Accuracy Evaluation Based on Simulation for Finite Precision Systems Using Inferential Statistics

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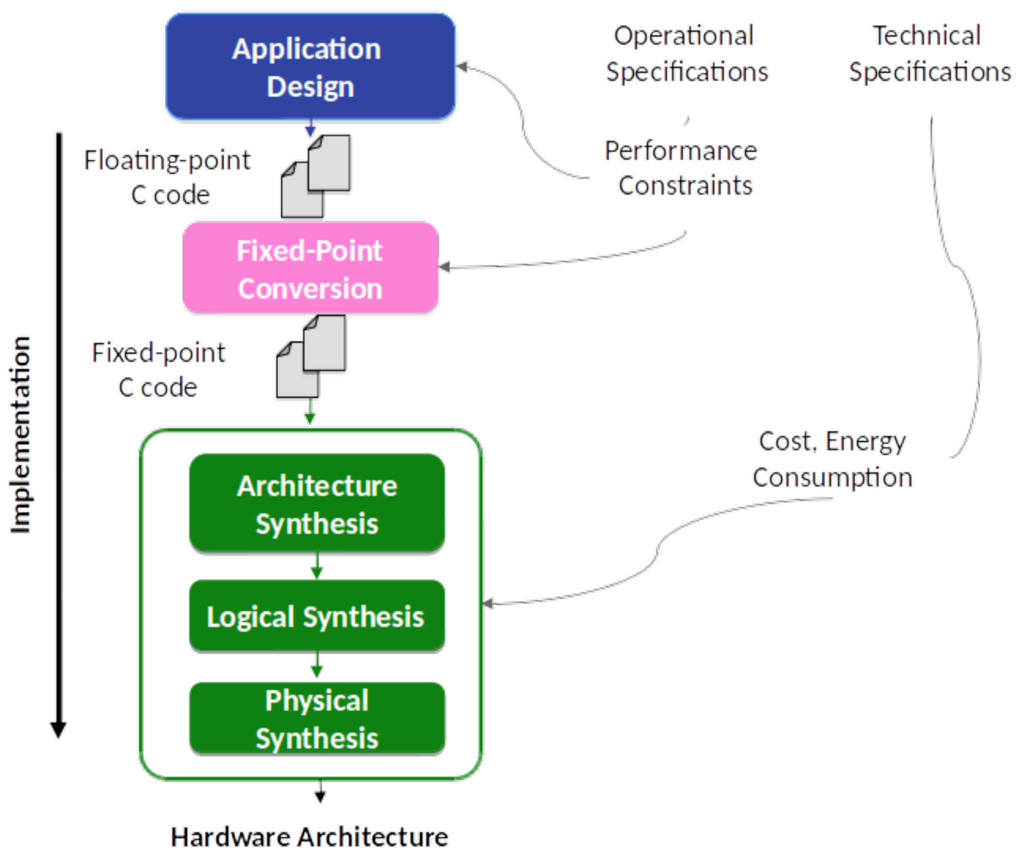
Context and Motivations

- Embed high complexity algorithms on over-**constrained** platforms
  - Memory, energy, computation time constraints
- Approximate computing (ACx) trades-off quality and cost
  - Data level ACx: **Fixed-point** arithmetic



Objectives

- Fixed-point refinement process
  - NP-hard optimization problem

$$\min(C(\mathbf{w})) \quad \text{subject to} \quad P(\mathbf{w}) < P_m$$

  - C: implementation cost
  - **w**: word-length vector
  - P(**w**): noise power for word-lengths **w**
  - P_m: noise power constraint
- Challenge : accuracy evaluation
  - Noise power $P(\mathbf{w})$ evaluation
  - Simulation-based methods with a fixed number of samples $[10^5;10^{12}]$

Quantization noise power characterization

- Quantization noise power $P = E[e_x^2]$
- Error distance $e_x = |x_Q - x_\infty|$
  - x_Q: application output with fixed-point data types
  - x_∞: application output with floating-point data types
- Mean and standard deviation estimators

$$\overline{\mu_{e_x^2}} = \frac{1}{T}\sum_{i=1}^{T} e_{x,i}^2$$

$$\tilde{S}^2 = \frac{1}{T}\sum_{i=1}^{T}(e_{x,i}^2 - \overline{\mu_{e_x^2}})^2$$

- Inferential statistics
  - Confidence interval $\text{IC}_{\mu_{e_x^2}}$
    - Estimation of $\mu_{e_x^2}$ included in $\text{IC}_{\mu_{e_x^2}}$ with a probability p

$$\text{IC}^p_{\mu_{e_x^2}} = [\overline{\mu_{e_x^2}} - a^\alpha_{\mu_{e_x^2}}; \overline{\mu_{e_x^2}} + a^\alpha_{\mu_{e_x^2}}] \qquad a^\alpha_{\mu_{e_x^2}} = z_\alpha(p)\cdot\frac{\tilde{S}}{\sqrt{N_P - 1}}$$

  - Minimal number of samples to simulate for a desired accuracy h

$$N_{\mu_{e_x^2}} > \frac{z_\alpha^2 \cdot \tilde{S}^2}{h^2}$$

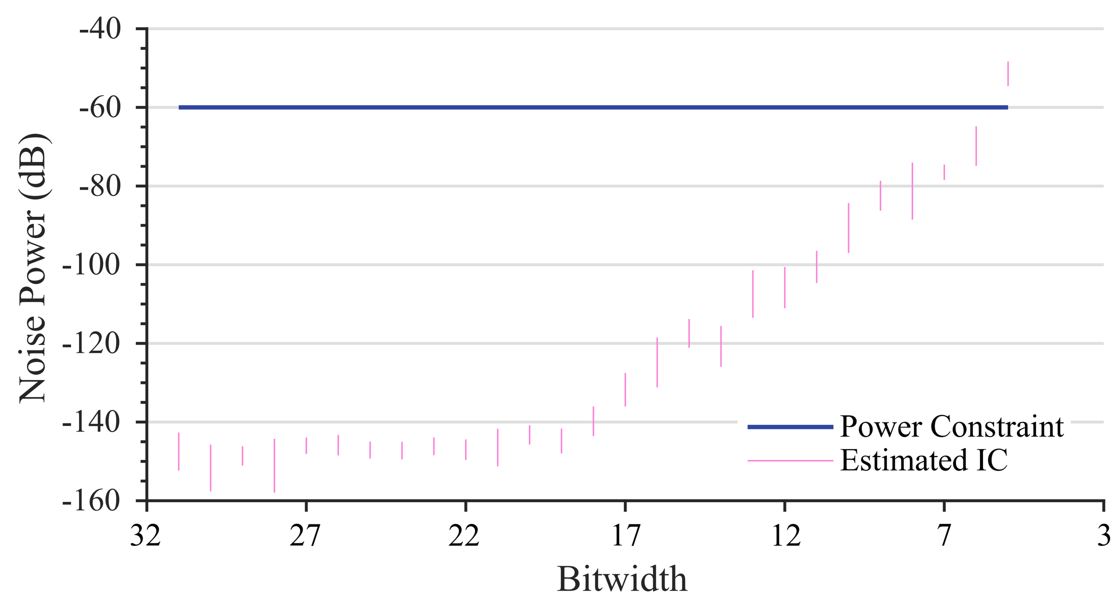
Algorithm to compute N

Algorithm 1 Proposed Computation of N_P

```
procedure COMPUTEN_P(ℰ, h, p, T)
    E ← ∅
    n ← 0
    repeat
        (e_n^2, .., e_{n+T}^2) ← sampling(ℰ, T)
        E ← E ⋃ (e_n^2, .., e_{n+T}^2)
        mean(μ_{e_x^2}) ← computeMean(E, n + T)
        S̃^2 ← computeSD(E, n + T, mean(μ_{e_x^2}))
        N_P ← computeN(S̃^2, h)
        n ← n + T
    until n ≥ N_P
    return N_P
end procedure
```

Exploitation for Word-Length Optimization

- Adaptation of the number of samples to simulate
  - Comparison with a threshold (constant) c
    - Minimal number of samples to take a decision i.e. $c \notin \text{IC}_{\mu_{e_x^2}}$



Experiments: quality results

- N_P for varied elementary blocks and (h,p).

		Accuracy of estimation %			
		0.01		0.001	
	IC%	N_P	ICExpe	N_P	ICExpe
FIR(64)	68	45	68.4	55	69.7
	95	45	95	55.5	95.3
	98	30	98.7	77.63	98.8
	99	55	99	93.8	98.8
Quantization 8-bit to 6-bit	68	65	68	474	69
	95	145	94.5	1785.8	95.4
	98	145	98.4	2973	99
	99	205	99.2	3012	99.4
Quantization float to 8-bit	68	18	69.6	857.8	70.3
	95	42.7	95.1	3253.6	95.3
	98	39	98	5766	98.9
	99	54	99	5565	99.1

- Exploitation to accelerate Word-Length Optimization (WLO).
  - Reference WLO : simulation on $N_{\text{ref}} = 100000$ samples
  - G_N: gain in terms of number of simulated samples
  - G_t: gain in terms of simulation time
  - α: average ratio between the effective number of simulated samples and the minimal number of samples $N = 30$.

Applications	N_v	Cost	G_N	G_t	α
FIR	5	0.99	1108	879	1.03
IIR	15	1.01	1479	1333	1.04
FFT	10	1.01	775	610	1.17
HEVC	23	1.01	824	769	1.01
Stereo	10	1	123.2	89.9	1.3

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