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Minimax Magnitude Response Approximation of Pole-Radius Constrained IIR Digital Filters

Introduction

- The magnitude-response approximation of a desired IIR digital filter is highly nonconvex with respect to the filter coefficients.
- When no pole radius constraint is imposed, the problem can be transformed into a convex one for magnitude square function approximation. But resultant maximum pole radiuses may be larger than the prescribed value.
- Ref. [6] imposes a stability constraint and formulates the magnitude square function approximation as a semi-definite programming, resulting in optimal filters with guaranteed maximum pole radius.
- The iirlpnormc() function in Matlab also allows a pole-radius constraint. It minimizes the L_p-norm of the magnitude-response approximation error subject to the pole-radius constraint.
- This paper converts the pole-radius constrained magnitude-response approximation into an approximation of the desired magnitude response and an accompanied phase response.
- The accompanied phase response is iteratively updated until a solution to the original magnitude approximation problem is obtained.

Problem formulation

- Pole-radius constrained magnitude minimax design:

$$\min_{g,a\in S(\rho),b}\ \max_{\omega\in\Omega} W(\omega)\left|E(\omega,g,a,b)\right| \quad (1)$$

- $H(z,g,\boldsymbol{a},\boldsymbol{b}) = g\,B(z,\boldsymbol{b})\,/\,A(z,\boldsymbol{a})$ is transfer function of the filter, $\boldsymbol{a}$ and $\boldsymbol{b}$ are the filter's denominator and numerator coefficient vectors;
- $S(\rho)=\left\{\boldsymbol{a}\in\mathrm{R}^N \,\middle|\, \begin{array}{l}\text{all zeros of } A(z,\boldsymbol{a}) \text{ lie inside a circle}\\ \text{of radius } \rho \text{ centered at the origin}\end{array}\right\}$ is the filter's stability domain;
- $D(\omega)$ is the desired magnitude response;
- $W(\omega) > 0$ is a weight function;
- $\Omega=\Omega_p\cup\Omega_s$ is a subset of $[0, \pi]$ with Ω_p and Ω_s being the passband and stopband.

Solution algorithm

- Introduce a frequency-response minimax approximation problem as follows:

$$\min_{g,a\in S(\rho),b,\delta}\ \delta, \quad (2a)$$

$$\text{s.t.: } W(\omega)\left|H(e^{j\omega},g,a,b)-D(\omega)e^{j\phi(\omega)}\right|\le\delta,\ \omega\in\Omega \quad (2b)$$

- where $\phi(\omega)$ is some given phase function.
- ***Theorem***. Assume $H(z, g^*, \boldsymbol{a}^*, \boldsymbol{b}^*)$ is a solution filter of the magnitude-response approximation problem. Denote by $\phi^*(\omega)$ the phase of $H(e^{j\omega}, g^*, \boldsymbol{a}^*, \boldsymbol{b}^*)$. Then the solution of problem (2) with $\phi(\omega) = \phi^*(\omega)$ is also a solution of problem (1).

Solution algorithm

- The unknown phase function $\phi^*(\omega)$ is determined iteratively. To speed up the iterative algorithm, the frequency-response error constraint in (2b) is replaced by an elliptic-error constraint described as follows

$$W(\omega)\left|\mathrm{Re}[\tilde{E}_\phi(\omega,g,a,b)]+\tfrac{j}{\lambda}\mathrm{Im}[\tilde{E}_\phi(\omega,g,a,b)]\right|\le\delta,\omega\in\Omega_p$$

where

$$\tilde{E}_\phi(\omega,g,a,b)=e^{-j\phi(\omega)}H(e^{j\omega},g,a,b)-D(\omega)$$

is a transformed frequency-response error and $\lambda\ge1$ is an algorithm parameter.
- The iterative algorithm for solving the minimax magnitude-response approximation problem (1) is presented as Algorithm 1 below.

Algorithm 1

Step 1. Given an initial phase function $\phi_0(\omega)$, and $\boldsymbol{a}_0=\boldsymbol{0}$. Let $k=0$.

Step 2. Solve for $\boldsymbol{a}_{k+1}$ and $\boldsymbol{b}_{k+1}$ the following nonconvex minimax complex approximation subproblem

$$(g_{k+1},a_{k+1},b_{k+1},\delta_{k+1})=\operatorname*{argmin}_{g,a\in S(\rho),b,\delta}\ \delta, \quad (3a)$$

$$\text{s.t.: } W(\omega)\left|H(e^{j\omega},g,a,b)\right|\le\delta,\ \omega\in\Omega_s, \quad (3b)$$

$$W(\omega)\left|\mathrm{Re}[\tilde{E}_{\phi_k}(\omega,g,a,b)]+\tfrac{j}{\lambda}\mathrm{Im}[\tilde{E}_{\phi_k}(\omega,g,a,b)]\right|\le\delta,\omega\in\Omega_p. \quad (3c)$$

Step 3. Compute the phase response $\phi_{k+1}(\omega)$ of the filter $H(z,g_{k+1},a_{k+1},b_{k+1})$.

Step 4. If some stop condition is satisfied, terminate. Otherwise, let $k=k+1$ and go back to Step 2.

- To solve the nonconvex minimax problem (3), the Gauss-Newton strategy is applied. That is, let

$$H(e^{j\omega},x)=H(e^{j\omega},x(\ell))+\nabla H(e^{j\omega},x(\ell))^T[x-x(\ell)]$$

$$\partial H(e^{j\omega},x)/\partial x\,[x-x(\ell)]$$

where $x=[g,\boldsymbol{a}^T,\boldsymbol{b}^T]^T$.
- To guarantee the stability, the GPR-based constraint is imposed, i.e.,

$$-\mathrm{Re}[e^{-j\theta_\ell(\omega)}A(\rho e^{j\omega},a)]<0,\ \ \omega\in[0,\pi]$$

where $\theta_\ell(\omega)$ is the phase of $A(\rho e^{j\omega},\boldsymbol{a}(\ell))$.
- Concrete algorithm for problem (3) is described in Algorithm 2 of the paper.

Simulation & comparison

- ***Example 1***. Order-4 low-pass filters with a passband $[0,0.2\pi]$ and a stopband $[0.45\pi,\pi]$. $W(\omega)$ = 1.0 in the passband and 188.5 in the stopband.

Method	ellip()	Proposed 1	Proposed 2
R_p(dB)	1.88463	1.88463	2.48335
R_s(dB)	65.724	65.724	63.6146
r_{max}	0.9422	0.9423	0.9178

Method	R_p (dB)	R_s (dB)	r_{max}	z_{max}
Proposed	2.9866	62.2503	0.9177	0.92
Method [6]	3.0	60.0	0.9061	0.9192

Simulation & comparison

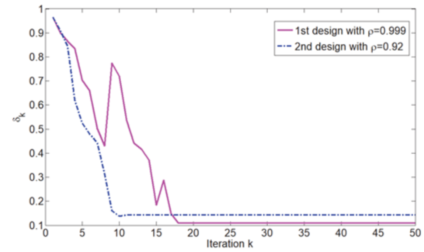


Fig. 1 Convergence of δ_k by the proposed Algorithm 1 in the first two designs of Example 1

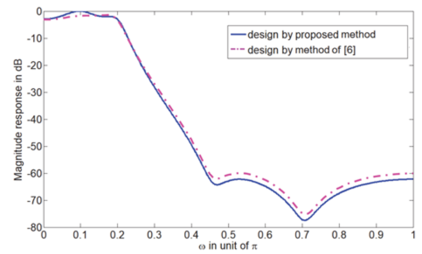
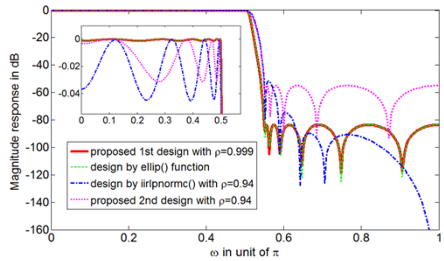


Fig. 2 Magnitude responses of the two filters with pole/zero radiuses smaller than 0.92

- ***Example 2***. Order-12 low-pass filters with passband $[0,0.5\pi]$ and stopband $[0.6\pi,\pi]$. $W(\omega)$ = 1.0 in both the passband and stopband.

Method	Proposed	ellip()	Proposed	iirlpnormc
R_p(dB)	0.00116	0.00116	0.03125	0.04532
R_s(dB)	83.525	83.525	54.914	51.464
r_{max}	0.9781	0.9781	0.9377	0.94



Key references

1. A. Antoniou, *Digital Signal Processing: Signals, Systems, and Filters*, McGraw-Hill, New York, 2005.
2. L. R. Rabiner, N. Y. Graham, and H. D. Helms, "Linear programming design of IIR digital filters with arbitrary magnitude function," *IEEE TASSP*, vol. ASSP-22, no. 2, pp. 117-123, 1974.
3. B. Limketkai, W. Ng, and T. Rockwood, "Optimization-based synthesis of photonic IIR filters accounting for internal losses in microresonators," *Journal of Lightwave Technology*, vol. 35, no. 20, pp. 4459-4467, 2017.
4. X. Lai and Z. Lin, "Optimal Design of Constrained FIR Filters without Phase Response Specifications," *IEEE TSP*, vol. 62, no. 17, pp. 4532-4546, 2014.

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