ENLLVM: Ensemble based Nonlinear Bayesian Filtering using Linear Latent Variable Models

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Abstract

- Real-time nonlinear Bayesian filtering algorithms are overwhelmed by data volume, velocity and increasing complexity of computational models.
- Novel ensemble based nonlinear Bayesian filtering requires a small number of simulations and can be applied to high-dimensional systems in the presence of intractable likelihood functions.
- It uses linear latent projections to estimate the joint probability distribution between states, parameters, and observables using a mixture of Gaussian components generated by the reconstruction error for each ensemble member.

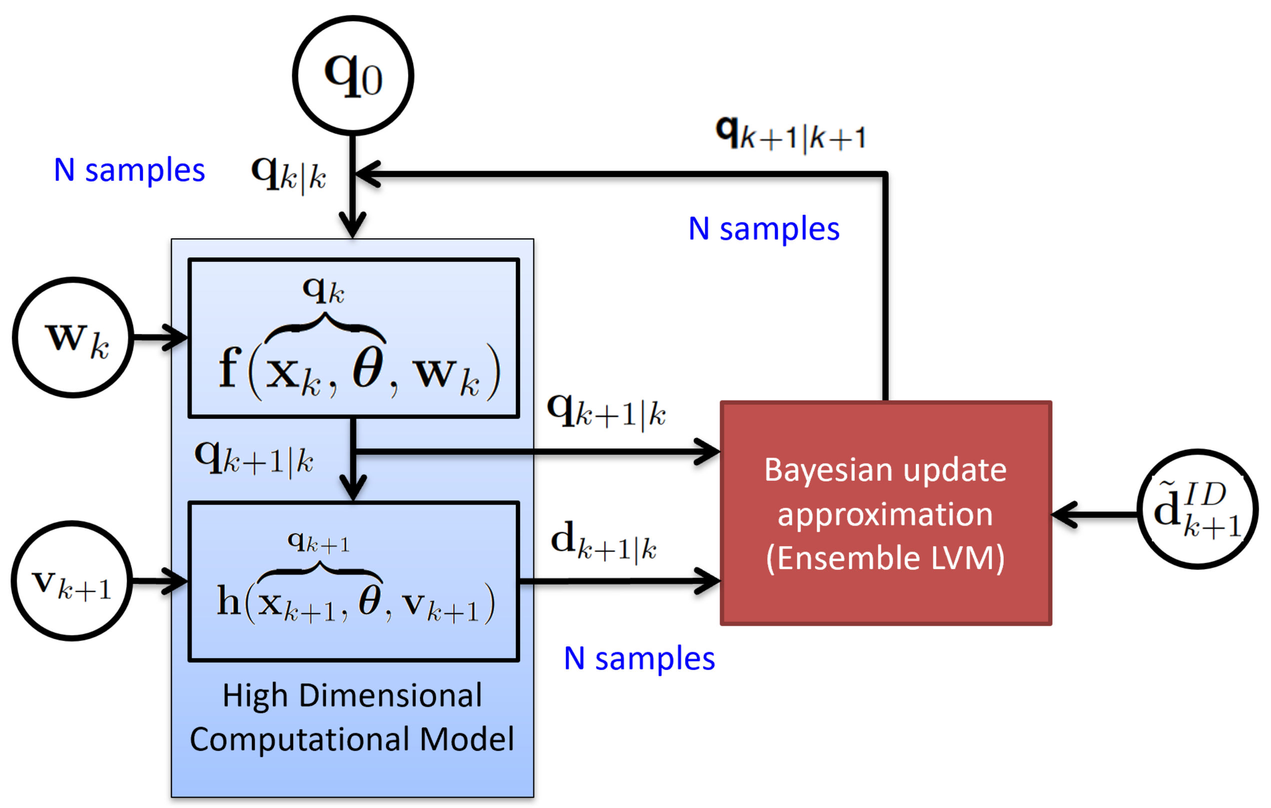
What do we want?

Nonlinear dynamical system

$$\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k, \mathbf{w}_k, \theta)$$
$$\mathbf{d}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k, \theta)$$

Uncertain initial condition

$$\mathbf{x}_0 \sim p(\mathbf{x}_0)$$
$$\theta \sim p(\theta)$$

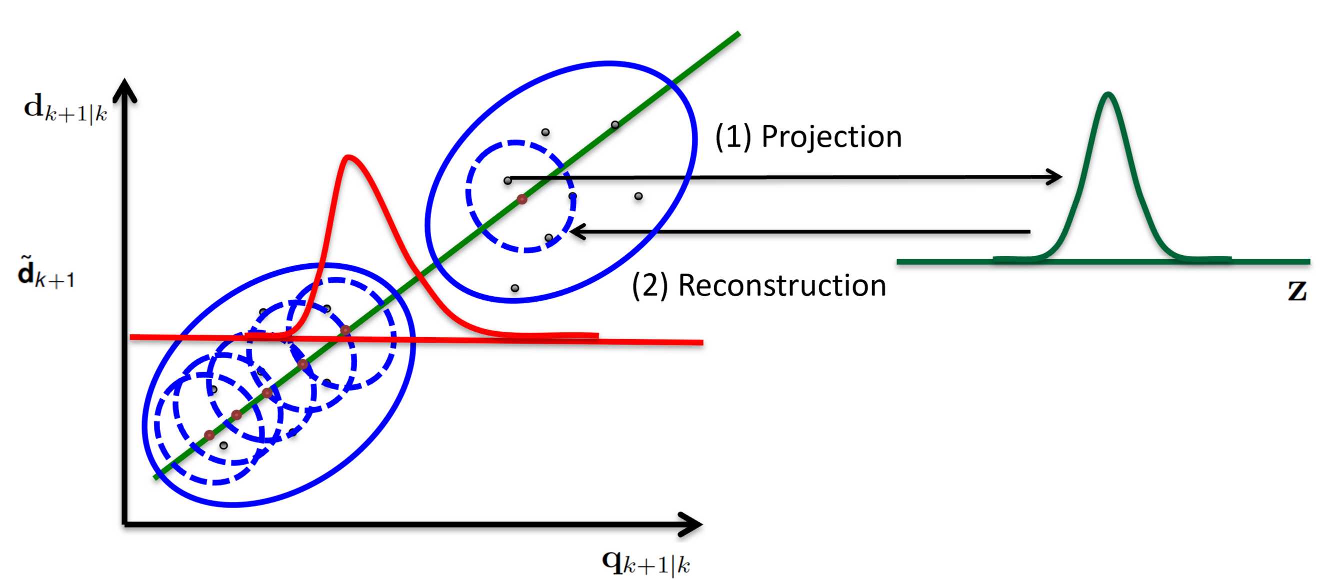


Goal - posterior pdf

$$p(\mathbf{x}_k, \theta \mid \mathbf{D}_k) = \frac{p(\tilde{\mathbf{d}}_k \mid \mathbf{x}_k, \theta, \mathbf{D}_{k-1}) p(\mathbf{x}_k, \theta \mid \mathbf{D}_{k-1})}{p(\tilde{\mathbf{d}}_k \mid \mathbf{D}_{k-1})}$$

Methodology

$$\mathbf{q}_{k+1|k} = \mathbf{W}_q \mathbf{z} + \mu_q + \eta_q$$
$$\mathbf{d}_{k+1|k} = \mathbf{W}_d \mathbf{z} + \mu_d + \eta_d$$
$$\mathbf{z} \sim \mathcal{N}(0, \mathbf{I}_{MxM})$$
$$\eta_q \sim \mathcal{N}(0, \mathbf{\Psi}_q)$$
$$\eta_d \sim \mathcal{N}(0, \alpha \mathbf{\Psi}_d)$$
$$\alpha > 1.0$$



LLVM as a density estimator

$$p(\mathbf{q}_{k+1|k}, \mathbf{d}_{k+1|k}) = \mathcal{N}(\mu, \mathbf{W}\mathbf{W}^T + \mathbf{\Psi})$$

LLVM and Ensemble LLVM provide the same estimate for the mean and covariance of the samples.

Ensemble LLVM as density estimator

$$p(\mathbf{q}_{k+1|k}, \mathbf{d}_{k+1|k}) = \frac{1}{N} \sum_{i=1}^{N} \mathcal{N}(\mathbf{W}E[\mathbf{z}|\mathbf{q}^i_{k+1|k}, \mathbf{d}^i_{k+1|k}] + \mu, \mathbf{W}Cov[\mathbf{z}|\mathbf{q}^i_{k+1|k}, \mathbf{d}^i_{k+1|k}]\mathbf{W}^T + \mathbf{\Psi})$$

However Ensemble LLVM capture higher order statistics as compared with LLVM.

Finding the hyper-parameter α

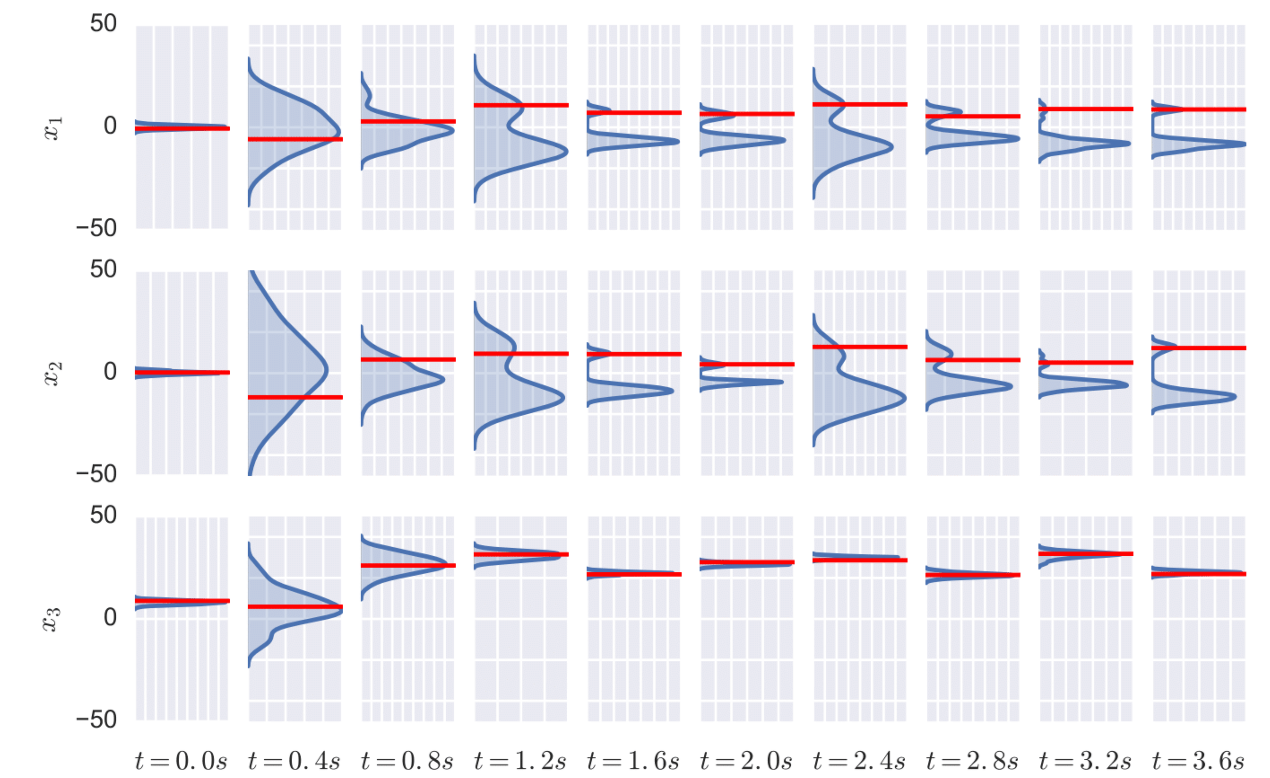
$$\alpha = \arg\max \mathcal{N}(\tilde{\mathbf{d}}_{k+1}; \mu_d, \mathbf{W}_d \mathbf{W}_d^T + \alpha \mathbf{\Psi}_d)$$

Numerical Results Lorenz 63

$$dx_1/dt = -cx_1 + cx_2$$
$$dx_2/dt = -x_1 x_3 + r x_1 - y_2$$
$$dx_3/dt = x_1 x_2 - b x_3$$

$$d_k = \sqrt{x_1(t_k)^2 + x_2(t_k)^2 + x_3(t_k)^2} + v_k, \; v_k \sim \mathcal{N}(0,1)$$

$$p(x(t_0)) \sim 0.5\mathcal{N}([-0.2,-0.2,8]^T, \sqrt{0.35}\mathbf{I}_3) + 0.5\mathcal{N}([0.2,0.2,8]^T, \sqrt{0.35}\mathbf{I}_3)$$



Numerical Results - Lorenz 96

$$dx_j(t)/dt = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + 8, \; j = 1 \dots 40$$

Data generation process:

$$M1: \; dx_j(t)/dt = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + 9$$
$$M2: \; dx_j(t)/dt = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + 10$$
$$M3: \; dx_j(t)/dt = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + 11$$
$$M4: \; dx_j(t)/dt = (x_{j+1} - x_{j-2}) x_{j-1} - x_j + 12$$

Linear measurement model:

$$d_j(t) = x_{2j-1}(t) + v_j(t), \quad v_j(t) \sim \mathcal{N}(0, I_{20})$$

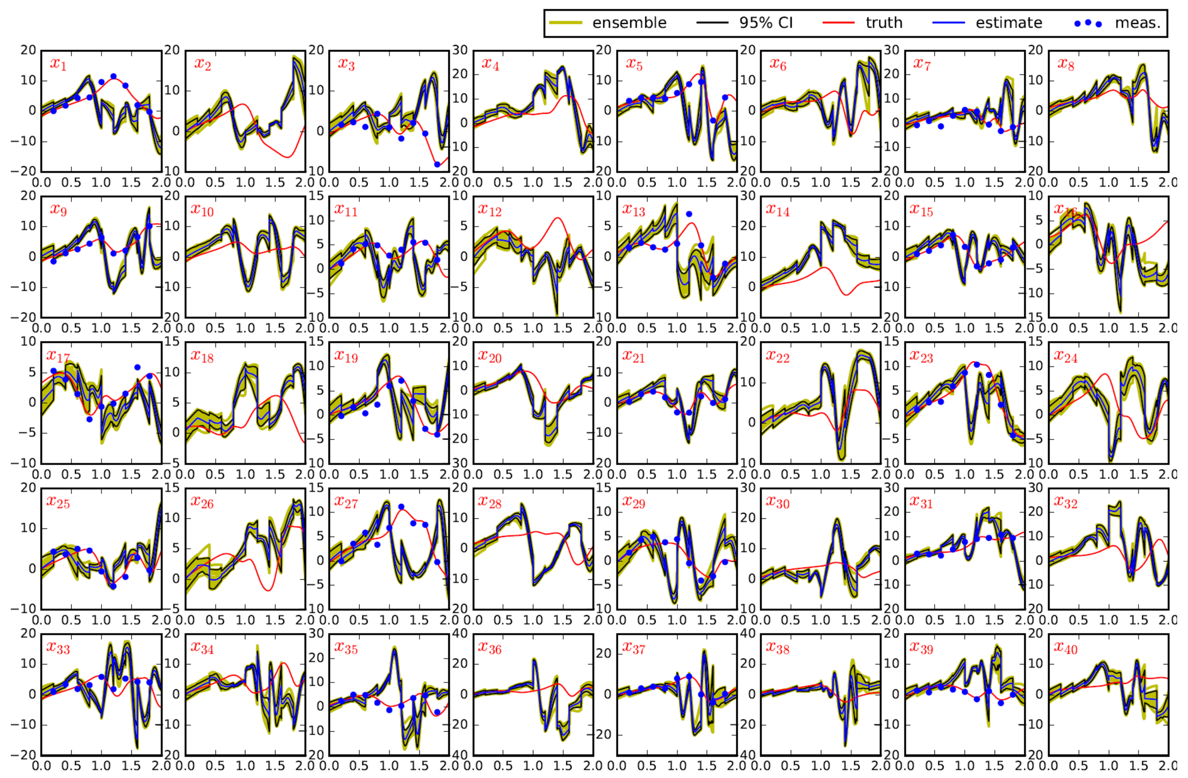
Nonlinear measurement models:

$$(I): \; d_j(t) = x_{2j-1}(t) x_{2j}(t) + v_j(t), \quad v_j(t) \sim \mathcal{N}(0, I_{20})$$
$$(II): \; d_j(t) = x_{2j-1}(t)^2 + v_j(t), \quad v_j(t) \sim \mathcal{N}(0, I_{20})$$

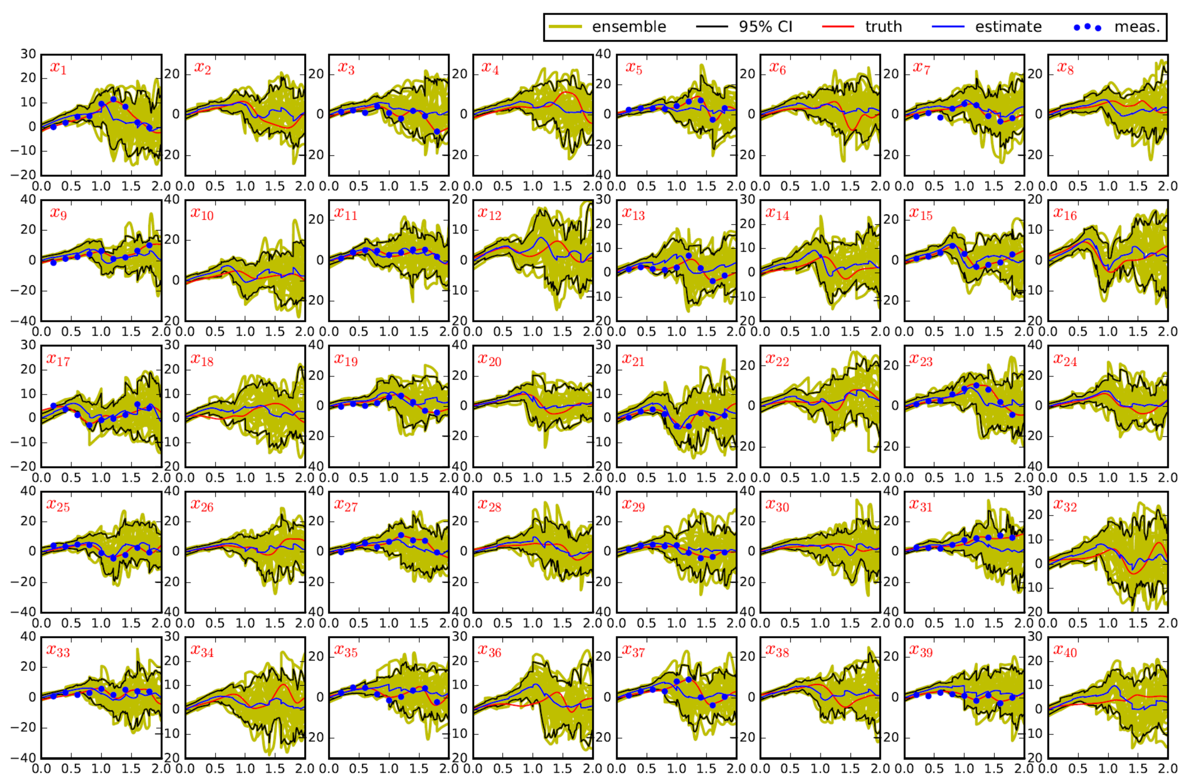
Table 1. EnKF vs EnPPCA: RMSE statistics of 100 trials

Model Error	EnKF		EnPPCA	
	Mean	STD	Mean	STD
Linear Measurement Model				
No err.	**2.71**	0.61	2.92	**0.23**
M1: 9	3.71	0.63	**2.99**	**0.23**
M2: 10	4.63	0.71	**3.07**	**0.22**
M3: 11	5.31	0.81	**3.18**	**0.21**
M4: 12	5.9	0.84	**3.27**	**0.20**
Nonlinear Measurement Model (I)				
No err.	**1.87**	**0.53**	2.85	0.26
M1: 9	3.3	0.63	**2.91**	**0.22**
M2: 10	4.37	0.47	**3.02**	**0.21**
M3: 11	5.01	0.44	**3.17**	**0.17**
M4: 12	5.51	0.49	**3.32**	**0.17**
Nonlinear Measurement Model (II)				
No err.	2.87	0.76	**2.84**	**0.24**
M1: 9	3.78	0.75	**2.94**	**0.22**
M2: 10	4.78	0.56	**3.03**	**0.20**
M3: 11	5.43	0.53	**3.16**	**0.19**
M4: 12	6.1	0.55	**3.37**	**0.17**

Joint space is 60 dimensional, only 30 samples are used and a latent space of dimension of 5.



EnKF - Model Error (forcing = 12)



EnPPCA - Model Error (forcing = 12)

- Model error - lazy approach - model uncertainty pushed into parameter/state uncertainty
- Model error - intractable likelihood - requires just simulations of forward model