridless Sound Field Decomposition Based on Structured Reciprocity Gap Functional in Spherical Harmonic Domain

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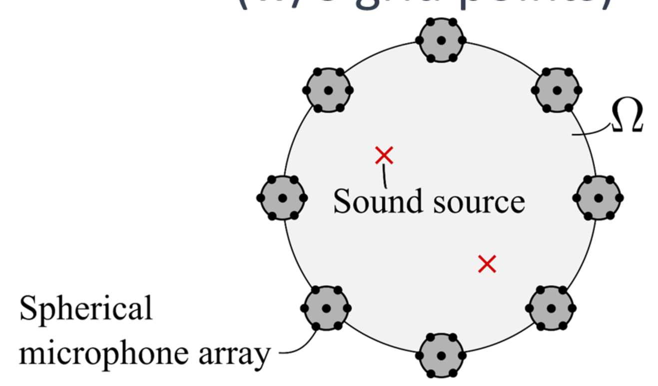
Abstract

Sound field decomposition

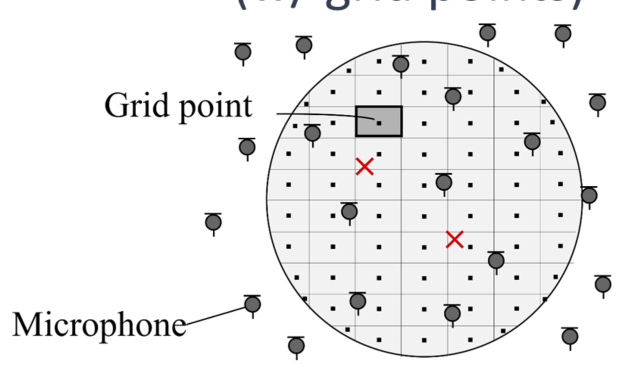
- Goal is to interpolate and reconstruct sound field inside region including sources (ill-posed problem!)
- Sound field should be decomposed into fundamental solutions of Helmholtz eq., i.e., point sources

Proposed method and its relation to prior works

Gridless sound field decomposition[1] (w/o grid points)



Sparse sound field decomposition[2] (w/ grid points)



- Based on spherical-harmonic-domain reciprocity gap functional (SHD-RGF)
- Reconstruction accuracy will be strongly affected by noise

- Based on discretization of possible source region into grid points
- Discrete set of point-source dictionary causes off-grid problem

Grouping time-frequency bins to improve robustness

▶ Proposed Structured RGF
- Formulation of structured SHD-RGF using annihilating filter (AF)
- Decomposition algorithm to find valid solution

▶ Group-sparse representation[2]
- Exploiting group-sparse structure in time-frequency domain
- Off-grid problem is still a major issue

[1] Y. Takida et al, "Gridless sound field decomposition based on reciprocity gap functional in spherical harmonic domain", in *IEEE SAM*, pp. 627—631, 2018.
[2] S. Koyama et al, "Sparse sound field decomposition for super-resolution in recording and reproduction", *JASA*, vol. 143, no.6, pp. 3780—3895, 2018.

Problem Statement

Sound field decomposition

- Source distribution $Q(\mathbf{r},k)$ inside Ω is approximated as a linear combination of J point sources

$$Q(\mathbf{r},k) \approx \sum_{j=1}^{J} c_j \delta(\mathbf{r}-\mathbf{r}_j) \quad \Rightarrow \quad u(\mathbf{r},k) \approx \sum_{j=1}^{J} c_j(k) G(\mathbf{r}|\mathbf{r}_j,k)$$

Spatial convolution of source distribution $Q(\cdot)$ with three-dimensional free-field Green's function $G(\cdot)$

- Pressure on $\partial\Omega$ is approximated in SHD by truncating harmonic order

$$u(\mathbf{r}) \approx \sum_{\nu=0}^{N} \sum_{\mu=-\nu}^{\nu} u_{\nu,\mu} h_\nu(kr) Y_{\nu,\mu}(\theta,\phi)$$

- Measurement model: Q N_mth-order microphone arrays are used

$$\boldsymbol{\alpha} = \mathbf{T}\mathbf{u} + \boldsymbol{\epsilon}$$

- $\boldsymbol{\alpha} \in \mathbb{C}^{Q(N_\mathrm{m}+1)^2}$: Measurements
- $\mathbf{T} \in \mathbb{C}^{Q(N_\mathrm{m}+1)^2 \times (N+1)^2}$: Translation matrix
- $\mathbf{u} \in \mathbb{C}^{(N+1)^2}$: Coefficients in SHD
- $\boldsymbol{\epsilon} \in \mathbb{C}^{Q(N_\mathrm{m}+1)^2}$: Measurements errors

Estimating c_j and $\mathbf{r}_j$ from $\boldsymbol{\alpha}$ makes it possible to reconstruct $u(\cdot)$

Sound Field Decomposition Based on SHD-RGF

Concept of RGF

- Test function $w_n(\cdot)$ and RGF $R(\cdot)$ for $w_n(\cdot)$ is defined as

$$w_n(\mathbf{r}) := p^n e^{ikz}, \quad p = x + iy$$

$$R(w_n) := \int_\Omega w_n(\mathbf{r}) Q(\mathbf{r}) d\mathbf{r}$$

- By applying point source assumption and Green's theorem to $R(w)$, the following equation holds:

$$\sum_{j=1}^{J} c_j w_n(\mathbf{r}_j) = \int_{\partial\Omega} \left(u(\mathbf{r}) \frac{\partial w(\mathbf{r})}{\partial \mathbf{n}} - w(\mathbf{r}) \frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} \right) dS \quad \cdots (*)$$

The parameter of sound sources J, $\mathbf{r}_j$ and c_j can be estimated from pressure and velocity values on $\partial\Omega$.

Localization based on SHD-RGF[1]

- Surface integral (Right-hand side of Eq. (*)) $s_n := R(w_n)$ can be analytically calculated in SHD[1]

$$s_n = \frac{i}{kR^2} \sum_{\nu=0}^{\infty} \sum_{\mu=-\nu}^{\nu} (-1)^{\mu+1} w_{\nu,-\mu}^{(n)} u_{\nu,\mu}$$

$w_{\nu,\mu}^{(n)}$: Analytical expansion coefficient of $w_n(\mathbf{r})$ by spherical harmonics[1]

- Compose Hankel matrices with $s_n := R(w_n)$ and estimate the source locations on x-y plane by eigenvalue decomposition.

Closed-form solutions can be greatly affected by measurement errors.

Structured SHD-RGF and Proposed Algorithm

Construction of AF for decomposition

- Construct AF represented by polynomial function

$$H(q) = \prod_{j=1}^{J} (1 - p_j q^{-1}) = \sum_{j=1}^{J} h_j q^{-j}$$

Its roots correspond to source locations $\{p_j\}_{j=1}^{J}$, $p_j = x_j + iy_j$

- Convolution coefficient sequence $\{h_j\}_{j=0}^{J}$ and elements $\{s_n\}_{n\in\mathbb{N}}$ $(n = 1, \ldots, J)$

$$h_{n+J} * s_{n+J} = \sum_{j=0}^{J} h_j s_{n+J-j}$$

$$= \sum_{j'=1}^{J} c_{j'} e^{-ikz_{j'}} p_{j'}^{n+J} \sum_{j=0}^{J} h_j p_{j'}^{-j} = 0 \quad \Rightarrow \quad \textbf{Annihilation: } \mathbf{h} * \mathbf{s} = \mathbf{0}$$

where $H(p_{j'}) = 0$.

- Annihilation holds for multiple time-frequency (T-F) bins when the source locations are assumed to be static for T time frames.

Annihilation using multiple T-F bins: $\mathbf{h} * \mathbf{s}_{f,t} = \mathbf{0}$

$f \in \{1, \ldots, F\}$: index of freq bins
$t \in \{1, \ldots, T\}$: index of time frames

Proposed AF-based Algorithm for SHD-RGF

- Optimization problem for SHD-RGF using AF

$$\underset{\mathbf{u}_{f,t},\mathbf{h}}{\text{minimize}} \quad \sum_{f,t} \|\boldsymbol{\alpha}_{f,t} - \mathbf{T}_f \mathbf{u}_{f,t}\|_2^2$$: Minimization of model error

$$\text{such that} \quad \mathbf{h} * \mathbf{s}_{f,t} = \mathbf{0}, \ \mathbf{h}^\mathsf{H}\mathbf{h} = 1$$: Annihilation and regularization

- Equivalent optimization problem

$$\underset{\mathbf{h}}{\text{minimize}} \quad \mathbf{h}^\mathsf{H}\boldsymbol{\Lambda}(\mathbf{h})\mathbf{h} \qquad \text{such that} \quad \mathbf{h}^\mathsf{H}\mathbf{h} = 1$$

$$\mathbf{h} * \mathbf{s}_{f,t} = \mathbf{W}_f(\mathbf{h})\mathbf{u}_{f,t} = \mathbf{V}_f(\mathbf{u}_{f,t})\mathbf{h}$$
$$\boldsymbol{\Lambda}(\mathbf{h}) := \sum_{f,t} \mathbf{V}_f(\mathbf{v}_{f,t})^\mathsf{H} \boldsymbol{\Sigma}(\mathbf{h})^{-1} \mathbf{V}_f(\mathbf{v}_{f,t})$$
$$\boldsymbol{\Sigma}(\mathbf{h}) := \mathbf{W}_f(\mathbf{h})(\mathbf{T}_f^\mathsf{H}\mathbf{T}_f)^{-1}\mathbf{W}_f(\mathbf{h})^\mathsf{H}$$
$$\mathbf{v}_{f,t} := (\mathbf{T}_f^\mathsf{H}\mathbf{T}_f)^{-1}\mathbf{T}_f^\mathsf{H}\boldsymbol{\alpha}_{f,t}$$

- Variable $\mathbf{h}$ is updated by solving above problem iteratively starting with initial value $\mathbf{h}^{(0)}$

$$\mathbf{h}^{(i+1)} = \arg\min_{\mathbf{h}} \mathbf{h}^\mathsf{H}\boldsymbol{\Lambda}(\mathbf{h}^{(i)})\mathbf{h}, \ \text{such that} \ \mathbf{h}^\mathsf{H}\mathbf{h} = 1,$$

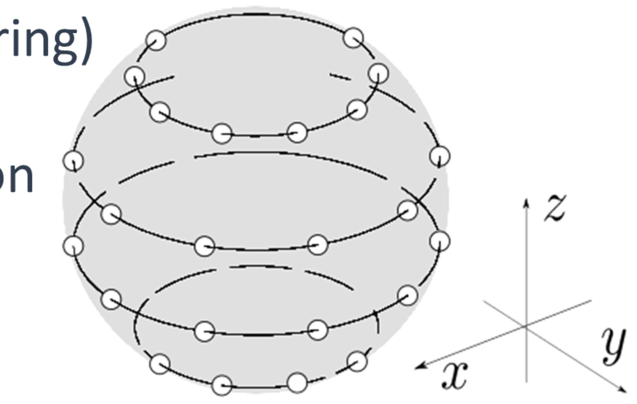
It can be computed in closed-form at each iteration

Numerical Simulations

Simulation conditions

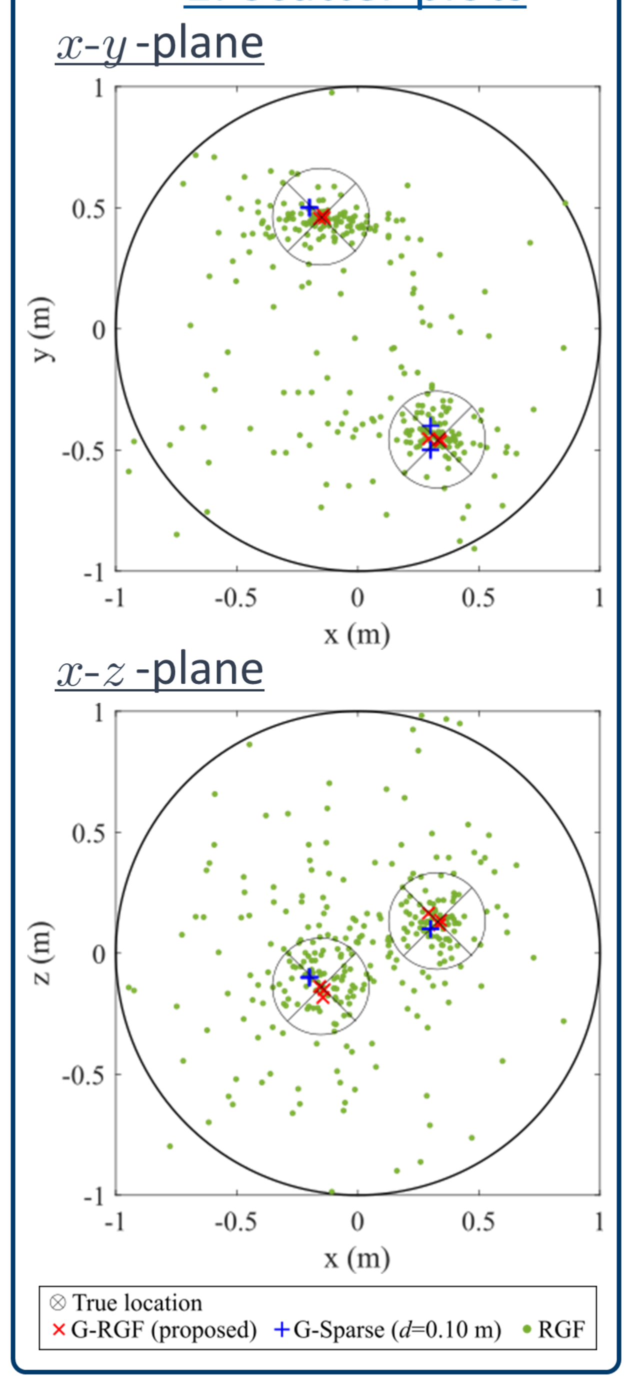
- Comparing proposed method (**G-RGF**) with
  - G-Sparse: group sparse sound field decomposition[2] (interval of grid points d of 0.10, 0.15, and 0.20 m)
  - RGF: RGF in spherical harmonic domain[1] for single time-freq. bin
- Microphone array
  - 24 second-order spherical microphone arrays
  - Four-ring geometry (nine arrays on each ring)
- Evaluation criteria
  - Root-mean-square error of source location

$$\mathrm{RMSE} = \sqrt{\frac{1}{J}\sum_{j=1}^{J} \|\mathbf{r}_{j,\mathrm{true}} - \hat{\mathbf{r}}_j\|_2^2}$$

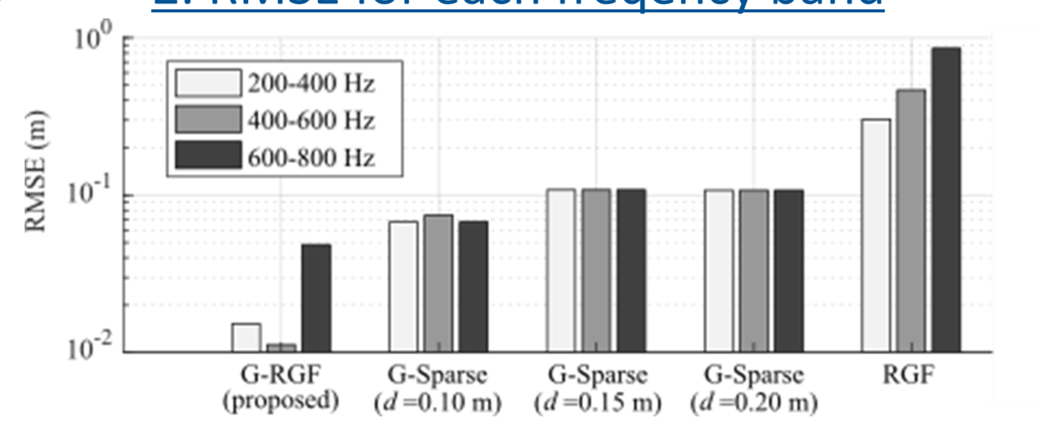


Results

1. Scatter plots

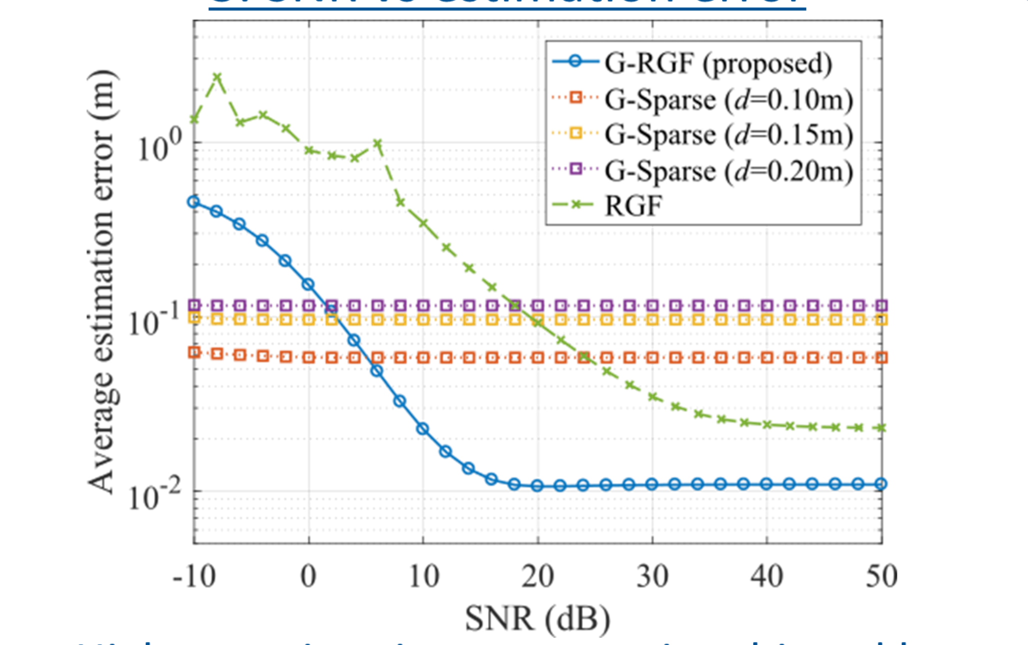


2. RMSE for each freqency band



Highest source localization accuracy is achieved by **G-RGF** for all group of time-frequency bands

3. SNR vs estimation error



Highest estimation accuracy is achieved by proposed **G-RGF** above 6 dB

▶ Off-grid problem is avoided on the basis of RGF
▶ Robustness against noise is improved by grouping T-F bins