Motion Artefact Removal in Functional Near-InfraRed Spectroscopy Signals based on Robust Estimation

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Introduction

- Functional Near Infrared Spectroscopy (fNIRS) is a noninvasive modality for monitoring functional brain activities
- Motion artefacts will lead to sudden changes in the measured light intensities (and in fNIRS HbO/HbR signals)
- We propose a motion artefact removal algorithm based on robust estimation
- Results show the proposed algorithm can successfully remove or reduce motion-related artefacts under different interference conditions

Background

fNIRS Signal Model

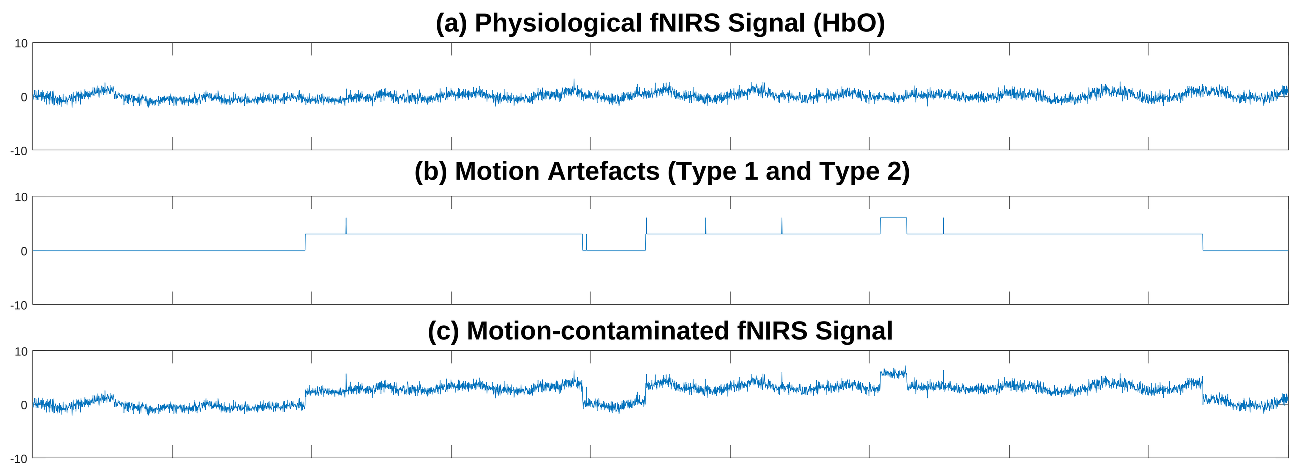

Figure 1. fNIRS Signal Model

Motion artefacts: 1) spike-shaped (short-term motion artefacts) artefacts; 2) square-shaped artefacts (long-term motion artefacts)

fNIRS signal = physiological response + artefact (motion artefact & noise)

$$\mathbf{y} = \mathbf{x} + \mathbf{e}, \quad \mathbf{y}, \mathbf{x}, \mathbf{e} \in \mathbb{R}^N$$

Bases representation

Using a linear model, $\mathbf{y} = \mathbf{B}\boldsymbol{\theta} + \mathbf{e}$, where $\boldsymbol{\theta}$ represents unknown parameters.

Reduced bases representation, $\mathbf{y} = \mathbf{B}_r\, \boldsymbol{\theta}_r + \mathbf{e}$, where $\boldsymbol{\theta}_r \in \mathbb{R}^r$ and $\mathbf{B}_r$ represent a selection of $\boldsymbol{\theta}$ and $\mathbf{B}$.

Difference-Based Estimation (DBE)

To remove long-term artefacts, first-order differences of $\mathbf{y}$, $\mathbf{B}_r$, $\mathbf{e}$ ($\tilde{\mathbf{y}}$, $\tilde{\mathbf{B}}_r$, $\tilde{\mathbf{e}}$) are used to estimate $\boldsymbol{\theta}_r$. $\tilde{\mathbf{y}} = \tilde{\mathbf{B}}_r\, \boldsymbol{\theta}_r + \tilde{\mathbf{e}}$ [1]

Traditional Least-Squares (LS) Estimation

$$\hat{\boldsymbol{\theta}}_{r-\mathrm{LS}} = (\tilde{\mathbf{B}}_r^\top \tilde{\mathbf{B}}_r)^{-1} \tilde{\mathbf{B}}_r^\top \tilde{\mathbf{y}}$$

The limitation of LS estimation: not robust to outliers
Motion artefacts: $\tilde{\mathbf{e}}$ is represented as positive and negative spikes, which yields outliers in $\tilde{\mathbf{y}}$. The LS solution is no longer appropriate in estimating parameters.

Robust Estimation

Starting with a robust estimate $\hat{\boldsymbol{\theta}}_\alpha$ by solving

$$\hat{\boldsymbol{\theta}}_\alpha = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, L_\alpha(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \frac{1}{N-1} \Sigma_{i=1}^{N-1} \rho_\alpha \left\{ \frac{\tilde{y}_i - \mathbf{b}_i^\top \boldsymbol{\theta}}{\sigma} \right\}$$

where ρ_α is robust loss function, $\rho_\alpha(v) = \alpha^{-1}\{1 - \exp(-\alpha v^2/2)\}$, $\alpha > 0$.

Weighted LS estimating equation is obtained by differentiating the objective

$$0 = \Sigma_{i=1}^{N-1} w_i(\boldsymbol{\theta}, \mathbf{B}; \alpha) \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\theta}} \left(\frac{\tilde{y}_i - \mathbf{b}_i^\top \boldsymbol{\theta}}{\sqrt{2}\sigma} \right)^2$$

where the weights are given by

$$w_i(\boldsymbol{\theta}, \mathbf{B}; \alpha) = \exp\left\{ -\frac{\alpha}{2} \left(\frac{\tilde{y}_i - \mathbf{b}_i^\top \boldsymbol{\theta}}{\sigma} \right)^2 \right\}$$

For $\alpha = 0$, uniform weights $w_1 = \cdots = w_{N-1} = 1$ correspond to LS estimator. For $\alpha > 0$, an observation $\tilde{y}_i$ far from the mean $\mathbf{b}_i^\top \boldsymbol{\theta}$ receives relatively low weight. Outliers are automatically downweighted. $\mathbf{b}_i$ is the i-th row of $\mathbf{B_r}$, $\hat{\sigma}^2 = 1$.

$$\hat{\boldsymbol{\theta}}_\alpha = (\tilde{\mathbf{B}}_r^\top \mathbf{W} \tilde{\mathbf{B}}_r)^{-1} \tilde{\mathbf{B}}_r^\top \mathbf{W} \tilde{\mathbf{y}}$$

where the weighted matrix $\mathbf{W}$ is the diagonal matrix with w_i being the diagonal entries. $\hat{\boldsymbol{\theta}}_\alpha$ can be calculated using a iterative procedure.

Algorithm Overview

Input: $\mathbf{y}$, $\mathbf{B}_{dct}$, r, α

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θ_dct = B_dct y
for n = 1 to r do
    Find B_{r_n}
end for
for i = 1 to N-1 do
    ỹ_i = y(i+1) − y(i)
    B̃_{r_i.} = B_{r_(i+1).} − B_{r_i.}
end for
LS Solution θ̂_LS = (B̃_r^T B̃_r)^{-1} B̃_r^T ỹ
Define the i-th row of B̃_r: B̃_{r_i.} = b̃_i;
Starting with: θ_α = θ̂_LS
for t = 1 to T Iterations do
    w_{i,t} = exp(−α(ỹ_i − b̃_i θ_α)^2)
    W_t = diag(w_{i,t})
    Update θ̂_α = (B̃_r^T W_t B̃_r)^{-1} B̃_r^T W_t ỹ
end for
x̂_{θ_α} = B_r θ̂_α
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Output: $\hat{\boldsymbol{\theta}}_\alpha$, $\hat{\mathbf{x}}_{\boldsymbol{\theta}_\alpha}$

Results and Discussion

Artefact Removal Results

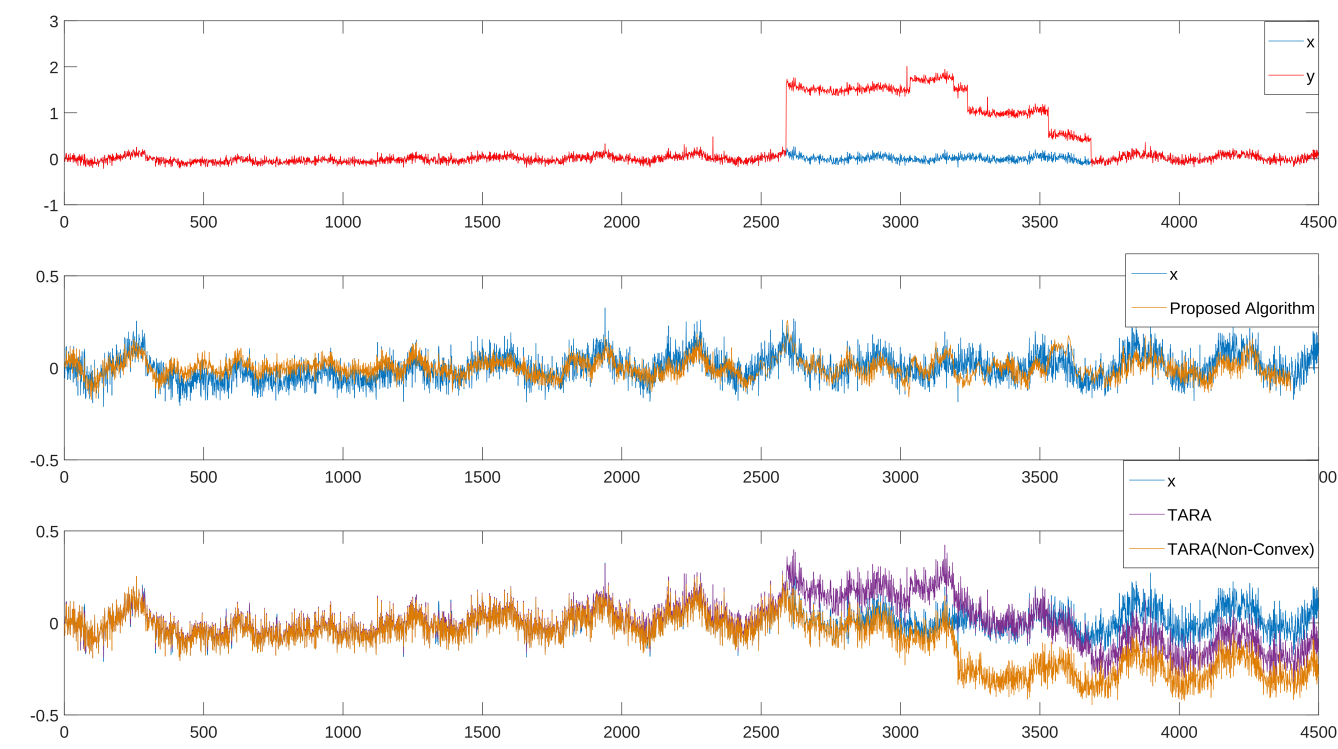

Figure 2. Example results. Top: Experimental data ($\mathbf{x}$) and experimental data with motion artefacts ($\mathbf{y}$). Middle: Experimental data ($\mathbf{x}$) and the estimated signal from the proposed algorithm ($\hat{\mathbf{x}}_{\boldsymbol{\theta}_\alpha}$). Bottom: Experimental data ($\mathbf{x}$) and the estimated signal from the Transient Artifact Reduction Algorithm (TARA) and TARA (Non-Convex) [2].

Robustness Results

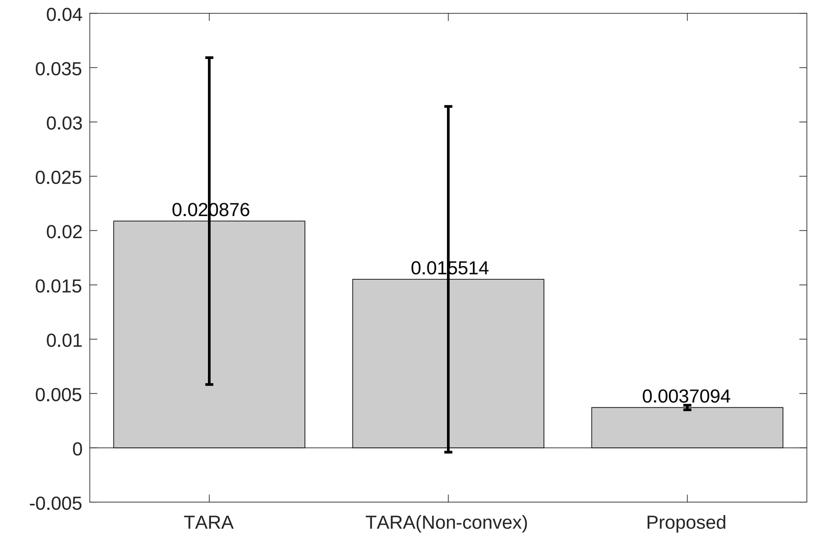

Figure 3. Mean and standard deviation of mean squared errors (MSEs) for TARA, TARA (Non-Convex) and the proposed algorithm using experimental data. The bar chart represent the mean values of MSEs, the error bars along each bars represent the standard deviation of MSEs.

Different Interference Conditions

The MSE results of the proposed algorithm under different conditions of signal-to-inference ratio (SIR). $\mathrm{SIR} = 10\log(P_{\tilde{\mathbf{x}}}/P_{\tilde{\mathbf{e}}})$.

		SIR=20 dB	SIR=18 dB	SIR=16 dB	SIR=14 dB	SIR=10 dB
TARA	Mean	0.0041	0.0114	0.0209	0.0499	0.1539
	Deviation	0.0040	0.0127	0.0150	0.0420	0.0917
TARA (Non-Convex)	Mean	0.0048	0.0119	0.0155	0.0440	0.1198
	Deviation	0.0060	0.0192	0.0159	0.0421	0.0999
Proposed	Mean	0.0031	0.0034	0.0037	0.0043	0.0058
	Deviation	0.0001	0.0002	0.0002	0.0004	0.0006

References

[1] A. Shah and A. K. Seghouane, "An integrated framework for joint HRF and drift estimation and HbO/HbR signal improvement in fNIRS data," *IEEE Transactions on Medical Imaging*, vol. 33, pp. 2086–2097, 2014.

[2] I. W. Selesnick, H. L. Graber, Y. Ding, T. Zhang, and R. L. Barbour, "Transient Artifact Reduction Algorithm (TARA) Based on Sparse Optimization," *IEEE Transactions on Signal Processing*, vol. 62, pp. 6596–6611, Dec. 2014.