Improved Set-Membership Partial-Update Affine Projection Algorithm

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Presentation Outline

1. Introduction
2. Set-Membership Filtering (SMF)
3. Set-Membership Partial-Update Affine Projection (SM-PUAP) Algorithm
4. Improved Set-Membership Partial-Update Affine Projection(I-SM-PUAP) Algorithm
5. Results
6. Conclusions

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Paper and Presentation Contents

- Powerful approaches to decrease computational complexity:
  1. Set estimation theory
  2. Partial-Update strategy
- Proposed algorithms in this paper:
  - Set-Membership Partial-Update Affine Projection (SM-PUAP)
  - Improved Set-Membership Partial-Update Affine Projection (I-SM-PUAP)

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Set Estimation Theory

- Finds a solution to a given optimization problem $\longrightarrow$ Any solution within the feasible set is acceptable
- Examples of estimators:
  - Batch processing: few techniques (usually too complex)
  - Iterative processing: optimal-bounding-ellipsoids (OBE) and set-membership (SM) algorithms

Partial-Update Strategy

- At each iteration of the algorithm, only part of the filter coefficients are updated
- Examples of PU strategy:
  - Partial-Update Least-Mean-Square Algorithm
  - Partial-Update Normalized LMS Algorithm
  - Partial-Update Affine Projection Algorithm

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Formulation

Main Sets

- Constraint set:

$$\mathcal{H}(k) \triangleq \left\{\mathbf{w} \in \mathbb{R}^N : |d(k) - \mathbf{w}^T\mathbf{x}(k)| \leq \overline{\gamma}\right\}$$

  where
  - Error: $e(k) \triangleq d(k) - \mathbf{w}^T\mathbf{x}(k)$
  - Uncertainties are modeled via $\overline{\gamma}$

- Feasibility set:

$$\Theta \triangleq \bigcap_{k\in\mathbb{N}} \mathcal{H}(k)$$

Problem Formulation

- Inputs: all data-pairs $(\mathbf{x}(k), d(k))$
- Target: find $\mathbf{w} \in \Theta$

Challenges

- Incomplete data
  - Impossible to guarantee that all input data-pairs are available
  - Online/iterative processing:
    - Must produce an estimate every time a new input data-pair arrives
    - Θ can be iteratively estimated via $\psi(k)$

$$\psi(k) \triangleq \bigcap_{i=0}^{k} \mathcal{H}(i)$$

    - $\psi(k)$ converges to Θ as $k \to \infty$
    - Problem: $k \to \infty \longrightarrow$ Infinite memory and prohibitive complexity
    - Solution: Use a finite number of constraint sets at each iteration $\longrightarrow$ SM affine projection algorithms

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SM-PU Algorithms: Overview

- Inputs (general case):
  - current data-pair $(\mathbf{x}(k), d(k))$, i.e., $\mathcal{H}(k)$
  - L previous data-pairs, i.e., L previous constraint sets

SM-PUAP Algorithm

- Inputs: L last data-pairs $\longrightarrow$ data is stacked sequentially

$$\mathbf{X}(k) \triangleq [\mathbf{x}(k)\ \mathbf{x}(k-1)\ \cdots\ \mathbf{x}(k-L+1)] \in \mathbb{R}^{N\times L}$$
$$\mathbf{d}(k) \triangleq [d(k)\ d(k-1)\ \cdots\ d(k-L+1)]^T \in \mathbb{R}^{L}$$

- At each iteration we have the estimator

$$\psi_{\mathrm{SM-PUAP}}(k) \triangleq \bigcap_{i=k-L+1}^{k} \mathcal{H}(i)$$

- At each iteration we update the coefficients determined by an index set

$$\mathcal{I}_M(k) = \{i_1(k), \cdots, i_M(k)\} \subset \{1, \cdots, N\}$$

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SM-PUAP Algorithm: Question

- At each iteration k we generate $\mathbf{w}(k+1) \in \psi_{\text{SM-PUAP}}(k)$
- We would like to have $\mathbf{w}(k+1) \in \psi(k) \rightarrow \Theta$, as $k \rightarrow \infty$
- Question: How to combine the elements of the sequence $\{\mathbf{w}(0), \mathbf{w}(1), \ldots, \mathbf{w}(k)\}$ to generate $\mathbf{w}(k+1) \in \psi(k)$?
  - We may focus on the transition of iterations and then expand the idea
  - At iteration $k-1 \longrightarrow \mathbf{w}(k) \in \psi_{\text{SM-PUAP}}(k-1) = \mathcal{H}(k-L) \bigcap \left(\bigcap_{i=k-L+1}^{k-1} \mathcal{H}(i) \right)$
  - At iteration $k \longrightarrow \mathbf{w}(k+1) \in \psi_{\text{SM-PUAP}}(k) = \mathcal{H}(k) \bigcap \left(\bigcap_{i=k-L+1}^{k-1} \mathcal{H}(i) \right)$
  - Answer:
    - Change coefficients as little as possible: $\min \|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2$

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SM-PUAP Algorithm: Optimization Problem

- Problem:

$$\begin{aligned} \min \quad & \|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2 \\ \text{s.t.} \quad & \mathbf{w}(k+1) \in \psi_{\text{SM-PUAP}}(k) \\ & \tilde{C}_{\mathcal{I}_M(k)}[\mathbf{w}(k+1) - \mathbf{w}(k)] = 0 \end{aligned}$$

- Solution:

$$\mathbf{w}(k+1) = \begin{cases} \mathbf{w}(k) + C_{\mathcal{I}_M(k)}\mathbf{X}(k)\mathbf{P}(k)\left[\mathbf{e}(k) - \boldsymbol{\gamma}(k)\right] & \text{if } |e_0(k)| > \overline{\gamma}, \\ \mathbf{w}(k) & \text{otherwise,} \end{cases}$$

where

$$\begin{aligned} \mathbf{P}(k) &\triangleq \left[\mathbf{X}^T(k)C_{\mathcal{I}_M(k)}\mathbf{X}(k) + \delta\mathbf{I}\right]^{-1} \in \mathbb{R}^{L\times L} \\ \mathbf{e}(k) &= [e_0(k)\ e_1(k)\ \cdots\ e_{L-1}(k)]^T \in \mathbb{R}^L \\ \boldsymbol{\gamma}(k) &= [\gamma_0(k)\ \gamma_1(k)\ \cdots\ \gamma_{L-1}(k)]^T \in \mathbb{R}^L, \text{ with } |\gamma_i(k)| \le \overline{\gamma} \end{aligned}$$

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Why I-SM-PUAP instead of SM-PUAP?

- Drawback of SM-PUAP algorithm
  - Decrease the convergence speed by increasing $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2$
- Suggestions to address the drawback
  - Decrease the error bound $\overline{\gamma}$ but causes more updates and high computational complexity
  - Control the increment of $\|\mathbf{w}(k+1) - \mathbf{w}(k)\|_2^2$

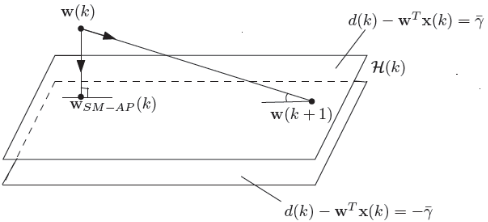

Figure: Projection in partial-update algorithm

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I-SM-PUAP Algorithm

How to control the increment of $\|\mathbf{w}(k+1)-\mathbf{w}(k)\|_2^2$?

Project $\mathbf{w}(k)$,with the same direction as SM-PUAP update, on a sphere centered at $\mathbf{w}(k)$ whose radius is the minimum of distances between the $\mathbf{w}(k)$ and its perpendicular projections on surfaces $d(k)-\mathbf{w}^T\mathbf{x}(k)=\pm\overline{\gamma}$

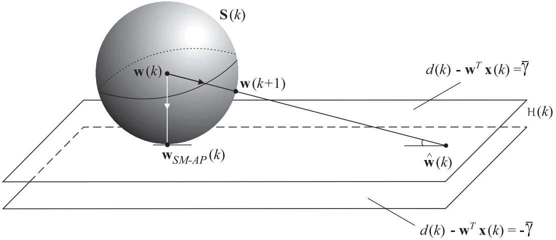

Figure: Projection in improved partial-update algorithm

I-SM-PUAP Update Equation

Intersect the segment $\overline{\mathbf{w}(k)\hat{\mathbf{w}}(k)}$ and the sphere $S(k)$ we get

$$\mathbf{w}(k+1) = \begin{cases} \mathbf{w}(k) + \frac{\mu(k)}{\|\mathbf{a}(k)\|_2}\mathbf{a}(k) & \text{if } |e_0(k)| > \overline{\gamma}, \\ \mathbf{w}(k) & \text{otherwise,} \end{cases}$$

where

$$\mathbf{a}(k) \triangleq C_{\mathcal{I}_{M(k)}}\mathbf{X}(k)\left[\mathbf{X}^T(k)C_{\mathcal{I}_{M(k)}}\mathbf{X}(k) + \delta\mathbf{I}\right]^{-1}\mathbf{e}(k) \in \mathbb{R}^N$$

$$\mu(k) \triangleq \min\left(\frac{|-e_0(k) \pm \overline{\gamma}|}{\|\mathbf{x}(k)\|_2}\right) \in \mathbb{R}_+$$

$$\mathbf{e}(k) = [e_0(k)\ e_1(k)\ \cdots\ e_{L-1}(k)]^T \in \mathbb{R}^L$$

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Simulations

- Learning (MSE) curves for SM-PUAP and I-SM-PUAP algorithms

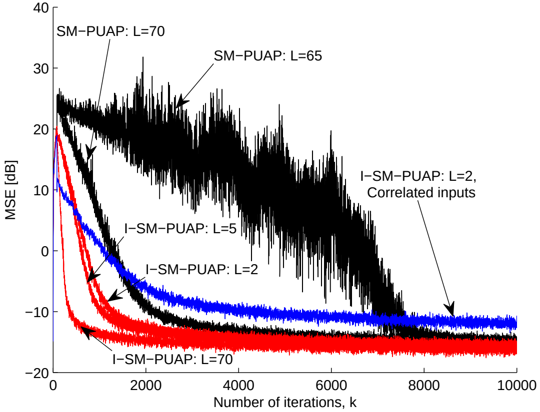

Figure: Learning curve for SM-PUAP and I-SM-PUAP algorithms, $N = 80$

Update Rate

-

Input	N	L	SM-PUAP
$\mathcal{N}(0,1)$	80	65	25%
$\mathcal{N}(0,1)$	80	70	14%

-

Input	N	L	I-SM-PUAP
$\mathcal{N}(0,1)$	80	2	8.3%
$\mathcal{N}(0,1)$	80	5	6.5%
$\mathcal{N}(0,1)$	80	70	2%
4th-AR	80	2	20%

Summary of observations for I-SM-PUAP vs. SM-PUAP

- I-SM-PUAP algorithm has lower MSE
- I-SM-PUAP algorithm has faster convergence rate
- I-SM-PUAP algorithm has lower computational complexity
- I-SM-PUAP algorithm has lower update rate

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Conclusions

- In this presentation:
  - Revisited set estimation theory with emphasis on set-membership filtering
  - Revisited set-membership partial-update affine projection algorithm
  - Presented improved set-membership partial-update affine projection algorithm

Thank You!