# MSE based Precoding Schemes for Partially Correlated Transmissions in Interference Channels

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## Introduction

- A wireless sensor network (WSN) in presence of interferes experiences MSE degradation at the fusion center.
- Modeling as an interference channel (IC) with known CSI, MSE based interference alignment (IA) methods [1] can be used to design precoding methods.
- Since the sensors transmit correlated observations [2], we present two iterative precoding methods based on MSE that achieve global convergence.
- The first method normalizes the precoder for the power constraint. However, only one of the correlated transmitters fulfill the constraint. Therefore, the second method jointly considers the MSE optimization problem subject to the constraint.

## System Model

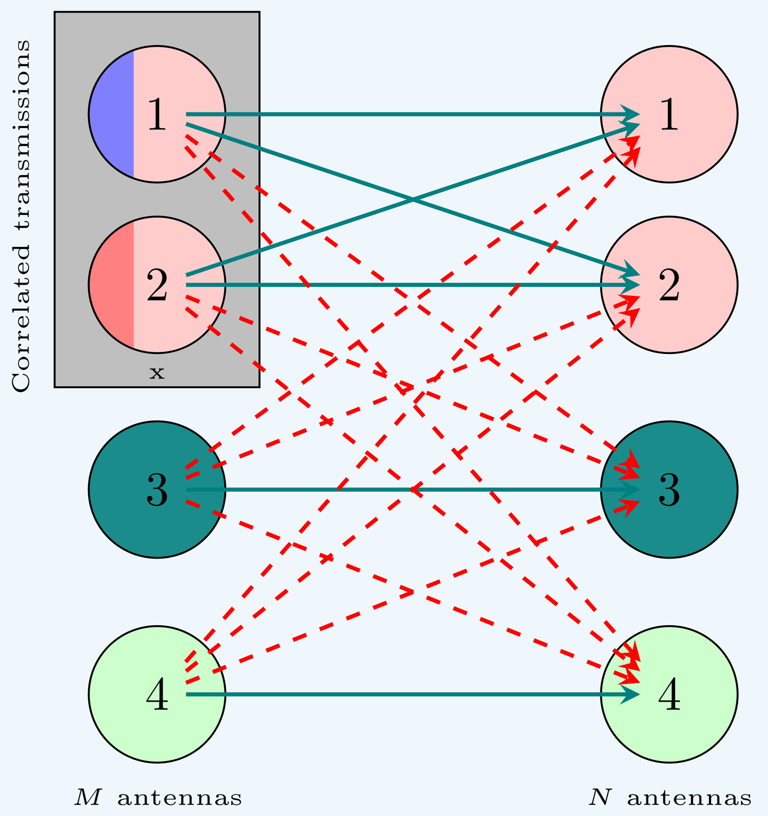


Figure 1: IC with correlated transmissions

- In IA-feasible MIMO IC $(M \times N, d)^K$, correlated transmitted signal of two or more users $(\mathcal{S} = \{1, 2\})$ is given as
$$\mathbf{x}_i = \sqrt{1-\alpha_i}\mathbf{x} + \sqrt{\alpha_i}\mathbf{w}_i, i \in \mathcal{S}, \quad (1)$$
where $\mathbf{w}_i \sim \mathcal{CN}(\mathbf{0}, \frac{P}{d}\mathbf{I}_d)$.
- For example, in a WSN with amplify and forward (AF) sensors, observations $\mathbf{x} + \sigma_i \mathbf{w}_i$ of a parameter $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_d))$ with $\sigma_i \mathbf{w}_i \sim \mathcal{CN}(\mathbf{0}, \sigma_i \mathbf{I})$ are scaled to satisfy unity transmit power constraint, i.e., with $\alpha_i = \frac{\sigma_i^2}{1+\sigma_i^2}$,
$$\frac{\mathbf{x} + \sigma_i \mathbf{w}_i}{\sqrt{1+\sigma_i^2}} = \sqrt{1-\alpha_i}\mathbf{x} + \sqrt{\alpha_i}\mathbf{w}_i.$$

- The received signal is given as $\mathbf{U}_k^\dagger \mathbf{y}_k = \sum_{j \in \mathcal{K}'} \mathbf{U}_k^\dagger \mathbf{H}_{kj} \mathbf{V}_j \mathbf{x}_j + \mathbf{U}_k^\dagger \tilde{\mathbf{n}}_k$, where $\mathcal{K}' = \{\mathrm{x}\} \cup (\mathcal{K} \setminus \mathcal{S})$, $\mathbf{H}_{k\mathrm{x}} = \left[\sqrt{1-\alpha_1}\mathbf{H}_{k1}, \sqrt{1-\alpha_2}\mathbf{H}_{k2}\right]$, $\mathbf{V}_{\mathrm{x}} = \begin{bmatrix}\mathbf{V}_1 \\ \mathbf{V}_2\end{bmatrix}$, and $\tilde{\mathbf{n}}_k = \sum_{j \in \mathcal{S}} \sqrt{\alpha_j} \mathbf{U}_k^\dagger \mathbf{H}_{kj} \mathbf{V}_j \mathbf{w}_j + \mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_k)$. The simplification $\sqrt{1-\alpha_1}\mathbf{H}_{11}\mathbf{V}_1\mathbf{x} + \sqrt{1-\alpha_2}\mathbf{H}_{12}\mathbf{V}_2\mathbf{x} = \mathbf{H}_{1\mathrm{x}}\mathbf{V}_{\mathrm{x}}\mathbf{x}$ is used.

## Global convergence conditions

- The equations for avoiding under-determined MSE optimization is expressed as
$$\mathbf{U}_k^\dagger \mathbf{H}_{kj} \mathbf{V}_j = \mathbf{0}, \ \forall (k, j) \in \mathcal{I} \quad (2)$$
$$\mathbf{U}_k^\dagger \mathbf{H}_{kk'} \mathbf{V}_{k'} \succ \mathbf{0}, \ \forall (k, k') \in \mathcal{D}, \quad (3)$$
where $\mathcal{D} = \left\{(k, k') \middle| k \in \mathcal{K}, k' = \begin{cases} \mathrm{x}, & k \in \mathcal{S} \\ k, & k \in \mathcal{K} \setminus \mathcal{S} \end{cases}\right\}$ and $\mathcal{I} = \mathcal{K} \times \mathcal{K}' \setminus \mathcal{D}$.
- The above equations result in the modified necessary conditions as
$$M + N - \left[K + 2 - (|\mathcal{S}| - 1)\left(1 + \frac{1}{K}\right)\right] d \geq 0. \quad (4)$$

## MSE receiver, Rate, and Loss of DoFs

- The decoder expression to minimize the $k^{th}$ receiver's MSE $\mathcal{E}_k$ is given as
$$\mathbf{U}_k = \arg\min_{\mathbf{U}_k} \mathbb{E}\left\{\|\mathbf{U}_k^\dagger \mathbf{y}_k - \mathbf{x}_k\|^2\right\} = \begin{cases} \mathbf{B}_k^{-1}\mathbf{H}_{k\mathrm{x}}\mathbf{V}_{\mathrm{x}}, & \forall k \in \mathcal{S} \\ \mathbf{B}_k^{-1}\mathbf{H}_{kk}\mathbf{V}_k, & k \in \mathcal{K} \setminus \mathcal{S} \end{cases} \quad (5)$$
where $\mathbf{B}_k = \sum_{j \in \mathcal{K}'} \mathbf{H}_{kj}\mathbf{V}_j\mathbf{V}_j^\dagger\mathbf{H}_{kj}^\dagger + \frac{d}{P}\mathbf{C}_k$.
- Information rate expression for $k^{th}$ user can be obtained as
$$R_k = \log_2\left|\mathbf{U}_k^\dagger \mathbf{B}_k \mathbf{U}_k\right| - \begin{cases} \log_2\left|\mathbf{U}_k^\dagger \mathbf{B}_k \mathbf{U}_k - \bar{\mathbf{H}}_{k\mathrm{x}}\bar{\mathbf{H}}_{k\mathrm{x}}^\dagger\right|, & \forall k \in \mathcal{S} \\ \log_2\left|\mathbf{U}_k^\dagger \mathbf{B}_k \mathbf{U}_k - \bar{\mathbf{H}}_{kk}\bar{\mathbf{H}}_{kk}^\dagger\right|, & \forall k \in \mathcal{K} \setminus \mathcal{S} \end{cases}$$
$$= \begin{cases} -\log_2\left|\mathbf{I}_d - \bar{\mathbf{H}}_{k\mathrm{x}}^\dagger\right|, & \forall k \in \mathcal{S} \\ -\log_2\left|\mathbf{I}_d - \bar{\mathbf{H}}_{kk}^\dagger\right|, & \forall k \in \mathcal{K} \setminus \mathcal{S} \end{cases}$$
- For conventional-IA satisfying $\mathbf{U}_k^\dagger \mathbf{H}_{kj} \mathbf{V}_j = \mathbf{0}, \forall k \neq j$ and $\mathbf{U}_k^\dagger \mathbf{H}_{kk} \mathbf{V}_k = \mathbf{D}_k, \forall k$, the simplified rate is $R_k = \sum_{i=1}^d R_{ki}$ with
$$R_{ki} = \begin{cases} \log_2\left(1 + \frac{P}{d}\frac{(1-\alpha_k)D_{ki}^2}{D_{ki}^2 \alpha_k \frac{P}{d} + \sigma^2}\right), & \forall k \in \mathcal{S} \\ \log_2\left(1 + \frac{P}{d}\frac{D_{ki}^2}{\sigma^2}\right), & \forall k \in \mathcal{K} \setminus \mathcal{S} \end{cases},$$
which yields that the higher $\alpha_k$ is, the lower $R_{ki}$ will be. For $k \in \mathcal{S}$, $\lim_{P \to \infty} R_{ki} = -\log_2 \alpha_k$, show the loss of DoF lost these users.

## MSE based Transmit Precoder (Method-I)

- The precoder expressions to minimize the sum MSE is obtained as
$$\{\mathbf{V}_{\mathrm{x}}, \mathbf{V}_j, j \in \mathcal{K} \setminus \mathcal{S}\} = \arg\min_{\mathbf{V}_{\mathrm{x}}, \mathbf{V}_j} \sum_{k \in \mathcal{K}} \mathcal{E}_k \quad (6)$$
$$\mathbf{V}_{\mathrm{x}} = (\mathbf{A}_{\mathrm{x}} + \mathcal{D}(\alpha_l \mathbf{A}_l, l \in \mathcal{S}))^{-1} \mathbf{H}_{\mathrm{xx}}^\dagger \mathbf{U}_{\mathrm{x}} \quad (7)$$
$$\mathbf{V}_{\mathrm{x}} \leftarrow \beta_{\mathrm{x}} \mathbf{V}_{\mathrm{x}} \quad (8)$$
$$\mathbf{V}_j = \mathbf{A}_j^{-1}\mathbf{H}_{jj}^\dagger \mathbf{U}_j, \ j \in \mathcal{K} \setminus \mathcal{S} \quad (9)$$
$$\mathbf{V}_j \leftarrow \beta_j \mathbf{V}_j \quad (10)$$
where $\mathbf{A}_j = \sum_{k \in \mathcal{K}} \mathbf{H}_{kj}^\dagger \mathbf{U}_k \mathbf{U}_k^\dagger \mathbf{H}_{kj}$; $\beta_{\mathrm{x}} = \frac{\sqrt{d}}{\max_{l \in \mathcal{S}} \|\mathbf{V}_l\|_F}$ and $\beta_j = \frac{\sqrt{d}}{\|\mathbf{V}_j\|_F}$ stands for the normalization to satisfy the power constraint.

## MSE based Transmit Precoder (Method-II)

- To minimize the sum MSE subject to individual transmit power constraint can be written as
$$\{\mathbf{V}_l, l \in \mathcal{S}\} = \arg\min_{\mathbf{V}_l, l \in \mathcal{S}} \mathrm{tr}\left[\mathbf{V}_{\mathrm{x}}^\dagger (\mathbf{A}_{\mathrm{x}} + \mathcal{D}(\alpha_l \mathbf{A}_l, l \in \mathcal{S}))\mathbf{V}_{\mathrm{x}}\right] - 2\mathrm{tr}\left[\Re \mathbf{U}_{\mathrm{x}}^\dagger \mathbf{H}_{\mathrm{xx}} \mathbf{V}_{\mathrm{x}}\right]$$
$$\text{subject to } \|\mathbf{V}_l\|_F^2 \leq d, \ l \in \mathcal{S} \quad (11)$$
and for $j \in \mathcal{K} \setminus \mathcal{S}$,
$$\mathbf{V}_j = \arg\min_{\mathbf{V}_j} \mathrm{tr}\left[\mathbf{V}_j^\dagger \mathbf{A}_j \mathbf{V}_j - 2\Re \mathbf{U}_j^\dagger \mathbf{H}_{jj} \mathbf{V}_j\right]$$
$$\text{subject to } \|\mathbf{V}_j\|_F^2 \leq d. \quad (12)$$

## Iterative Algorithms for Correlated Transmissions

1: Initialize the precoders $\mathbf{V}_{\mathrm{x}}$ and $\mathbf{V}_k$ for $k \in \mathcal{K} \setminus \mathcal{S}$.
2: Compute decoders using (5).
3: *For Method-I:* compute precoders using (8) and (10),
*For Method-II:* compute precoders using (11) and (12).
4: Go to step 2 until maximum iterations are reached.

## Simulation Results

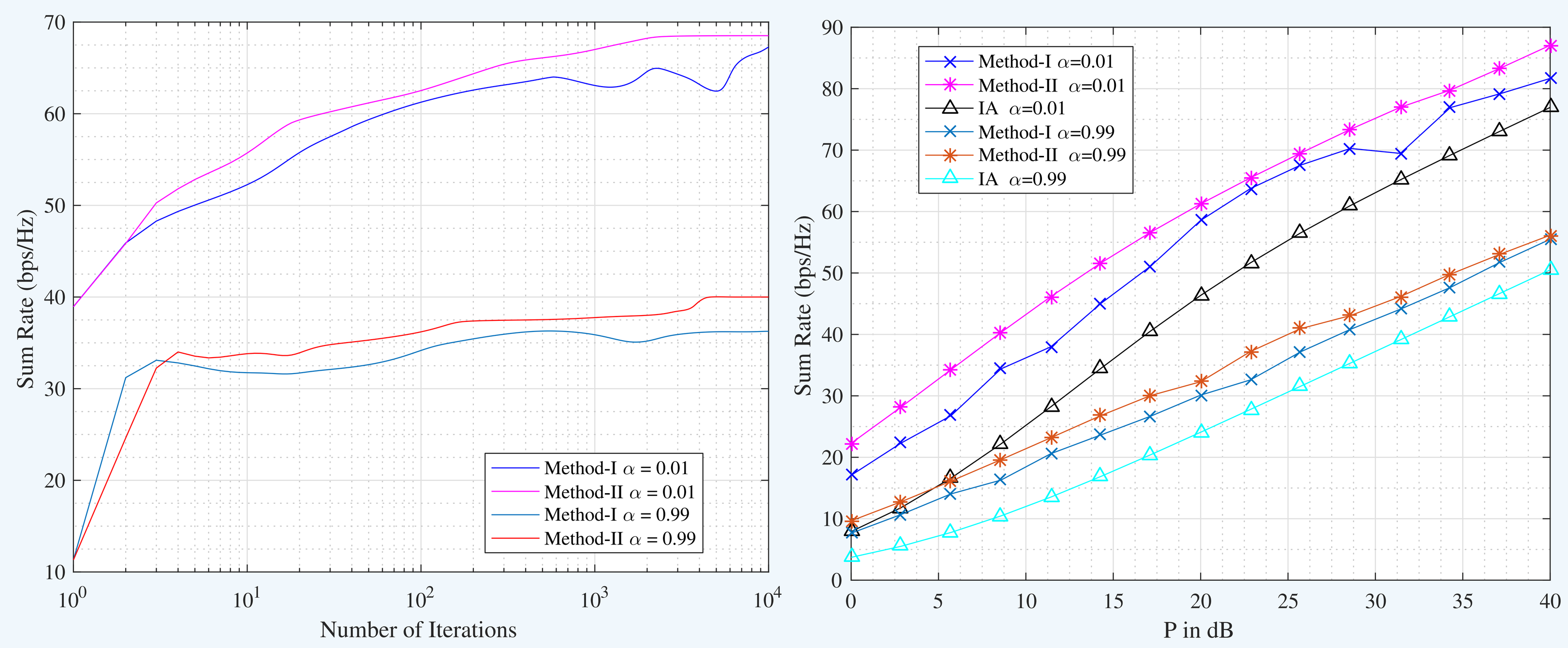


Figure 2: Figure illustrates (left) the sum rate convergence at 25dB SNR, and (right) sum rate versus SNR plots for $\alpha = 0.01, 0.99$ with $M = N = 5$, $K = 4$, $d = 2$ and $\mathcal{S} = \{1, 2\}$. Both methods yield better sum rates than conventional IA, and method-II is the best one.

## Conclusion

- When transmissions from two (or more users) are partially correlated, two MSE based precoding methods are derived exploiting the correlation.
- In particular, first method is based on joint precoding computation, while the other computes individual user's precoder.
- With an iterative convergent procedure similar to a typical IA algorithm, simulation results verify the sum rate global convergence and the improved sum rate performance of method-II over the method-I and typical IA applied to this system.

## References

[1] S. M. Razavi and T. Ratnarajah, "Adaptive LS and MMSE based beamformer design for multiuser MIMO interference channels," *IEEE TVT*, vol. 9545, pp. 1–1, 2015.

[2] J.-J. Xiao, S. Cui, Z.-Q. Luo, and A. J. Goldsmith, "Linear coherent decentralized estimation," *Signal Processing, IEEE Transactions on*, vol. 56, no. 2, pp. 757–770, 2008.