Iteratively Reweighted Penalty Alternating Minimization Methods with Continuation for Image Deblurring

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Overview

- We develop a convergent penalty method to a class of structured linearly constrained problems frequently used in deblurring.
- To speed up the convergence speed, we use a continuation strategy for the penalty parameter.

Problem

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$$\min_{x,y}\{\Psi(x,y) := f(x) + \sum_{i=1}^{N} h(g(y_i))\},$$
$$\text{s.t. } Ax + By = c.$$

Drawbacks of Nonconvex ADMM:

1. The convergence guarantees of nonconvex ADMMs require a very large Lagrange dual multiplier. Worse still, the large multiplier makes the nonconvex ADMM run slowly.
2. When applying nonconvex ADMMs to the nonconvex TV deblurring model, by direct checks, the convergence requires TV operator to be full row-rank; however, the TV operator cannot promise such an assumption.
3. The previous analyses show that the sequence converges to a critical point of an auxiliary function under several assumptions. But the relationship between the auxiliary function and the original one is unclear in the nonconvex settings.

Algorithm

The algorithm is designed by combining linearization techniques to the nonsmooth part and alternating minimization strategy.

Scheme : parameters $\bar{\gamma} > 0, a > 1, \delta > 0$

Initialization: $z^0 = (x^0, y^0)$, $\gamma_0 > 0$

for $k = 0, 1, 2, \ldots$

$x^{k+1} \in \arg\min_x\{f(x) + \frac{\gamma_k}{2}\|Ax + By^k - c\|_2^2\}$

$w_i^k \in -\partial(-h(g(y_i^k)))$, $i \in [1, 2, \ldots, N]$

$y^{k+1} \in \arg\min_y\{\sum_i^N w_i^k g(y_i) + \frac{\gamma_k}{2}\|Ax^{k+1} + By - c\|_2^2 + \frac{\delta\gamma_k\|y-y^k\|_2^2}{2}\}$

$\gamma_{k+1} = \min\{\bar{\gamma}, (a\gamma_k)\}$

end for

Output x^k

Convergence

Under several mild assumptions we can prove:

(1)

$$\Phi_{\bar{\gamma}}(x^k, y^k) - \Phi_{\bar{\gamma}}(x^{k+1}, y^{k+1})$$
$$\geq \min\{\bar{\gamma}, \nu\bar{\gamma}\} \cdot \|x^{k+1} - x^k\|_2^2 + \frac{\delta\bar{\gamma}\|y^{k+1} - y^k\|_2^2}{2}.$$

for $k > \ulcorner \log_a(\frac{\bar{\gamma}}{\gamma_0}) \urcorner$.

(2)

$$\sum_k(\|x^{k+1} - x^k\|_2^2 + \|y^{k+1} - y^k\|_2^2) < +\infty,$$

which implies that

$$\lim_k \|x^{k+1} - x^k\|_2 = 0, \quad \lim_k \|y^{k+1} - y^k\|_2 = 0.$$

Numerical results

We apply the proposed algorithm to image deblurring and compare the performance with the nonconvex ADMM. The Lena image is used in the numerical experiments.

Figure: Deblurring results for Lena under Gaussian operator by using the two algorithms. (a) Original image; (b) Blurred image; (c) IRPAMC 16.0dB; (d) nonconvex ADMM 14.4dB.