Transmit Radiation Pattern Invariance in MIMO Radar With Application to DOA Estimation

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Introduction

- Beamspace transformation and beamforming techniques are the key approaches in many different fields
- Having the same beampattern for different beamforming vectors often plays a key importance in practical applications
- Existing transmit beamspace design methods result in non-identical individual (per waveform) transmit radiational patterns

Signal Model

- Consider a radar system with M transmit antennas
- $\boldsymbol{\Theta}$ is the angular sector where desired targets are located.
- $\bar{\boldsymbol{\Theta}}$ is the out-of-sector region where interference is located.
- Design mother transmit weight vector $\mathbf{w}$ to focus the transmit power within $\boldsymbol{\Theta}$ (several methods can be used for designing $\mathbf{w}$).

Signal Model (Cont'd)

- The transmit array beampattern can be expressed as

$$p(\theta) = \|\mathbf{W}^H \mathbf{a}(\theta)\|^2$$

$\mathbf{a}^*(\theta)$: Tx array steering vector
$\mathbf{W}$: $M/ \times K$ transmit weight matrix

- The signals at the input of the transmit antennas

$$\mathbf{x}(t) = \sum_{k=1}^{K} \psi_k(t) \mathbf{w}_k^*$$

$\psi_k(t), \ k = 1, \ldots, K$: Orthogonal waveforms
$\mathbf{w}_k, \ k = 1, \ldots, K$: Transmit weight vectors

- How to design $\mathbf{w}_k, \ k = 1, \ldots, K$?

Transmit weight vector design

- Start with a single weight vector

$$p(\theta) = \|\mathbf{w}^H \mathbf{a}(\theta)\|^2$$

- Simple methods can be used (Spheroidal, FIR design, convex optimization, etc)

- The total number of other distinct beamforming vectors with the same exact beampattern is at most $2^{M-1} - 1$

- Similar results have been observed in Time series analysis

Transmit Radiation Pattern Invariance

- Consider the following function of a single variable x

$$f(x) \triangleq \overbrace{(w_1 + w_2 x + w_3 x^2 + \cdots + w_M x^{M-1})}^{\text{First Multiplicative Term}} \times \overbrace{(w_1^* + w_2^* x^{-1} + w_3^* x^{-2} + \cdots + w_M^* x^{-M+1})}^{\text{Second Multiplicative Term}}$$

- It can be immediately concluded that

$$p(\theta) = f(e^{j\pi \sin(\theta)})$$

- If x_0 is a root of the first term, then $1/x_o^*$ is a root of the second term!

Transmit Radiation Pattern Invariance

- $f(x)$ can be decomposed as

$$
\begin{aligned}
f(x) =& |w_M|^2 \left(\frac{w_1}{w_M} + \frac{w_2}{w_M} x + \frac{w_3}{w_M} x^2 + \cdots + x^{M-1} \right) \\
& \times \left(\frac{w_1^*}{w_M^*} + \frac{w_2^*}{w_M^*} x^{-1} + \frac{w_3^*}{w_M^*} + \cdots + x^{-M+1} \right) \\
=& |w_M|^2 \prod_{i=1}^{M-1} (x - x_i) \prod_{i=1}^{M-1} (x^{-1} - x_i^*)
\end{aligned}
$$

- 2^{M-1} different different combinations!
- $\mathbf{w}$ can be used to generate the population $\mathbf{w}_1 \ldots \mathbf{w}_{2^{M-1}}$

Transmit Radiation Pattern Invariance

- The selection of K weight vectors under uniform power constraint can be cast as

$$\min_{\mathbf{w}_1,\ldots,\mathbf{w}_K} \eta$$

$$\text{s.t.} \sum_{k=1}^{K} |\mathbf{w}_{[k,m]}|^2 \leq \eta, \quad m = 1,\ldots,M$$

$$\{\mathbf{w}_1,\ldots,\mathbf{w}_K\} \in \mathbf{W}_{\text{pop}}$$

  $\mathbf{W}_{\text{pop}}$: population of $2^{M-1} - 1$ associated weight vectors

- Additional requirements can be enforced to achieve additional benefits!

Transmit Radiation Pattern Invariance

- The larger the magnitude of x_i, the larger the deviation between the two vectors associated with x_i and $1/x_i^*$
- Partition into two groups

$$
\begin{aligned}
f(x) =& |w_M|^2 \prod_{i=1}^{Q} (x - x_i) \prod_{i=1}^{M-Q-1} (x - x_i) \\
& \times \prod_{i=1}^{Q} (x^{-1} - x_i^*) \prod_{i=1}^{M-Q-1} (x^{-1} - x_i^*) \\
=& |w_M|^2 h(x) \prod_{i=1}^{Q} (x - x_i) \prod_{i=1}^{Q} (x^{-1} - x_i^*)
\end{aligned}
$$

$h(x)$: Contains $M - Q - 1$ smallest roots

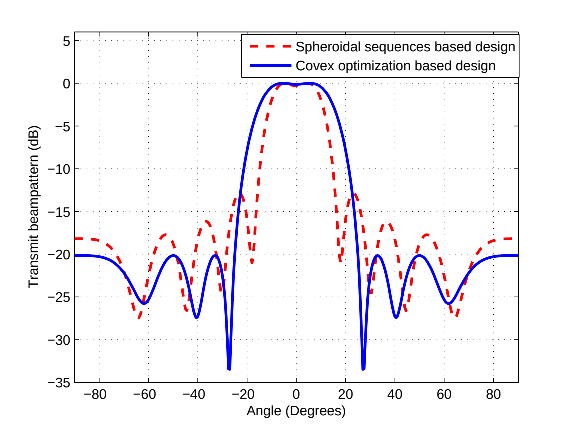
- A smaller population can be utilized

Simulation Results

- $M = 10$ transmit elements
- $\boldsymbol{\Theta} = [-10^\circ,\ 10^\circ]$
- Spheroidal based design: $\mathbf{w}_{\text{SPH}} = \sqrt{M/2}(\mathbf{u}_1 + \mathbf{u}_2)$
  $\mathbf{u}_1$ and $\mathbf{u}_2$: Two principle eigenvectors of the matrix
  $\mathbf{A} = \int_{\boldsymbol{\Theta}} \mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta$
- Convex optimization based design

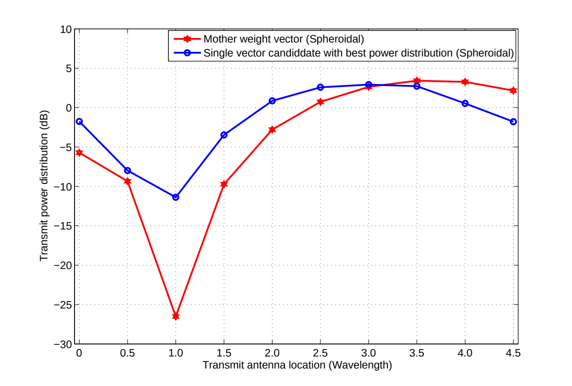
$$\min_{\mathbf{w}} \max_{i} \|\mathbf{w}^H\mathbf{a}(\theta_i) - e^{-j\phi_i}\|, \quad \theta_i \in \boldsymbol{\Theta},\ i = 1, \ldots, I$$
$$\text{subject to } \|\mathbf{w}^H\mathbf{a}(\theta_k)\| \leq \delta, \quad \theta_k \in \bar{\boldsymbol{\Theta}},\ k = 1, \ldots, K$$

Simulation Results (Cont'd)



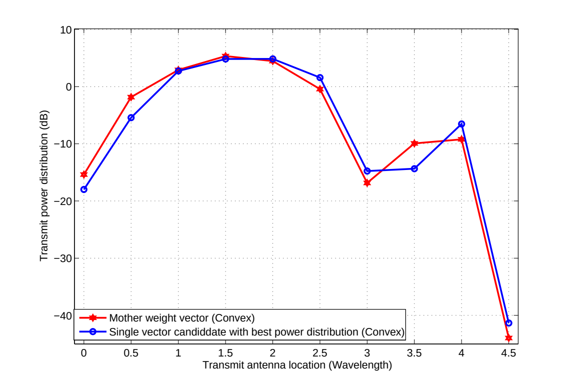
Transmit beampattern (One mother weight vector)

Simulation Results (Cont'd)



Transmit power distribution across array elements

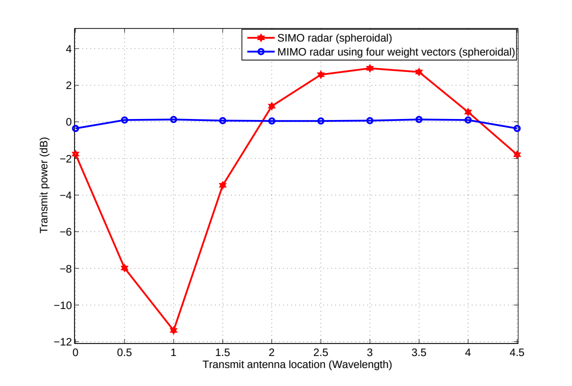
Simulation Results (Cont'd)



Transmit power distribution across array elements

Simulation Results (Cont'd)

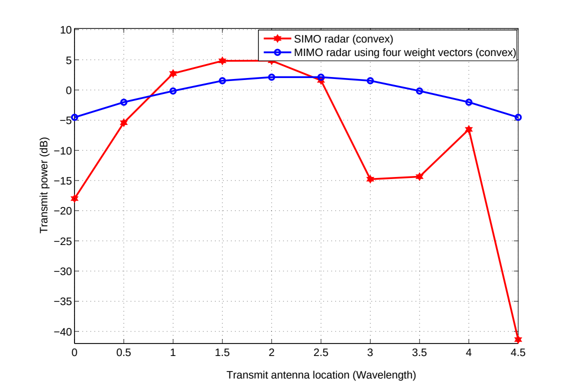
MIMO using $K = 4$ weight vectors



Transmit power distribution across array elements

Simulation Results (Cont'd)

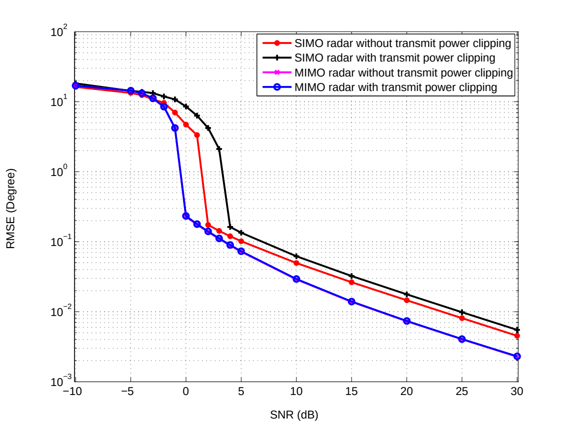
MIMO using $K = 4$ weight vectors.



Transmit power distribution across array elements

Simulation Results (Cont'd)

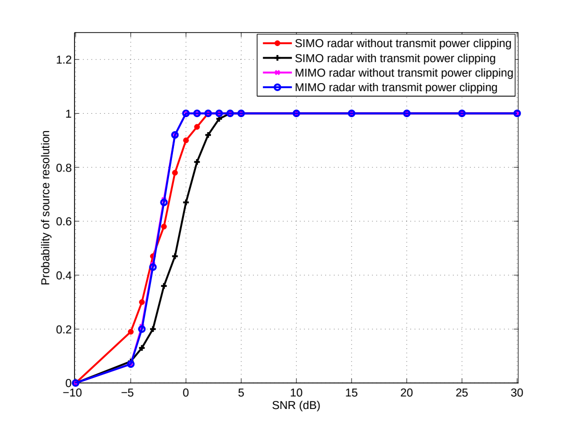
Two targets $-2^\circ,\ 2^\circ$



DOA estimation RMSE vs SNR

Simulation Results (Cont'd)

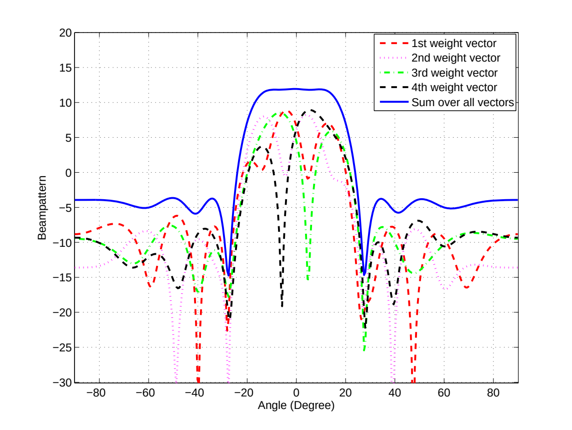
Two targets $-2^\circ,\ 2^\circ$



Probability of source resolution vs SNR

Simulation Results (Cont'd)

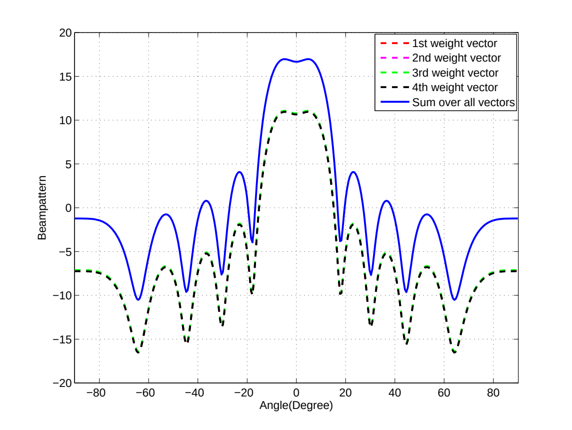
Joint design of $K = 4$ wight vectors



Transmit power distribution across array elements

Simulation Results (Cont'd)

$K = 4$ wight vectors drawn from population



Transmit power distribution across array elements

Conclusions

- An efficient approach for designing a transmit beamspace transformation in MIMO radar has been developed
- A principal beamforming vector is used to generate 2^{M-1} weight vectors with the same transmit pattern
- A computationally efficient sub-optimal approach for selecting best beamforming vectors has been developed
- The proposed approach has been tested by simulations in application to DOA estimation