CONVOLUTIONAL-SPARSE-CODED DYNAMIC MODE DECOMPOSITION AND ITS APPLICATION TO RIVER STATE ESTIMATION

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ABSTRACT

Extended dynamic mode decomposition (**EDMD**) with convolutional sparse coding(**CSC**) is proposed.

- EDMD is a data-driven analysis method for nonlinear dynamic systems.
- Conventional EDMD is difficult to reflect knowledge on spatial structure.
- Modified the analysis approach to convolutional synthesis one.
- Applied to estimate river bed state from water surface to reduce riverbank erosion.
- Solved by primal-dual splitting method (**PDS**).
- Experimental results show the significance of the estimation.

Index Terms - Convolutional sparse coding, Extended dynamic mode decomposition, Primal-dual splitting, NSOLT

Ⅰ. INTRODUCTION

- **Problem:** Existing EDMD is difficult to reflect knowledge on spatial structure.
- **Purpose:** Develop EDMD reflecting the knowledge on spatial structure.

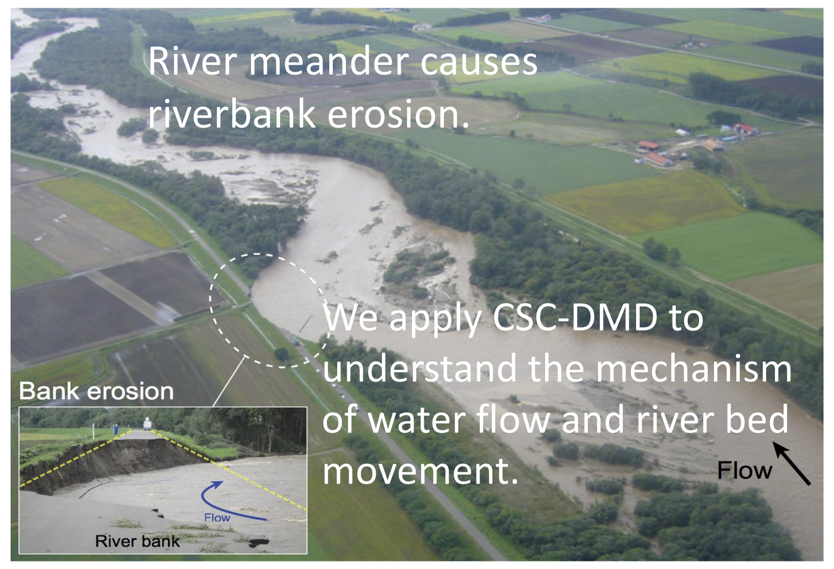

Fig. 1：River meandering

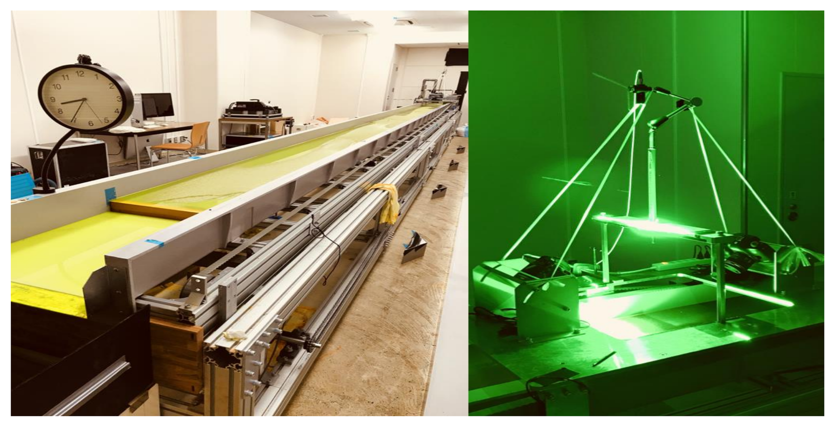

Fig. 2：River model(left), observation device(right)

Ⅱ. REVIEW OF EDMD & Ⅲ. CONVOLUTIONAL-SPARSE-CODED DMD

- Dynamic mode decomposition[1] (DMD) is a high-dimensional time-series data analysis method.
- Convolutional EDMD introduced an analysis dictionary to DMD.

➢ How can we choose the analysis map ?

State： $\mathbf{x}_k \in \mathcal{M} \subseteq \mathbb{R}^M$,
$\mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1})$

Feature： $\mathbf{y}_k \in \tilde{\mathcal{F}} \subseteq \mathbb{C}^L$,
$\mathbf{y}_k = \mathbf{\Psi}(\mathbf{x}_k)$

Nonlinear map： $\begin{Bmatrix} \mathbf{F}: \mathcal{M} \rightarrow \mathcal{M} \\ \mathbf{\Psi}: \mathcal{M} \rightarrow \tilde{\mathcal{F}} \end{Bmatrix}$

$\mathbf{y}_k$ time development： $\mathbf{y}_k \approx \mathbf{K}\mathbf{\Psi}(\mathbf{x}_{k-1})$

Time evolution ：$\mathbf{x}(t) = \tilde{\mathbf{D}}\mathbf{\Phi}e^{\mathbf{\Omega}t}\mathbf{\Phi}^{\dagger}\mathbf{y}_{o}$

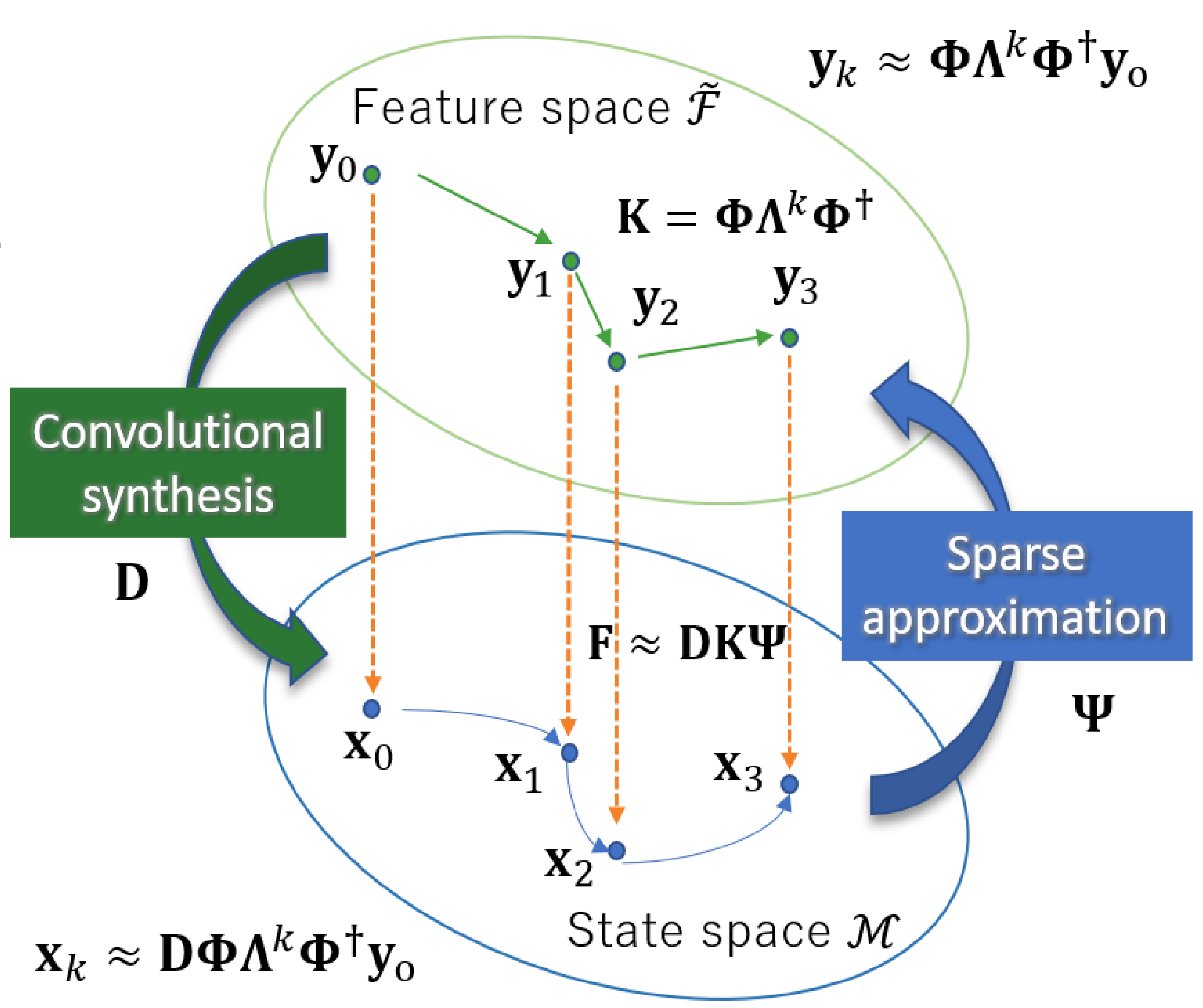

Fig. 3：Basic concept of CSC-DMD

- We modify the analysis approach to conventional synthesis one.
- Adopt NSOLT[2], a learnable Parseval tight filter bank as $\mathbf{D}$.

Design of synthesis dictionary D

$$\left\{\hat{\mathbf{D}}, \{\hat{\mathbf{y}}_k\}\right\} = \arg\min_{\mathbf{D},\{\mathbf{y}_k\}} J(\mathbf{D}, \{\mathbf{y}_k\} \,|\{\mathbf{x}_k\})$$

$$J(\mathbf{D}, \{\mathbf{y}_k\} \,|\{\mathbf{x}_k\}) \triangleq \sum_{k=0}^{N-1}\left(\frac{1}{2}\|\mathbf{x}_k - \mathbf{D}\mathbf{y}_k\|_2^2 + \lambda\rho(\mathbf{y}_k)\right)$$

Sparse coding Ψ

$$\mathbf{\Psi}(\mathbf{x}) = \arg\min_{\mathbf{y}\in\mathbb{R}^L} \frac{1}{2}\left\|\mathbf{x} - \hat{\mathbf{D}}\mathbf{y}\right\|_2^2 + \lambda\rho(\mathbf{y})$$

Application to river bed estimation

- Assumption: water surface shape $\mathbf{x}_{Sk} \in \mathbb{R}^{m_S}$ can be measured.
- Estimate river bed shape $\mathbf{x}_{Bk} \in \mathbb{R}^{m_B}$ from $\mathbf{x}_{Sk}$.

State vector： $\mathbf{x}_k \triangleq \begin{pmatrix} \mathbf{x}_{Sk} \\ \mathbf{x}_{Bk} \end{pmatrix} \in \mathbb{R}^M$

Extraction matrix： $\begin{Bmatrix} \mathbf{Q} \triangleq (\mathbf{O} \quad \mathbf{I}) \\ \mathbf{P} \triangleq (\mathbf{I} \quad \mathbf{O}) \end{Bmatrix}$

Estimation of river bed： $\hat{\mathbf{x}}_{Bk} = \mathbf{Q}\hat{\mathbf{D}}\hat{\mathbf{y}}_k = \mathbf{Q}\hat{\mathbf{D}}\mathcal{P}_{\hat{\mathbf{D}},\mathbf{\Phi},\mathbf{\Lambda}}(\mathbf{x}_{Sk}, \hat{\mathbf{y}}_{k-1})$

Time evolution equation： $\hat{\mathrm{x}}_{\mathrm{B}}(t) = \mathrm{Q}\hat{\mathrm{D}}\Phi e^{\Omega(t-k\Delta t)}\Phi^{\dagger}\hat{\mathrm{y}}_k, \quad \ell \geq k$

Estimator $\mathcal{P}_{\hat{\mathbf{D}},\mathbf{\Phi},\mathbf{\Lambda}}(\mathbf{x}_{Sk}, \hat{\mathbf{y}}_{k-1})$： $\hat{\mathbf{y}}_k = \arg\min_{\mathbf{y}\in\mathbb{R}^L} \frac{1}{2}\left\|\mathbf{x}_{Sk} - \mathbf{P}\hat{\mathbf{D}}\mathbf{y}\right\|_2^2 + \lambda\rho(\mathbf{y})$,

s.t. $\left\|\hat{\mathbf{D}}(\mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^{\dagger}\hat{\mathbf{y}}_{k-1} - \mathbf{y})\right\|_2^2 \leq \varepsilon^2$ ：Tolerance of the prediction error

<u>Solved by primal-dual splitting method</u> (PDS)[3]

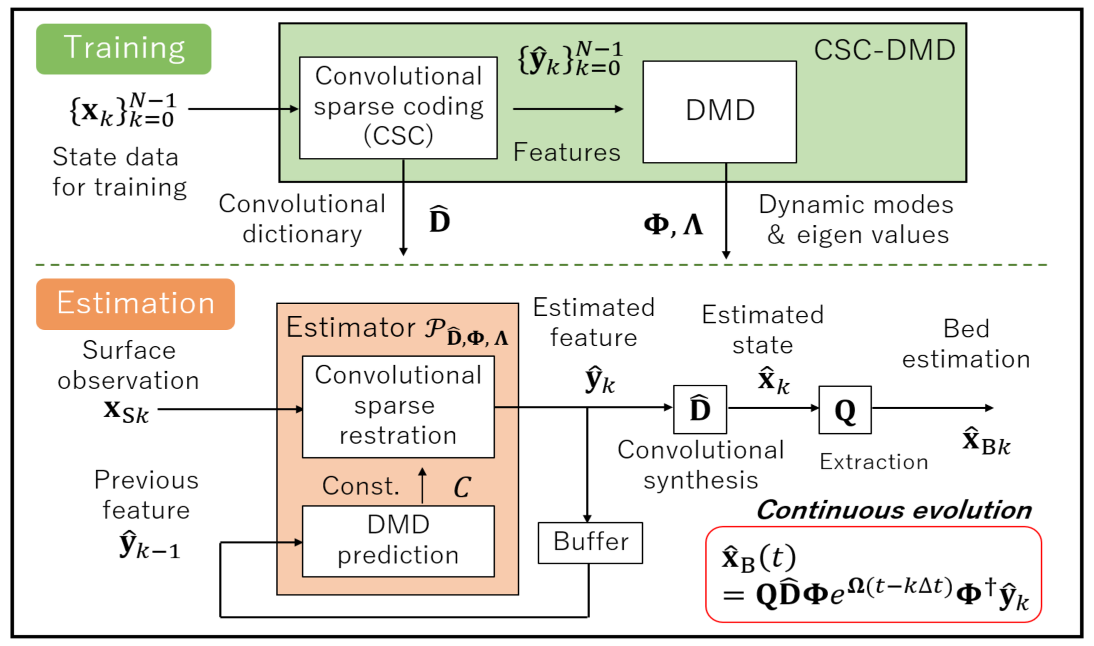

Fig. 4：Proposed river bed estimation system

Ⅳ. PERFORMANCE EVALUATION

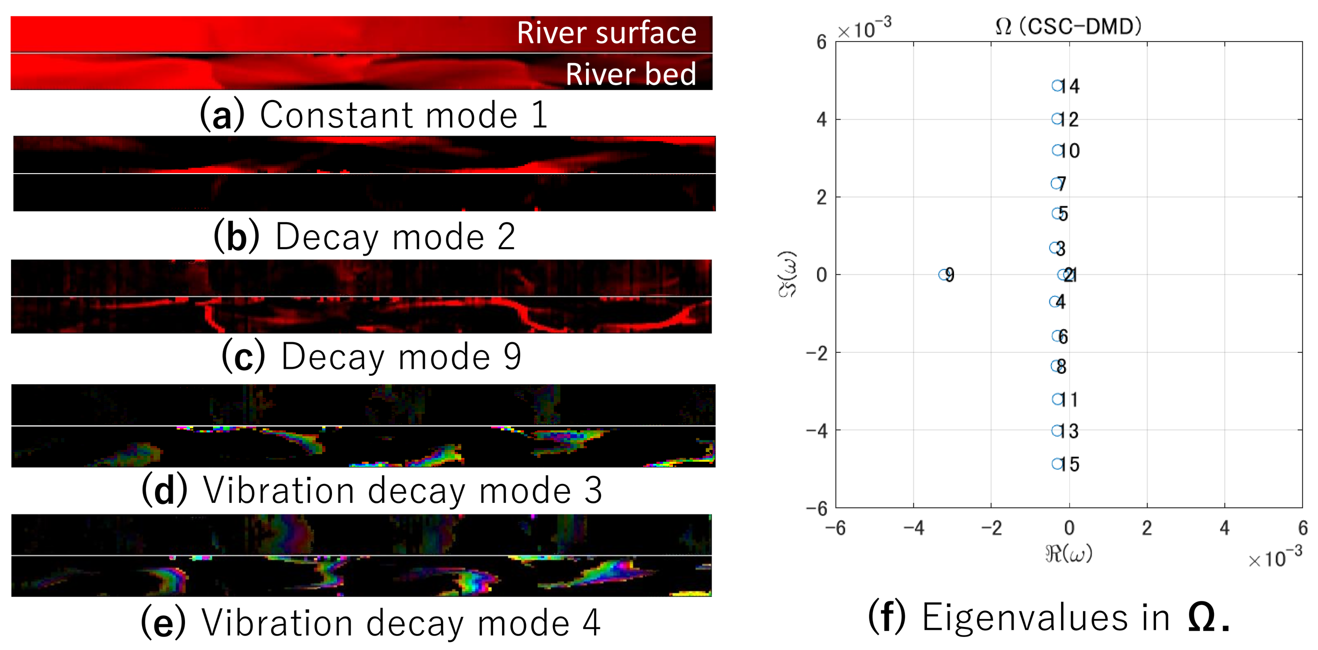

Fig. 5：CSC-DMD Examples: dynamic modes in $\hat{\mathbf{D}}\mathbf{\Phi}$ and placement of the eigenvalues.

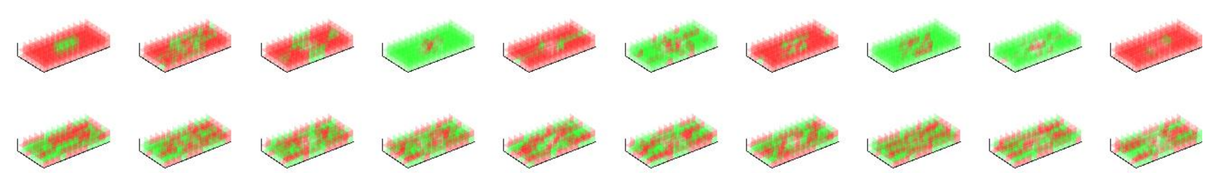

Fig. 6：Convolutional dictionary kernels (NSOLT)[2]. Decimation factor $[M_y, M_x, M_z] = [2, 4, 2]$, Polyphase order $[N_y, N_x, N_z] = [2, 2, 0]$.

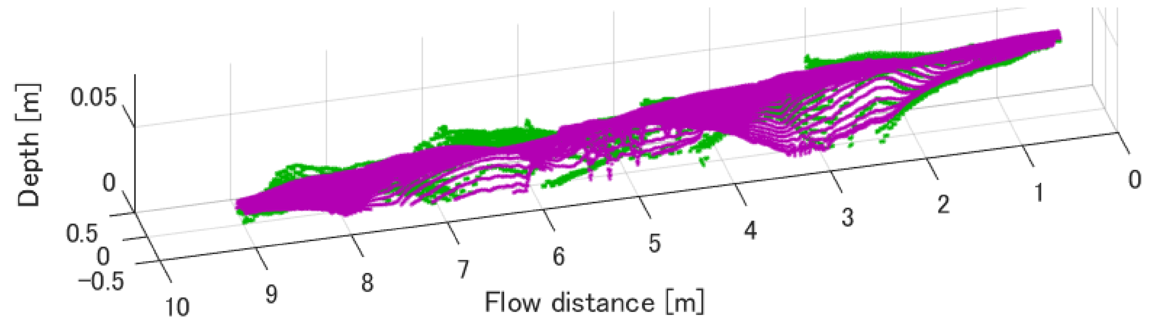

Fig. 7：Observation of river bed and its estimation at 440 min after inflow start.

Ⅴ. EXPERIMENTAL RESULT

- Training for 100 to 250 min and evaluation for 260 to 460 min.
- CSC-DMD (green) performs better than normal DMD (red).

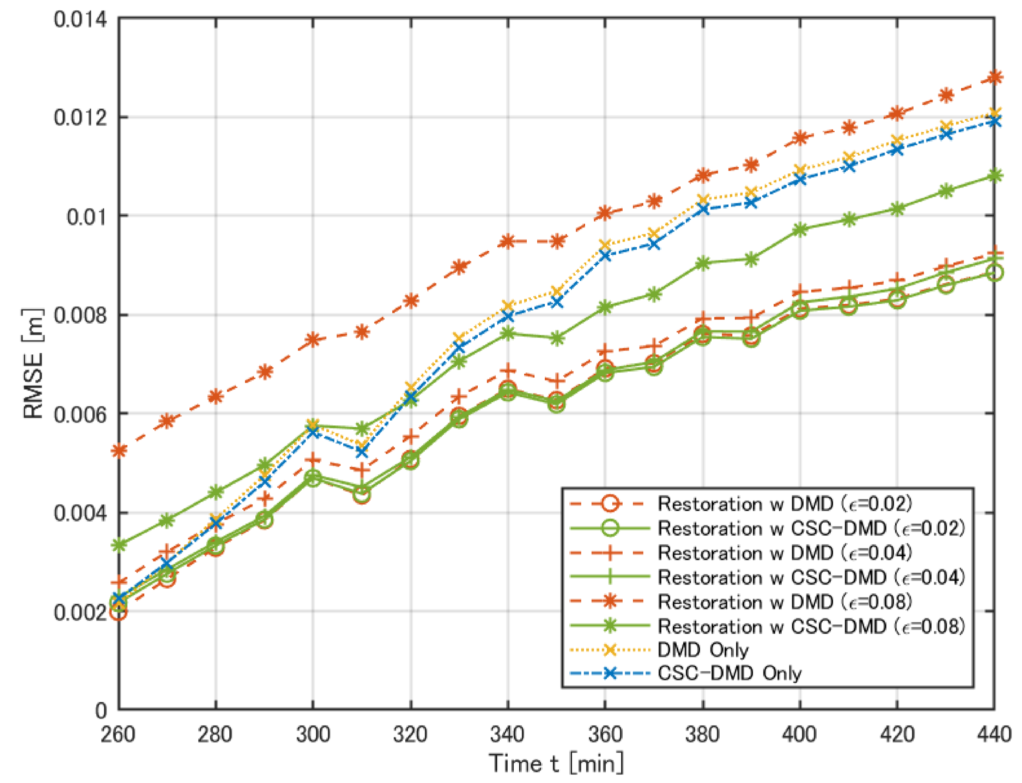

Fig. 8：Experimental results of river bed estimation

Ⅵ. CONCLUSIONS

- Proposed applying CSC to EDMD.
- Demonstrated the proposed method performs better than DMD.
- Future work: investigate the tolerance to state change, spatial transition and multi-scalability.

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