Graph Filtering with Multiple Shift Matrices

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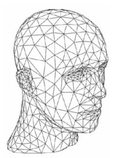
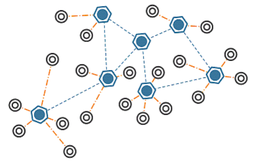
MOTIVATION

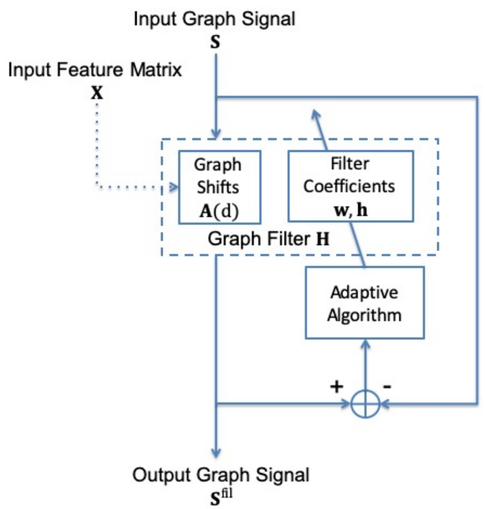
- Graphs can capture complex relational characteristics.
- Graph signal processing has advantage in dealing with datasets with irregular and complex structures.
- Adopting multiple shift matrices provides more flexibility in graph filter design.

POTENTIAL APPLICATIONS

- A classifier for data labeling.
- An error detector for network analysis.
- A pre-process of neural networks for reducing computation and mitigating overfitting risk.

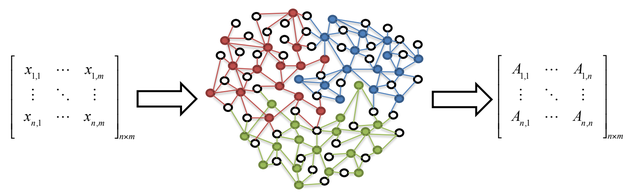




GRAPH FILTER DIAGRAM



PROBLEM STATEMENT

- A partially labeled dataset with graph encoded inner interaction.
  Graph vertices: data points.
  Graph edges: similarities among the vertices.
- The feature qualities of vertices are uneven.
- Graph shift matrices are generated from the dataset.
- A graph filter is designed as the classifier.



GRAPH FILTERING PROCEDURE

- Graph Filtering:

$$\mathbf{S}^{\text{fil}} = \mathbf{HS}$$

- Conventional Graph Filter Design Method:

$$A_{i,j} = \frac{\exp\left(-\frac{\rho(\mathbf{x}_i, \mathbf{x}_j)}{\sigma}\right)}{\sum_{i=1}^{N}\exp\left(-\frac{\rho(\mathbf{x}_i, \mathbf{x}_j)}{\sigma}\right)}$$

$$\mathbf{H} = h_0\mathbf{I} + h_1\mathbf{A} + h_2\mathbf{A}^2 + \cdots + h_L\mathbf{A}^L$$

- Proposed Graph Filter Design Method:

$$A(d)_{i,j} = \frac{\exp\left(-\frac{(x_{i,d} - x_{j,d})^2}{\sigma}\right)}{\sum_{i=1}^{N}\exp\left(-\frac{(x_{i,d} - x_{j,d})^2}{\sigma}\right)}$$

$$\mathbf{H} = \sum_{d=1}^{D}\sum_{l=1}^{L} w_d h_l \boldsymbol{A}(d)^l$$

ACKNOWLEDGEMENTS

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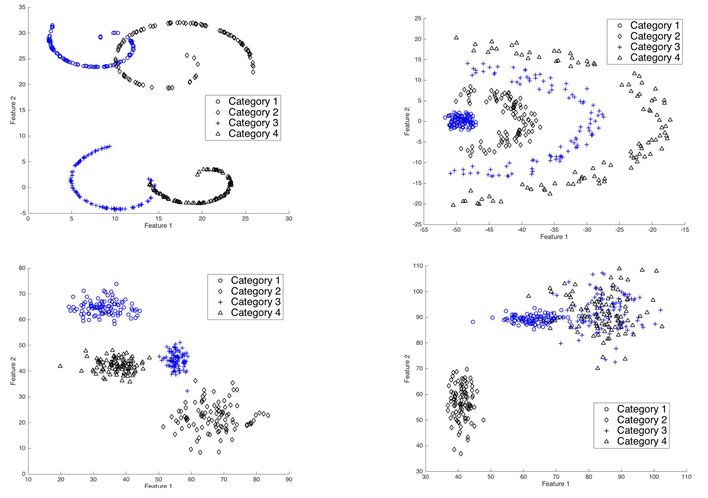
SIMULATION DATA WITH UNEVEN FEATURES



CONCLUSION

- A well designed graph filter can work as a semi-supervised classifier.
- The proposed filter designing method provides lower error rate than the conventional one when feature qualities are uneven.
- Our method is especially suitable for practical applications whose initial information is encrypted or insufficient due to privacy policies and measuring difficulties.

REFERENCES

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