Acoustic Equalization for Headphones Using a Fixed Feed-Forward Filter

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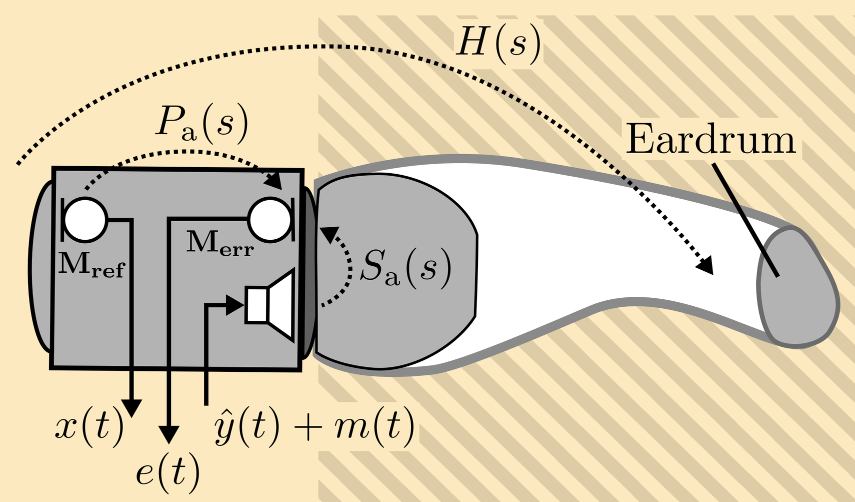
1 Introduction

Novel application: Active acoustic equalization (AAE) for headphones

- Equalization of ambient sound with fixed but arbitrary target transfer function $H(s)$ for, e.g., reconfigurable hearing protection
- Evaluated using a parametric equalizer as an example
- Utilize principles of active noise cancellation

Goal: Design fixed feed-forward FIR filter $\hat{W}(z)$ based on minimum mean square error (MMSE) criterion to approximate $H(s)$ with overall discrete-time transfer function $\hat{H}(z) = P(z) - \hat{W}(z)S(z)$



The discrete-time transfer functions $P(z)$ and $S(z)$ comprise the acoustic transfer functions $P_{\mathrm{a}}(s)$ and $S_{\mathrm{a}}(s)$, the ADCs and DAC, as well as the microphones and the loudspeaker

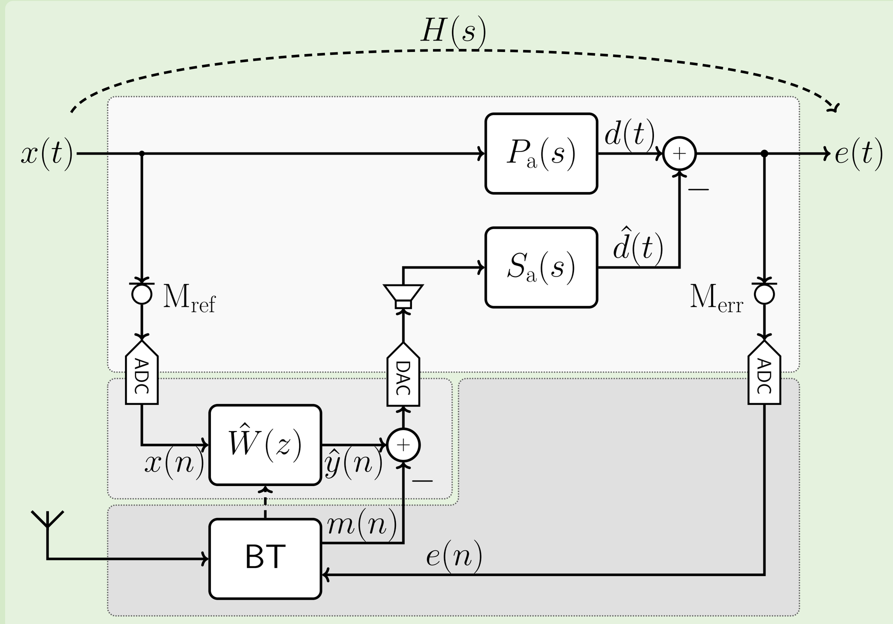
2 System Overview

- Time-domain formulation

$$\hat{\boldsymbol{h}} = \boldsymbol{p} - \boldsymbol{S}\hat{\boldsymbol{w}} \stackrel{!}{=} \boldsymbol{h}$$

with convolution matrix

$$\boldsymbol{S} = \begin{bmatrix} s_0 & 0 & \dots & 0 \\ s_1 & s_0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ s_{L-1} & s_{L-2} & \dots & s_0 \\ 0 & s_{L-1} & \dots & s_1 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & s_{L-1} \end{bmatrix}$$



- Cost function $\mathcal{C}_{\mathrm{wb}}$ and causal Wiener solution [1, 2]

$$\min_{\hat{\boldsymbol{w}}} \mathcal{C}_{\mathrm{wb}} = \left\|\hat{\boldsymbol{h}} - \boldsymbol{h}\right\|^2$$

$$\hat{\boldsymbol{w}}_{\mathrm{wb}} = \left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)^{-1}\boldsymbol{S}^{\mathrm{T}}\left(\boldsymbol{p} - \boldsymbol{h}\right)$$

$\boldsymbol{p} = [p_0, p_1, \dots, p_{L-1}, 0, \dots, 0]^{\mathrm{T}} \in \mathbb{R}^{2L-1}$
$\boldsymbol{h} = [h_0, h_1, \dots, h_{L-1}, 0, \dots, 0]^{\mathrm{T}} \in \mathbb{R}^{2L-1}$
$\hat{\boldsymbol{w}} = [\hat{w}_0, \hat{w}_1, \dots, \hat{w}_{L-1}]^{\mathrm{T}} \in \mathbb{R}^{L}$
$\boldsymbol{S} \in \mathbb{R}^{2L-1 \times L}$

- Optional Bluetooth (BT) module to configure $\hat{W}(z)$ or calibrate $S(z)$

3 Generalized Solution

- For target function $\boldsymbol{h} = g\tilde{\boldsymbol{h}}$ we lose influence of $\tilde{\boldsymbol{h}}$ for high attenuation ($g \ll 1$) as

$$\hat{\boldsymbol{h}} = \left[\boldsymbol{I} - \boldsymbol{S}\left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)^{-1}\boldsymbol{S}^{\mathrm{T}}\right]\boldsymbol{p} + g\boldsymbol{S}\left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)^{-1}\boldsymbol{S}^{\mathrm{T}}\tilde{\boldsymbol{h}}$$

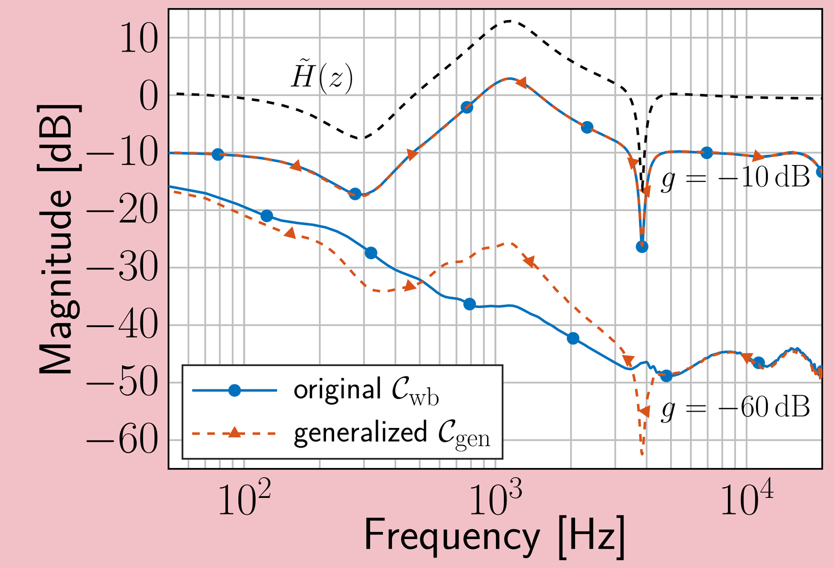
- Regain control by extending cost function $\mathcal{C}_{\mathrm{wb}}$ to

$$\mathcal{C}_{\mathrm{gen}} = \left\|\hat{\boldsymbol{X}}\left(\hat{\boldsymbol{h}} - \boldsymbol{h}\right)\right\|^2$$

with convolution matrix $\hat{\boldsymbol{X}}$ based on signal [3]

$$\hat{\boldsymbol{x}} = \mathcal{Z}^{-1}\left\{\frac{1}{|H(z)|}\right\}$$

Example for 3-band equalizer



- Causal Wiener solution for cost function $\mathcal{C}_{\mathrm{gen}}$ yields

$$\hat{\boldsymbol{w}}_{\mathrm{gen}} = \left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\Psi}_{\hat{x}\hat{x}}\boldsymbol{S}\right)^{-1}\boldsymbol{S}^{\mathrm{T}}\boldsymbol{\Psi}_{\hat{x}\hat{x}}\left(\boldsymbol{p} - \boldsymbol{h}\right)$$

with auto-correlation matrix $\boldsymbol{\Psi}_{\hat{x}\hat{x}}$

4 Robust Approach with Respect to Variations in Acoustic Paths

- Variations of $P(z)$ and $S(z)$ due to
  - Physiology of ear
  - Direction of arrival of ambient sound
  - Headphone fitting
- Filter-design based on J previous measurements of $P(z)$ and $S(z)$

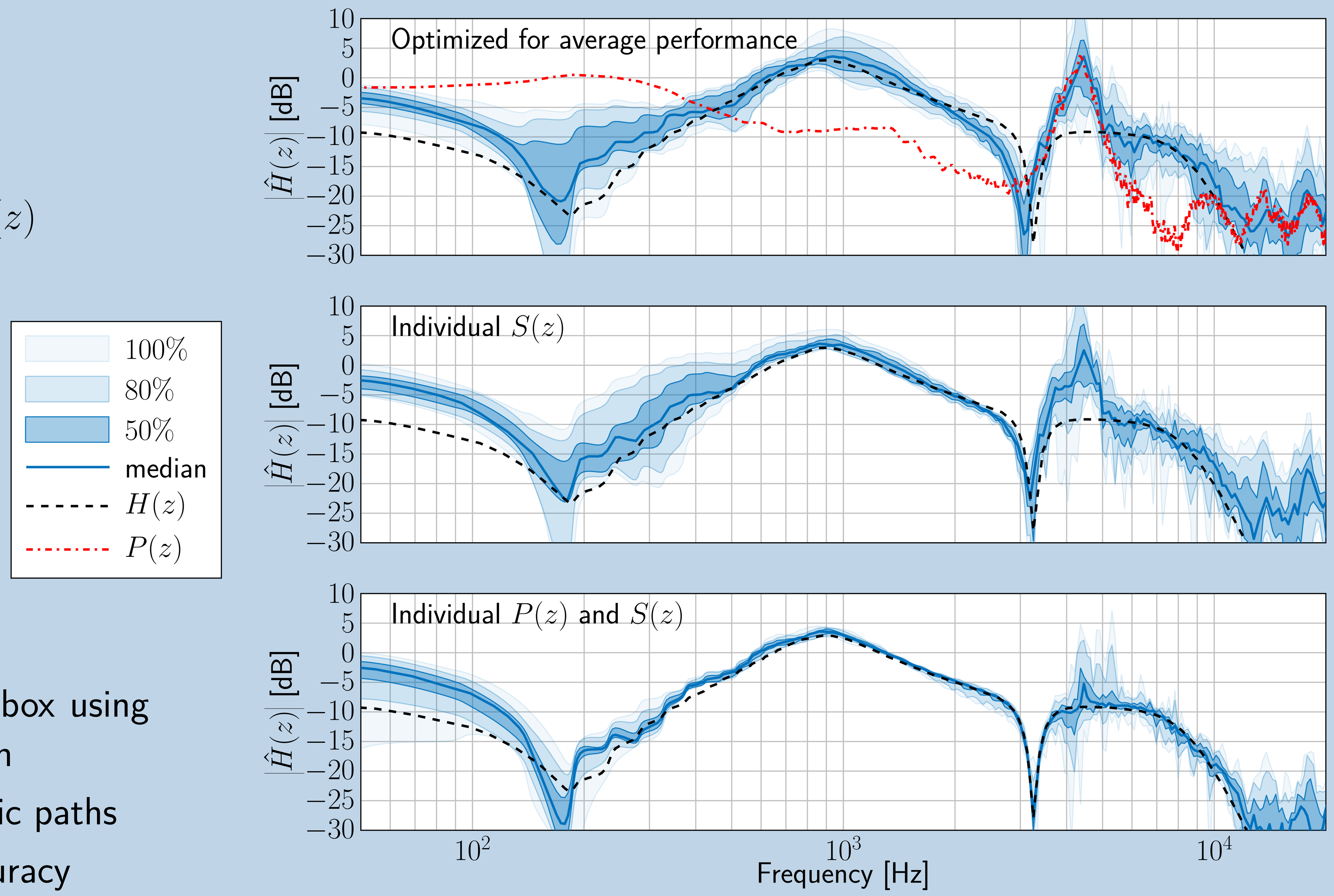
$$\hat{\boldsymbol{h}}_j = \boldsymbol{p}_j - \boldsymbol{S}_j\hat{\boldsymbol{w}} \stackrel{!}{=} \boldsymbol{h}, \quad j = 0, 1, \dots, J$$

- Averaged cost function $\mathcal{C}_{\mathrm{avg}}$ and causal Wiener solution

$$\min_{\hat{\boldsymbol{w}}} \mathcal{C}_{\mathrm{avg}} = \sum_{j=0}^{J-1}\left\|\hat{\boldsymbol{X}}\left(\hat{\boldsymbol{h}}_j - \boldsymbol{h}\right)\right\|^2$$

$$\hat{\boldsymbol{w}}_{\mathrm{avg}} = \left(\sum_{j=0}^{J-1}\boldsymbol{S}_j^{\mathrm{T}}\boldsymbol{\Psi}_{\hat{x}\hat{x}}\boldsymbol{S}_j\right)^{-1}\sum_{j=0}^{J-1}\boldsymbol{S}_j^{\mathrm{T}}\boldsymbol{\Psi}_{\hat{x}\hat{x}}\left(\boldsymbol{p}_j - \boldsymbol{h}\right)$$

Evaluation of real-time AAE system



Evaluation

- Measured transfer functions for $J = 12$ subjects in studio box using Bose QC20 acoustic front-end and dSPACE real-time system
- Performance shown for different knowledge of actual acoustic paths
- Individualization of $P(z)$ and $S(z)$ can further improve accuracy

5 Conclusions

- ▶ Novel **active acoustic equalization (AAE)** system 2
- ▶ Generalized **MMSE based** solution allows frequency shaping even for high target attenuation 3
- ▶ Considering variability of acoustic paths 4 has huge impact on performance
- ▶ **Individualization** of acoustic paths further improves the accuracy
- ▶ Evaluation of **real-time** AAE system with 12 subjects 4 confirms results

References

[1] Sen M. Kuo and Dennis R. Morgan, *Active Noise Control Systems: Algorithms and DSP Implementations*, Wiley, Hoboken, 1996.

[2] Colin Hansen, Scott Snyder, Xiaojun Qiu, Laura Brooks, and Danielle Moreau, *Active Control of Noise and Vibration*, CRC Press, Boca Raton, 2012.

[3] Swen Müller and Paulo Massarani, "Transfer-function measurement with sweeps," *J. Audio Eng. Soc.*, vol. 49, no. 6, pp. 443–471, 2001.