JSR-NET

A DEEP NETWORK FOR JOINT SPATIAL-RADON DOMAIN CT RECONSTRUCTION FROM INCOMPLETE DATA

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INTRODUCTION

CT image reconstruction from incomplete data, such as sparse views and limited angle reconstruction, is an important and challenging problem in medical imaging. This work proposes a new deep convolutional neural network (CNN), called JSR-Net, that jointly reconstructs CT image and the associated Radon domain projection. JSR-Net combines the traditional model-based approach with deep architecture design of deep learning. A hybrid loss function is adopted to improve the performance of JSR-Net.

HIGHLIGHTS

1. A new end-to-end deep model for CT image reconstruction.
2. Uniform model for sparse-view CT and limited-angle CT.
3. Intuitive interpretation of the deep neural network(DNN) by unrolling dynamics.
4. A new hybrid loss function–contains structure similarity and semantic segmentation loss.

MAIN IDEA

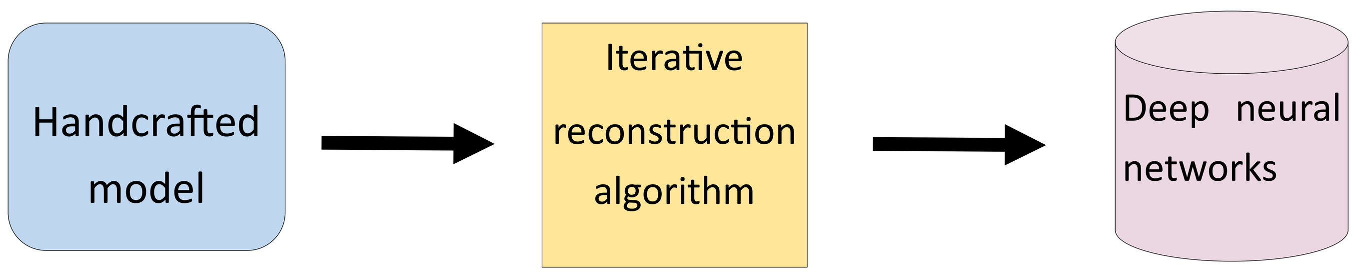

Figure 1: From handcrafted modeling to deep modeling.

MATERIALS AND METHODS

JSR model

The Joint Spatial-Radon domain image reconstruction(JSR) model [2] is

$$\min_{\boldsymbol{u},\boldsymbol{f}} \mathcal{F}(\boldsymbol{u},\boldsymbol{f},\boldsymbol{Y}) + \|\boldsymbol{\lambda}_1 \cdot \boldsymbol{W}_1\boldsymbol{u}\|_{1,2} + \|\boldsymbol{\lambda}_2 \cdot \boldsymbol{W}_2\boldsymbol{f}\|_{1,2}, \quad (1)$$

where the data fidelity term is defined by

$$\mathcal{F}(\boldsymbol{u},\boldsymbol{f},\boldsymbol{Y}) = \frac{1}{2}\|R_{\boldsymbol{\Gamma}^c}(\boldsymbol{f}-\boldsymbol{Y})\|^2 + \frac{\alpha}{2}\|R_{\boldsymbol{\Gamma}}(\boldsymbol{\mathcal{P}}\boldsymbol{u}-\boldsymbol{f})\|^2 + \frac{\gamma}{2}\|R_{\boldsymbol{\Gamma}^c}(\boldsymbol{\mathcal{P}}\boldsymbol{u}-\boldsymbol{Y})\|^2$$

Notations	
$R_{\boldsymbol{\Gamma}}$	restriction operator with respect to missing data region $\boldsymbol{\Gamma}$
$\boldsymbol{\Gamma}^c$	complement of $\boldsymbol{\Gamma}$
$\boldsymbol{\mathcal{P}}$	Radon transform
$\boldsymbol{Y}$	measured projection data
$\boldsymbol{W}_i, i=1,2$	wavelet frame transform
$\boldsymbol{f}$	repaired projection data
$\boldsymbol{u}$	desired CT image

Solution of the JSR model is obtained by ADMM algorithm as the following:

Algorithm for JSR model

1: Initialization: $\boldsymbol{b}_1^0 = \boldsymbol{b}_2^0 = 0$
2: **While** stop criterion is not met **do**
3: update $\boldsymbol{u}$:
$$\boldsymbol{u}^{k+1} = \mathcal{A}^{-1}\left[\alpha\boldsymbol{\mathcal{P}}^\top R_{\boldsymbol{\Gamma}}\boldsymbol{f}^k + \mathcal{B} + \mu_1\boldsymbol{W}_1^\top(\boldsymbol{d}_1^k - \boldsymbol{b}_1^k)\right]$$
$$\boldsymbol{d}_1^{k+1} = \mathcal{T}_{\boldsymbol{\lambda}_1/\mu_1}(\boldsymbol{W}_1\boldsymbol{u}^{k+1} + \boldsymbol{b}_1^k)$$
$$\boldsymbol{b}_1^{k+1} = \boldsymbol{b}_1^k + (\boldsymbol{W}_1\boldsymbol{u}^{k+1} - \boldsymbol{d}_1^{k+1})$$
where $\mathcal{A} = P^\top(\alpha R_{\boldsymbol{\Gamma}} + \gamma R_{\boldsymbol{\Gamma}^c})\boldsymbol{\mathcal{P}} + \mu_1$ and $\mathcal{B} = \gamma\boldsymbol{\mathcal{P}}^\top R_{\boldsymbol{\Gamma}^c}\boldsymbol{Y}$
4: update $\boldsymbol{f}$:
$$\boldsymbol{f}^{k+1} = \mathcal{C}^{-1}\left[\alpha R_{\boldsymbol{\Gamma}}\boldsymbol{\mathcal{P}}\boldsymbol{u}^{k+1} + \mathcal{D} + \mu_2\boldsymbol{W}_2^\top(\boldsymbol{d}_2^k - \boldsymbol{b}_2^k)\right]$$
$$\boldsymbol{d}_2^{k+1} = \mathcal{T}_{\boldsymbol{\lambda}_2/\mu_2}(\boldsymbol{W}_2\boldsymbol{f}^{k+1} + \boldsymbol{b}_2^k)$$
$$\boldsymbol{b}_2^{k+1} = \boldsymbol{b}_2^k + (\boldsymbol{W}_2 f^{k+1} - \boldsymbol{d}_2^{k+1})$$
where $\mathcal{C} = \alpha R_{\boldsymbol{\Gamma}} + R_{\boldsymbol{\Gamma}^c} + \mu_2$ and $\mathcal{D} = R_{\boldsymbol{\Gamma}^c}\boldsymbol{Y}$.
5: **end while**
6: Output: $\boldsymbol{u}^*$

JSR-Net

Based on the **Algorithm for JSR model**, JSR-Net is designed as the following:

Architecture of JSR-Net

1: Initialization: $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{u}, \boldsymbol{f}, \boldsymbol{W}_1, \boldsymbol{W}_2, \mathcal{N}(\cdot)$
2: **For** k=0:N
3: block $\boldsymbol{u}$:
$$\boldsymbol{u}^{k+1} = \mathcal{N}_u(\left[\boldsymbol{\mathcal{P}}^\top R_{\boldsymbol{\Gamma}}\boldsymbol{f}^k, \mathcal{B}, \boldsymbol{W}_1^\top(\boldsymbol{d}_1^k - \boldsymbol{b}_1^k)\right]; \Theta_u^k)$$
$$\boldsymbol{d}_1^{k+1} = \mathcal{N}_{d_1}(\boldsymbol{W}_1\boldsymbol{u}^{k+1} + \boldsymbol{b}_1^k; \Theta_{d_1}^k)$$
$$\boldsymbol{b}_1^{k+1} = \boldsymbol{b}_1^k + (\boldsymbol{W}_1\boldsymbol{u}^{k+1} - \boldsymbol{d}_1^{k+1})$$
where $\mathcal{B} = \gamma\boldsymbol{\mathcal{P}}^\top R_{\boldsymbol{\Gamma}^c}\boldsymbol{Y}$
4: block $\boldsymbol{f}$:
$$\boldsymbol{f}^{k+1} = \mathcal{N}_f(\left[R_{\boldsymbol{\Gamma}}\boldsymbol{\mathcal{P}}\boldsymbol{u}^{k+1}, \mathcal{D}, \boldsymbol{W}_2^\top(\boldsymbol{d}_2^k - \boldsymbol{b}_2^k)\right]; \Theta_f^k)$$
$$\boldsymbol{d}_2^{k+1} = \mathcal{N}_{d_2}(\boldsymbol{W}_2\boldsymbol{f}^{k+1} + \boldsymbol{b}_2^k; \Theta_{d_2}^k)$$
$$\boldsymbol{b}_2^{k+1} = \boldsymbol{b}_2^k + (\boldsymbol{W}_2\boldsymbol{f}^{k+1} - \boldsymbol{d}_2^{k+1})$$
where $\mathcal{D} = R_{\boldsymbol{\Gamma}^c}\boldsymbol{Y}$.
5: **end for**
6: Output: $\boldsymbol{u}^*$

NETWORK TRAINING

Loss function

Structure-Semantic-ℓ_2 (SS2) hybrid loss function is defined as

$$\mathcal{L}_{SS2} = \theta_1\mathcal{L}_{SSIM} + \mathcal{L}_{MSE} + \theta_3\mathcal{L}_{sem}, \quad (2)$$

Notations in SS2 Loss

$$\mathcal{L}_{SSIM} = \sum(1 - SSIM(\boldsymbol{u}_{rec}, \boldsymbol{u}_{truth}))$$
$$\mathcal{L}_{MSE} = \mathcal{L}_S + \theta_2\mathcal{L}_R$$
$$\mathcal{L}_S = \sum\|\boldsymbol{u}_{rec} - \boldsymbol{u}_{truth}\|^2$$
$$\mathcal{L}_R = \sum\|R_{\boldsymbol{\Gamma}^c}\boldsymbol{\mathcal{P}}(\boldsymbol{u}_{rec} - \boldsymbol{u}_{truth})\|^2$$
$$\mathcal{L}_{sem} = \sum\|sem(\boldsymbol{u}_{rec}) - sem(\boldsymbol{u}_{truth})\|^2$$

EXPERIMENTS

Sparse view CT image reconstruction

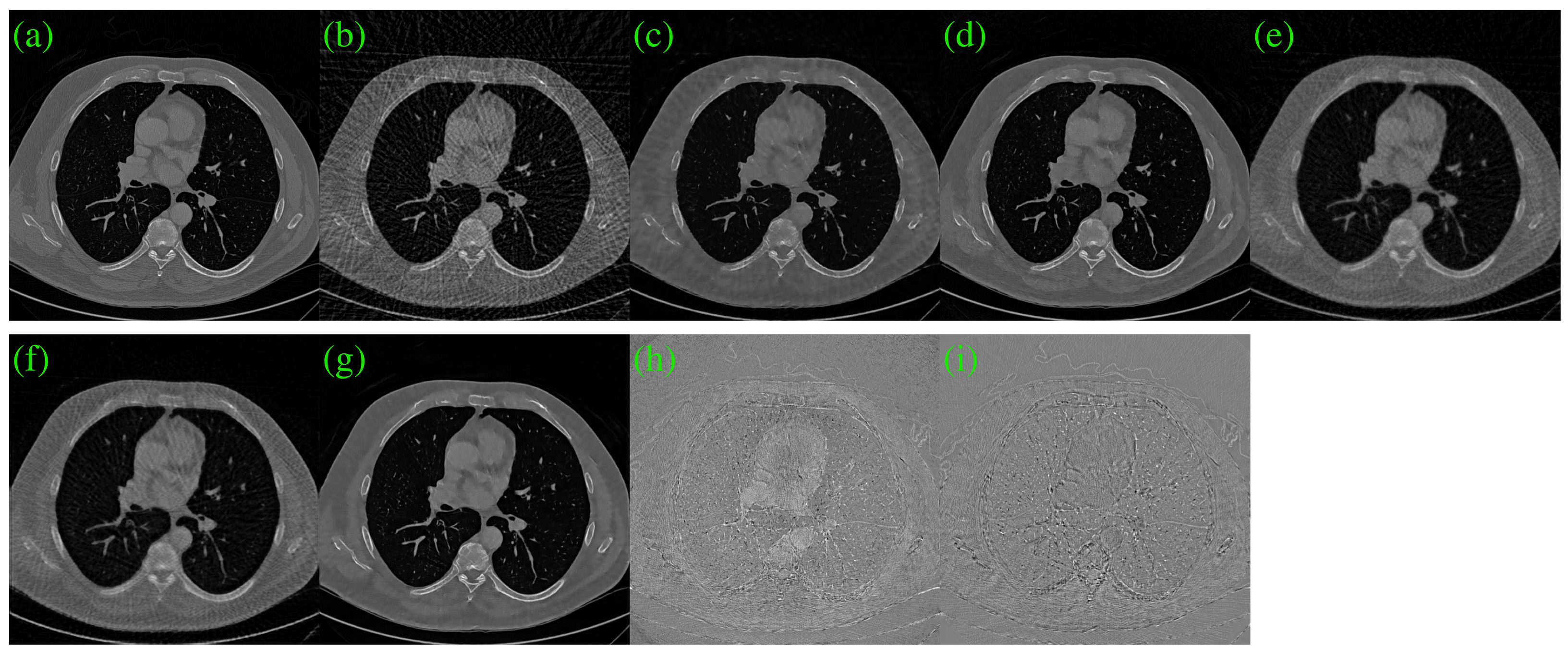

Figure 1. Sparse view CT image reconstruction. (a)Ground truth; (b)FBP; (c)PD-Net [1], ℓ_2; (d)PD-Net, SS2; (e)JSR model; (f) JSR-Net, ℓ_2; (g)JSR-Net, SS2; (h)Error map of PD-Net, SS2; (i)Error map of JSR-Net, SS2.

Limited angle CT image reconstruction

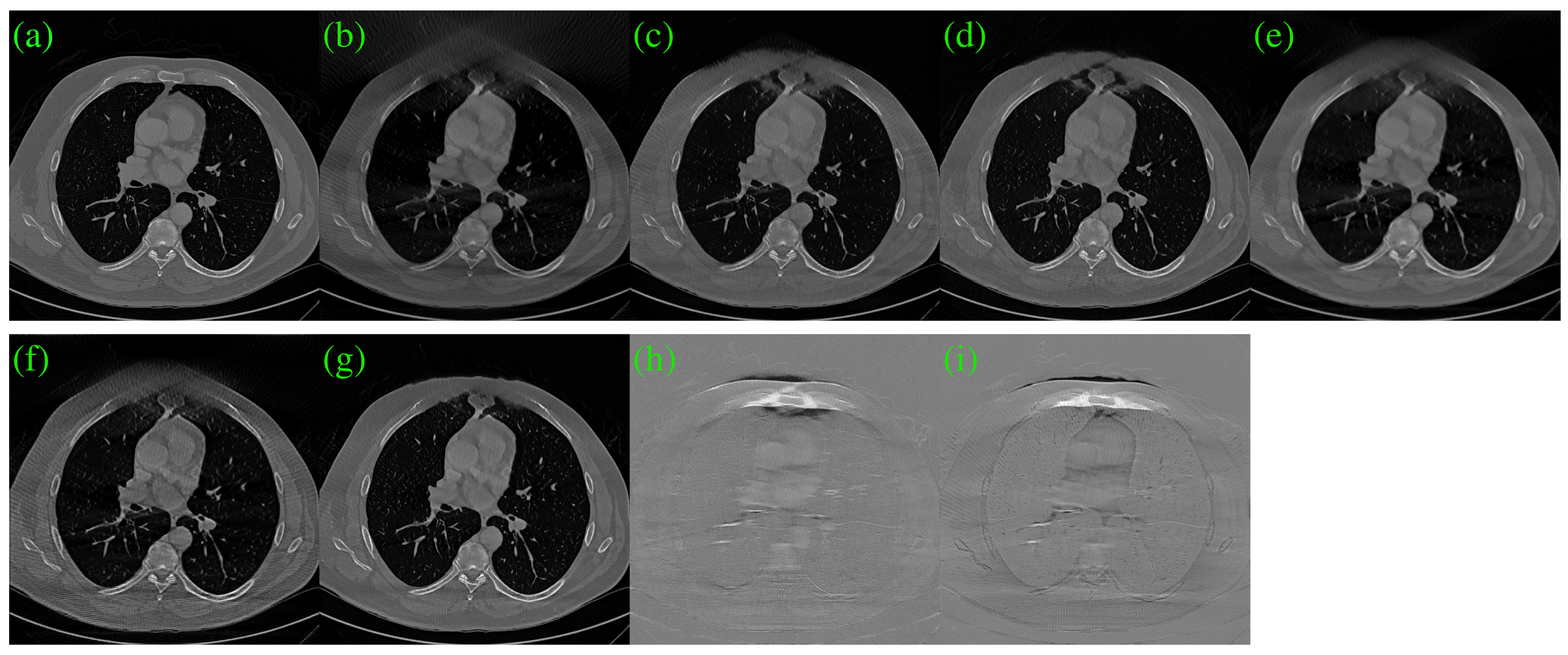

Figure 2. Limited angle CT image reconstruction. (a)Ground truth; (b)FBP; (c)PD-Net [1], ℓ_2; (d)PD-Net, SS2; (e)JSR model; (f) JSR-Net, ℓ_2; (g)JSR-Net, SS2; (h)Error map of PD-Net, SS2; (i)Error map of JSR-Net, SS2.

Quantitative results

Tasks	Models	Qual. Meas.			
		SSIM	PSNR	NRMSE	MSE
Sparse view CT	FBP	0.6173	17.25	1.078	0.0189
	PD-Net, ℓ_2	0.8709	28.54	0.1453	0.0014
	PD-Net, SS2	0.8844	30.68	0.1134	0.0009
	JSR model	0.8088	26.64	0.1866	0.0022
	JSR-Net,ℓ_2	0.8271	27.68	0.1604	0.0017
	JSR-Net,SS2	**0.9081**	**31.59**	**0.1022**	**0.0007**
Limited angle CT	FBP	0.4826	15.91	1.5143	0.0257
	PD-Net, ℓ_2	0.8778	26.43	0.1852	0.0023
	PD-Net, SS2	0.88	**27.44**	**0.1648**	**0.0018**
	JSR model	0.8317	25.38	0.2174	0.0029
	JSR-Net,ℓ_2	0.7337	23.72	0.253	0.0042
	JSR-Net, SS2	**0.9076**	27.31	0.1674	0.0019

Future work

1. Designing new loss function that is more effective in preserving tiny structures.
2. Designing new network architecture.
3. Extending JSR-Net to interior/exterior CT.
4. Extending JSR-Net to 3D Cone beam CT imaging.

References

[1] Jonas Adler and O. Öktem. *IEEE transactions on medical imaging*, 37(6):1322–1332, 2018.

[2] Bin Dong et.al. *Journal of Scientific Computing*, 54(2-3):333–349, 2013.

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