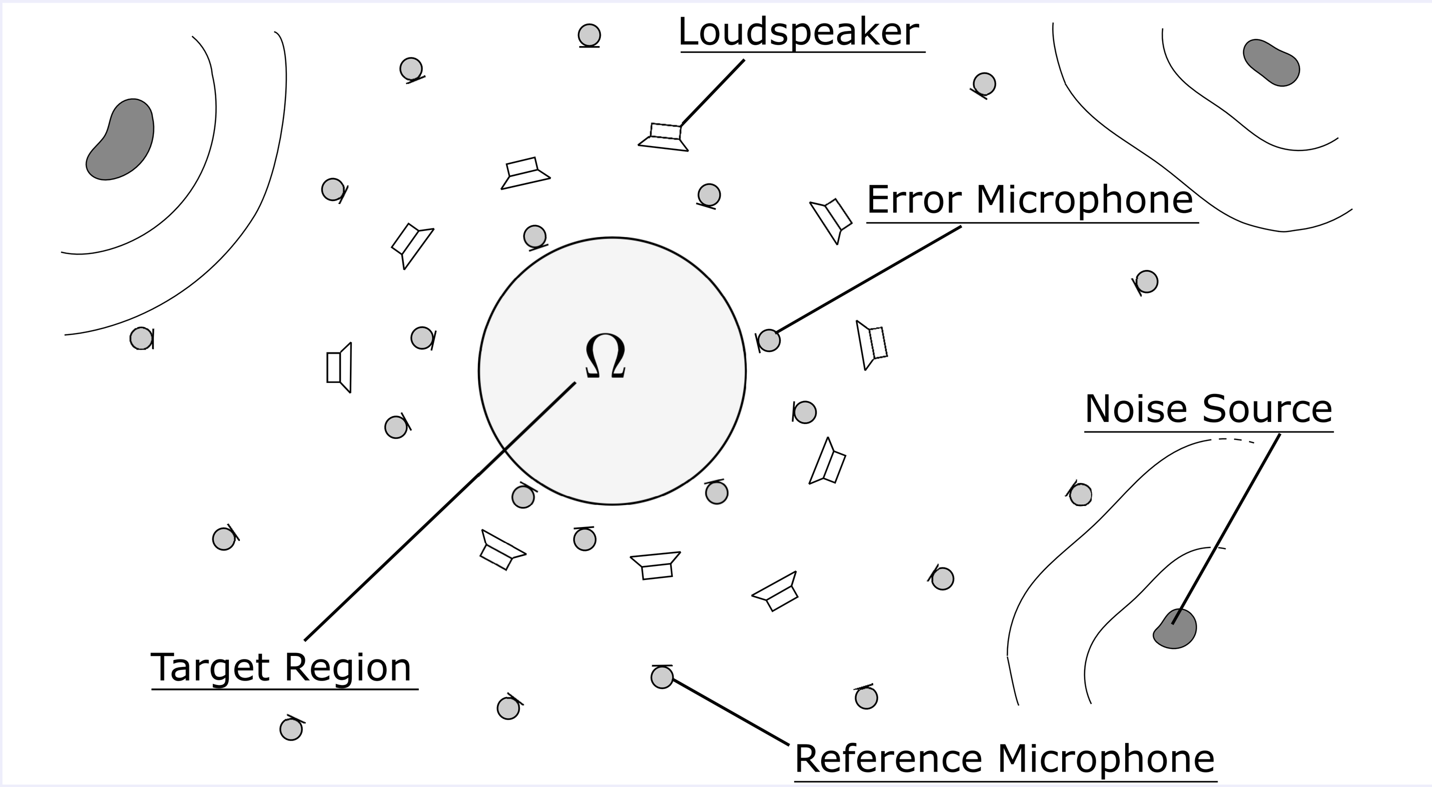
AASP-P7.1

Feedforward Spatial Active Noise Control Based on Kernel Interpolation of Sound Field

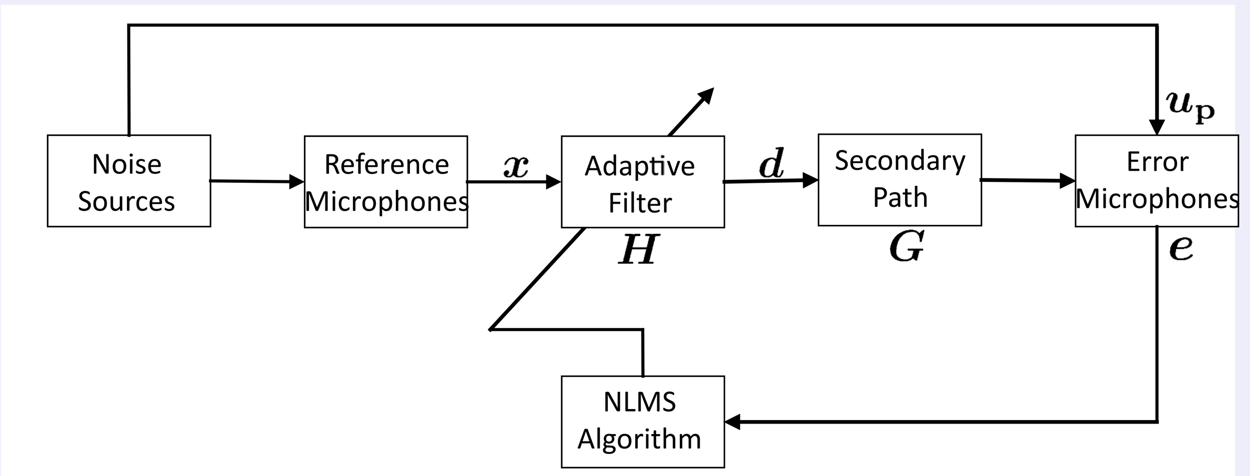
Hayato Ito, Shoichi Koyama, Natsuki Ueno, Hiroshi Saruwatari (University of Tokyo)

Summary

- ✓ Goal of spatial active noise control (ANC) is to cancel incoming primary noise in entire target region by secondary loudspeakers
- ✓ Conventional multipoint pressure control (MPC):
  - ➢ Noise cancellation is limited at multiple discrete points
- ✓ Proposed method:
  - ➢ Feedforward spatial ANC for regional noise cancellation by using distributed microphones and loudspeakers
  - ➢ Based on kernel ridge regression for sound field interpolation [1] (equivalent to harmonic analysis of infinite order [2])
  - ➢ Resulting algorithm is normalized least mean squares (NLMS) with weighting matrix and its computational cost for filter update is exactly same as that of MPC



Adaptive filter for feedforward control



- ✓ NLMS for feedforward control
- ✓ Conventional MPC:
  - ➢ Cost function: $\underset{\boldsymbol{H}}{\text{minimize}}\ \|\boldsymbol{e}\|_2^2 = \underset{\boldsymbol{H}}{\text{minimize}}\ \boldsymbol{e}^{\mathsf{H}}\boldsymbol{e}$
  - ➢ Filter update: $\boldsymbol{H}(n+1) = \boldsymbol{H}(n) - \frac{\mu_0}{\|\boldsymbol{G}^{\mathsf{H}}\boldsymbol{G}\|_2 \|\boldsymbol{x}(n)^{\mathsf{H}}\boldsymbol{x}(n)\|_2 + \beta} \boldsymbol{G}^{\mathsf{H}}\boldsymbol{e}(n)\boldsymbol{x}(n)^{\mathsf{H}}$
- ✓ Proposed method: NLMS with weighting matrix
  - ➢ Cost function: $\underset{\boldsymbol{H}}{\text{minimize}}\ \boldsymbol{e}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{e}$
  - ➢ Filter update: $\boldsymbol{H}(n+1) = \boldsymbol{H}(n) - \frac{\mu_0}{\|\boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{G}\|_2 \|\boldsymbol{x}(n)^{\mathsf{H}}\boldsymbol{x}(n)\|_2 + \beta} \boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{e}(n)\boldsymbol{x}(n)^{\mathsf{H}}$
- ✓ Computational cost is same as that of MPC by calculating $\boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}$ and $\|\boldsymbol{G}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{G}\|_2$ in advance

Kernel-induced sound field interpolation

- ✓ Discussion is in 2D case (extension to 3D case is straightforward)
- ✓ Regional noise power as cost function:

$$\mathcal{L} = \int_{\Omega} |u_{\mathrm{e}}(\boldsymbol{r})|^2 \,\mathrm{d}\boldsymbol{r} \quad (1)$$

Continuous pressure field at $\boldsymbol{r}$

- ✓ Kernel ridge regression with constraint that function to be estimated satisfies Helmholtz eq. to estimate $u_{\mathrm{e}}(\boldsymbol{r})$ from error signals $\boldsymbol{e}$

$$\hat{u}_{\mathrm{e}}(\boldsymbol{r}) = (\boldsymbol{P}\boldsymbol{e})^{\mathsf{T}} \boldsymbol{\kappa}(\boldsymbol{r}) \quad (2)$$

$$\boldsymbol{P} = (\boldsymbol{K} + \lambda \boldsymbol{I})^{-1}, \quad \boldsymbol{K} = \begin{bmatrix} J_0(k\|\boldsymbol{r}_1 - \boldsymbol{r}_1\|) & \cdots & J_0(k\|\boldsymbol{r}_1 - \boldsymbol{r}_M\|) \\ \vdots & \ddots & \vdots \\ J_0(k\|\boldsymbol{r}_M - \boldsymbol{r}_1\|) & \cdots & J_0(k\|\boldsymbol{r}_M - \boldsymbol{r}_M\|) \end{bmatrix}$$

$$\boldsymbol{\kappa}(\boldsymbol{r}) = [J_0(k\|\boldsymbol{r} - \boldsymbol{r}_1\|), \ \dots, \ J_0(k\|\boldsymbol{r} - \boldsymbol{r}_M\|)]^{\mathsf{T}}$$

Position of error microphones

Weighting matrix based on sound field interpolation

- ✓ Cost function can be rewritten by substituting Eq. (2) into (1) as

$$\mathcal{L} = \boldsymbol{e}^{\mathsf{H}}\boldsymbol{A}\boldsymbol{e}$$

Same as cost function of NLMS with weighting matrix

$$\boldsymbol{A} = \boldsymbol{P}^{\mathsf{H}} \left(\int_{\Omega} \boldsymbol{\kappa}^*(\boldsymbol{r}) \boldsymbol{\kappa}^{\mathsf{T}}(\boldsymbol{r}) \mathrm{d}\boldsymbol{r} \right) \boldsymbol{P}$$

- ✓ Numerical computation for calculating elements of $\boldsymbol{A}$ is possible, but numerical integral can be avoided when region Ω is circular
- ✓ Weighting matrix $\boldsymbol{A}$ for circular target region Ω:

$$\boldsymbol{A} = \boldsymbol{P}^{\mathsf{H}}\boldsymbol{S}^{\mathsf{H}}\boldsymbol{\Gamma}\boldsymbol{S}\boldsymbol{P}$$

$$\boldsymbol{S} = \begin{bmatrix} \vdots & \vdots & \vdots \\ J_{-1}(kr_1)e^{i\phi_1} & \cdots & J_{-1}(kr_1)e^{i\phi_1} \\ J_0(kr_1) & \cdots & J_0(kr_M) \\ J_1(kr_1)e^{-i\phi_1} & \cdots & J_1(kr_1)e^{-i\phi_1} \\ \vdots & \vdots & \vdots \end{bmatrix}, \quad \boldsymbol{\Gamma} = \mathrm{diag}(\cdots \ \gamma_{-1}, \ \gamma_0, \ \gamma_1, \ \cdots)$$

$$\gamma_\mu = 2\pi \int_0^R (J_\mu(kr))^2 \, r \, \mathrm{d}r$$
$$= \pi R^2 \left((J_\mu(kR))^2 - J_{\mu-1}(kR) J_{\mu+1}(kR) \right)$$

Radius of Ω

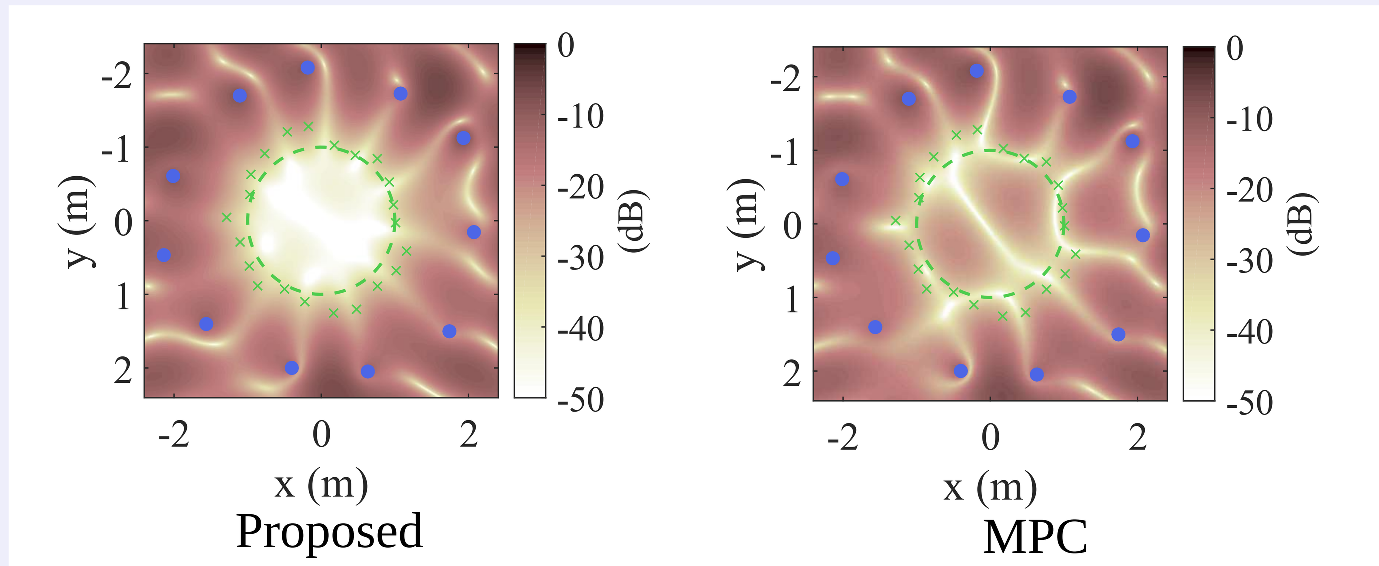
Numerical Simulation

- ✓ Experimental condition
  - ➢ Target region Ω : circular region with radius 1.0 m
  - ➢ Error mics: randomly shifted from equiangular placement, omnidirectional, 22 elements
  - ➢ Loudspeakers: randomly shifted from equiangular placement, point source, 11 elements
  - ➢ Reference mics: randomly shifted from equiangular placement, omnidirectional, 22 elements
  - ➢ Noise sources: 3 point sources
  - ➢ Transfer function: 2-D free field
  - ➢ SNR: 40 dB
- ✓ Performance measure: regional noise power reduction inside Ω

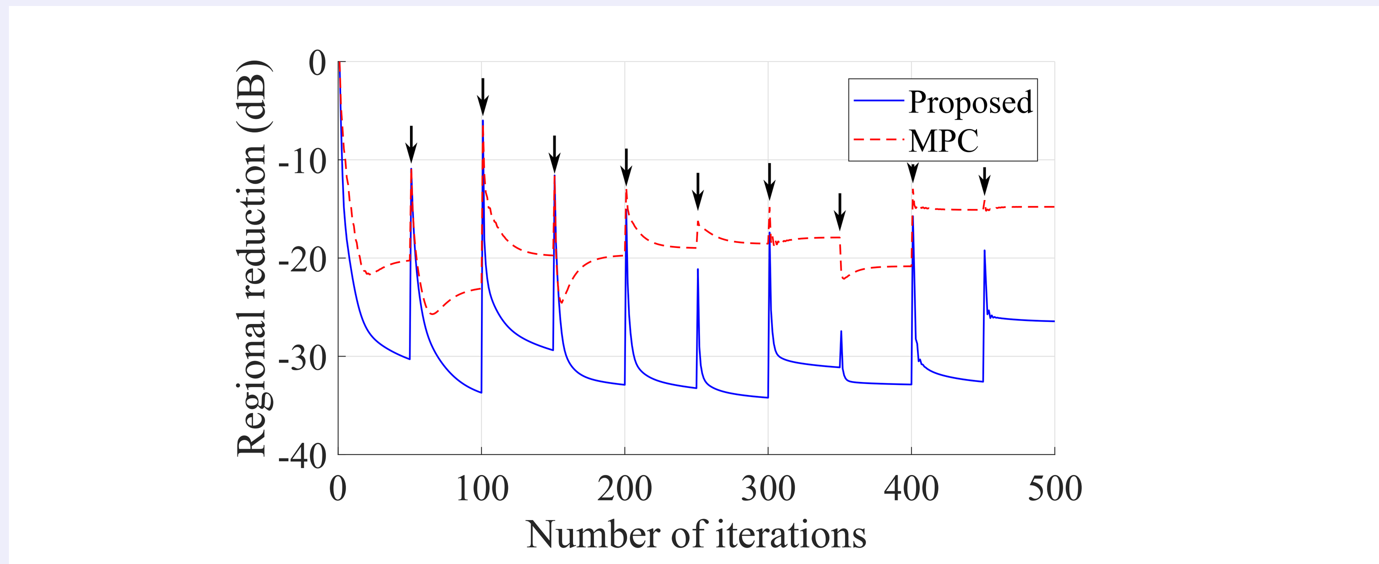
$$P_{\mathrm{red}} = 10 \log_{10} \frac{\sum_i \left| u_{\mathrm{e}}^{(n)}(\boldsymbol{r}_i) \right|^2}{\sum_i \left| u_{\mathrm{e}}^{(0)}(\boldsymbol{r}_i) \right|^2}$$

Pressure field after n iterations

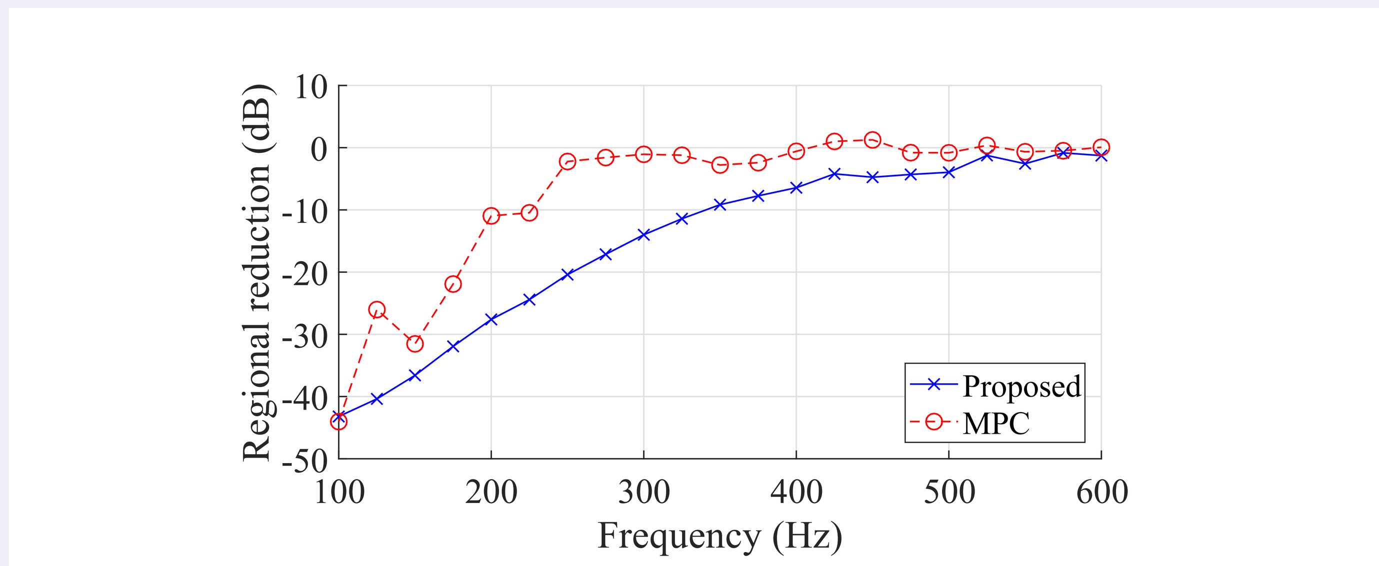
Initial pressure field



Power distribution of $u_{\mathrm{e}}^{(n)}(\boldsymbol{r}_i)$ after 500 iterations



P_{red} as function of # of iterations when noise amplitude changes at iteration numbers indicated by black arrows



P_{red} after 500 iterations as function of frequency

References

1. N. Ueno *et al.*, "Kernel ridge regression with constraint of Helmholtz equation for sound field interpolation", Proc. Int. Workshop Acoust. Signal Enhancement, pp. 436-440, Sep. 2018.
2. N. Ueno *et al.*, "Sound field recording using distributed microphones based on harmonic analysis of infinite order", IEEE Signal Process. Lett, vol. 25 (1) pp. 135-139, 2018.