# Frequency-Selective Hybrid Precoding and Combining for mmWave MIMO Systems with Per-Antenna Power Constraints

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## CHALLENGE

MIMO OFDM link

Fully-connected hybrid MIMO

Each antenna in the array has its own power amplifier

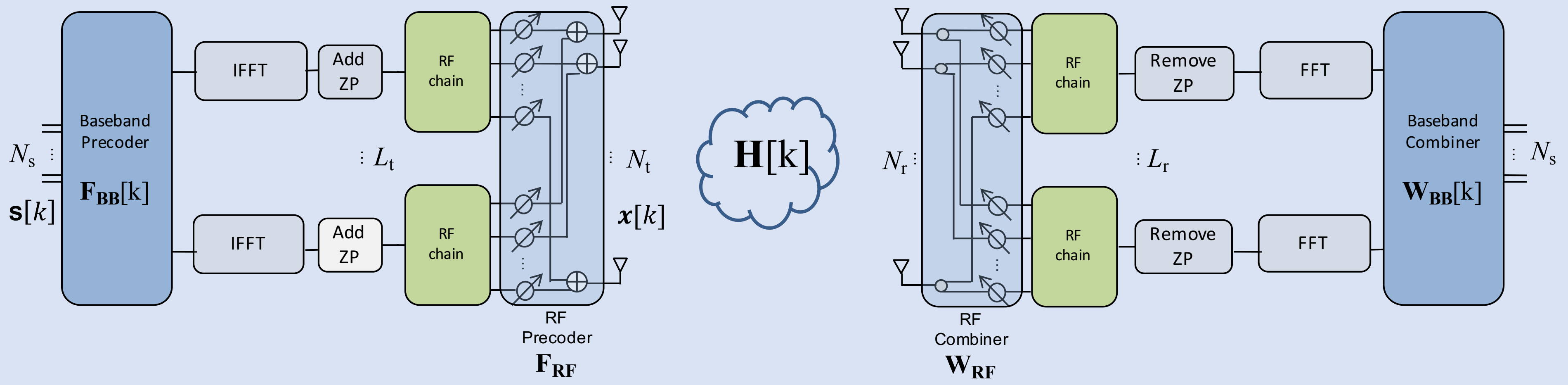

**Fig. 1**: Illustration of the structure of a hybrid MIMO architecture, which include analog and digital precoders and combiners.

Large # of antennas makes per-antenna-constrained design much more important at mmWave MIMO

Goal: design hybrid precoders and combiners under per-antenna power constraints (PPC)

## SYSTEM MODEL

Sampled response for the $d$th delay tap

Pulse shaping filter evaluated at the delays of each cluster

$$\mathbf{H}_d = \sqrt{\frac{N_t N_r}{\rho_L \sum_{c=1}^{C} R_c}} \sum_{c=1}^{C}\sum_{r=1}^{R_c} \alpha_{c,r} p_{\mathrm{rc}}(dT_s - \tau_{c,r}) \times \mathbf{a}_R(\phi_{c,r})\mathbf{a}_T^*(\theta_{c,r}),$$

Complex gains for the different paths

TX and RX array response vectors

$$\mathbf{H}[k] = \sum_{d=0}^{N_c-1} \mathbf{H}_d e^{-\mathrm{j}\frac{2\pi k}{K}d} = \mathbf{A}_R \mathbf{G}[k] \mathbf{A}_T^*$$

TX/RX array matrices

$$\underset{k=0,\ldots,K-1}{\mathcal{R}(\mathbf{F}[k],\mathbf{W}[k])} = \frac{1}{K}\sum_{k=0}^{K-1}\log_2\left|\mathbf{I}_{N_s} + \frac{\mathrm{SNR}}{N_s}(\mathbf{W}^*[k]\mathbf{W}[k])^{-1}\mathbf{W}^*[k]\mathbf{H}[k]\mathbf{F}[k]\mathbf{F}^*[k]\mathbf{H}^*[k]\mathbf{W}[k]\right|$$

Goal: maximize spectral efficiency

$$\{\{\mathbf{F}^\star[k]\}_{k=0}^{K-1}, \{\mathbf{W}^\star[k]\}_{k=0}^{K-1}\} = \underset{\{\mathbf{F}[k]\}_{k=0}^{K-1},\{\mathbf{W}[k]\}_{k=0}^{K-1}}{\arg\max} \mathcal{R}(\mathbf{F}[k],\mathbf{W}[k])$$

$$\text{subject to} \quad \frac{1}{N_s}\mathbf{e}_j^*\left(\sum_{k=0}^{K-1}\mathbf{F}[k]\mathbf{F}^*[k]\right)\mathbf{e}_j \leq p_j, \quad j = 1,\ldots,N_t$$

Hybrid precoders

Hybrid combiners

j-th Per-antenna power constraint

## PROPOSED HYBRID DESIGN

1. Use an upper bound to all-digital solution [1] → 2. Factorize all-digital solution into hybrid filters

$$\mathbf{F}[k] = \tilde{\mathbf{V}}_H[k]\mathbf{Q}_F[k]\boldsymbol{\Sigma}_F[k]$$

**Minimize Euclidean distance**

$$\mathbf{W}[k] = \tilde{\mathbf{U}}_H[k]\mathbf{R}_W[k]$$

### Low-complexity Hybrid precoder design

$$\min_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}} \|\mathbf{F} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

$$\text{subject to} \quad \begin{cases} \mathbf{F}_{\mathrm{RF}} \in \mathcal{M}^{N_t\times L_t}(Q_t), \\ \frac{1}{N_s}\mathbf{e}_j^*\left(\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^*\mathbf{F}_{\mathrm{RF}}^*\right)\mathbf{e}_j \leq p_j, \\ j = 1,\ldots,N_t, \end{cases}$$

Stacked precoders

$$\mathbf{F} \triangleq \underbrace{[\mathbf{F}[0]\ldots\mathbf{F}[K-1]]}_{\text{size } N_t\times KN_s} = [\underbrace{\mathbf{U}_{F,1}}_{N_t\times L_t} \quad \underbrace{\mathbf{U}_{F,2}}_{N_t\times(N_t-L_t)}]\underbrace{\boldsymbol{\Sigma}_F}_{N_t\times N_t}\underbrace{\mathbf{V}_F^*}_{N_t\times KN_s}$$

Analog precoder

$$[\mathbf{F}_{\mathrm{RF}}]_{j,i} = e^{\mathrm{j}Q(\measuredangle[\mathbf{U}_{F,1}]_{j,i})}, 1\leq j\leq N_t, 1\leq i\leq L_t$$

$$\mathbf{G} \triangleq \mathbf{F}_{\mathrm{RF}}^*\mathbf{F} = \mathbf{U}_G\boldsymbol{\Sigma}_G\mathbf{V}_G^* \qquad \mathbf{F}_{\mathrm{BB}} = \mathbf{U}_{\mathrm{BB}}\boldsymbol{\Sigma}_{\mathrm{BB}}\mathbf{V}_{\mathrm{BB}}^*$$

$$\mathbf{U}_{\mathrm{BB}} = \mathbf{U}_G \qquad \mathbf{V}_{\mathrm{BB}} = \mathbf{V}_G$$

Baseband precoder

$$\max_{\{\sigma_{\mathrm{BB},i}\}_{i=1}^{L_t}} \sum_{i=1}^{L_t}\sigma_{G,i}\sigma_{\mathrm{BB},i}$$

$$\text{subject to} \quad \frac{1}{N_s}\sum_{i=1}^{L_t}|\mathrm{g}_{j,i}|^2\sigma_{\mathrm{BB},i}^2 \leq p_j, \quad j = 1,\ldots,N_t,$$

### Hybrid combiner design

$$\min_{\mathbf{W}_{\mathrm{RF}},\{\mathbf{W}_{\mathrm{BB}}[k]\}} \sum_{k=0}^{K-1}\|\mathbf{W}[k] - \mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}}[k]\|_F^2$$

$$\text{subject to} \quad \begin{cases} \mathbf{W}_{\mathrm{RF}} \in \mathcal{M}^{N_t\times L_t}(Q_t), \\ \mathbf{W}_{\mathrm{BB}}^*[k]\mathbf{W}_{\mathrm{RF}}^*\mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}}[k] = \mathbf{I}_{N_s}, \\ k = 0,\ldots,K-1 \end{cases}$$

Stacked combiners

$$\mathbf{W} \triangleq \underbrace{[\mathbf{W}[0]\ldots\mathbf{W}[K-1]]}_{\text{size } N_r\times KN_s} = [\underbrace{\mathbf{U}_{W,1}}_{N_r\times L_r} \quad \underbrace{\mathbf{U}_{W,2}}_{N_r\times(N_r-L_r)}]\underbrace{\boldsymbol{\Sigma}_W}_{N_r\times N_r}\underbrace{\mathbf{V}_W^*}_{N_r\times KN_s}$$

Analog combiner

$$[\mathbf{W}_{\mathrm{RF}}]_{m,n} = e^{\mathrm{j}Q(\measuredangle[\mathbf{U}_{W,1}]_{m,n})} = \mathbf{U}_{\mathrm{RF}}\boldsymbol{\Sigma}_{\mathrm{RF}}\mathbf{V}_{\mathrm{RF}}^*$$

$$\mathbf{W}_{\mathrm{BB}}^*[k]\mathbf{V}_{\mathrm{RF}}\boldsymbol{\Sigma}_{\mathrm{RF}}^2\mathbf{V}_{\mathrm{RF}}^*\mathbf{W}_{\mathrm{BB}}[k] = \mathbf{I}_{N_s}, \quad k = 0,\ldots,K-1.$$

$$\mathbf{W}_{\mathrm{BB}}[k] = \mathbf{V}_{\mathrm{RF}}\boldsymbol{\Sigma}_{\mathrm{RF}}^{-1}\mathbf{Z}_W[k] \qquad \mathbf{W}_{\mathrm{RF}}\mathbf{W}_{\mathrm{BB}}[k] = \mathbf{U}_{\mathrm{RF}}\mathbf{Z}_W[k]$$

$$\min_{\mathbf{Z}[k]} \sum_{k=0}^{K-1}\|\mathbf{W}[k] - \mathbf{U}_{\mathrm{RF}}\mathbf{Z}_W[k]\|_F^2$$

$$\text{subject to} \quad \mathbf{Z}_W^*[k]\mathbf{Z}_W[k] = \mathbf{I}_{N_s}.$$

Baseband combiner (orthogonal Procrustes problems [2])

$$\mathbf{C}[k] \triangleq \mathbf{U}_{\mathrm{RF}}^*\mathbf{W}[k] = \mathbf{U}_C[k]\boldsymbol{\Sigma}_C[k]\mathbf{V}_C^*[k]$$

$$\mathbf{Z}_W[k] = \mathbf{U}_C[k]\mathbf{V}_C^*[k]$$

## RESULTS

Channel samples from 3GPP 5G NR UMa (Rice factor -10 dB)

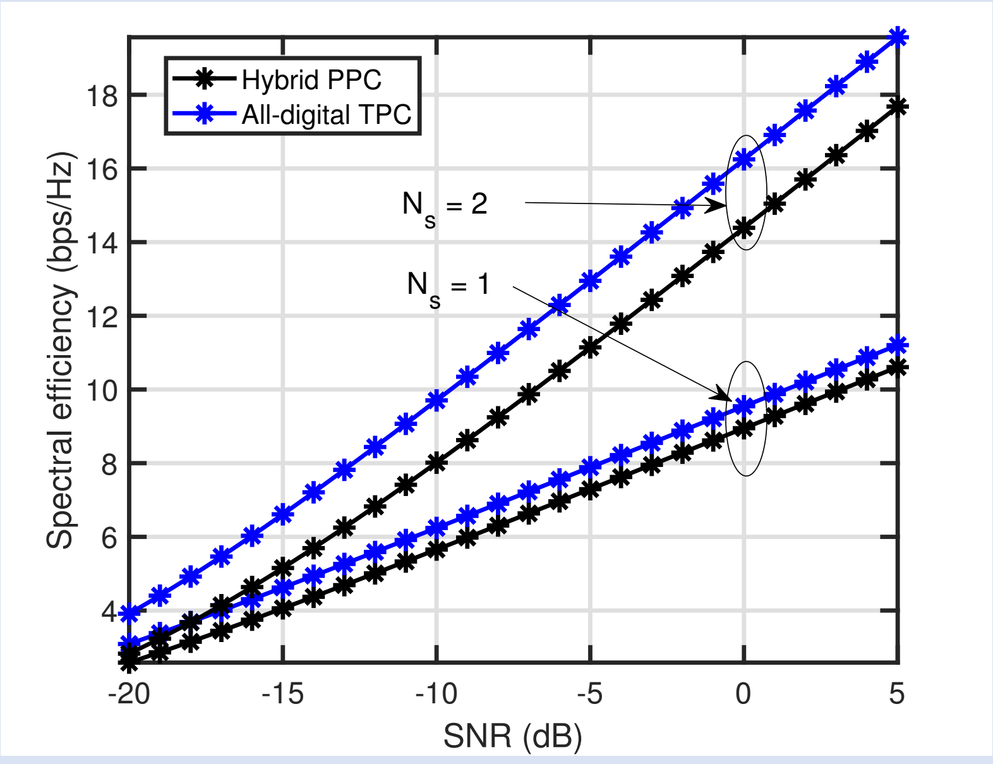

**Fig. 2**: Average spectral efficiency obtained with the proposed PPC and TPC designs. The number of transmit and receive antennas is $N_t = 64$ and $N_r = 16$, and the number of RF chains is $L_t = 4$ and $L_r = 2$.

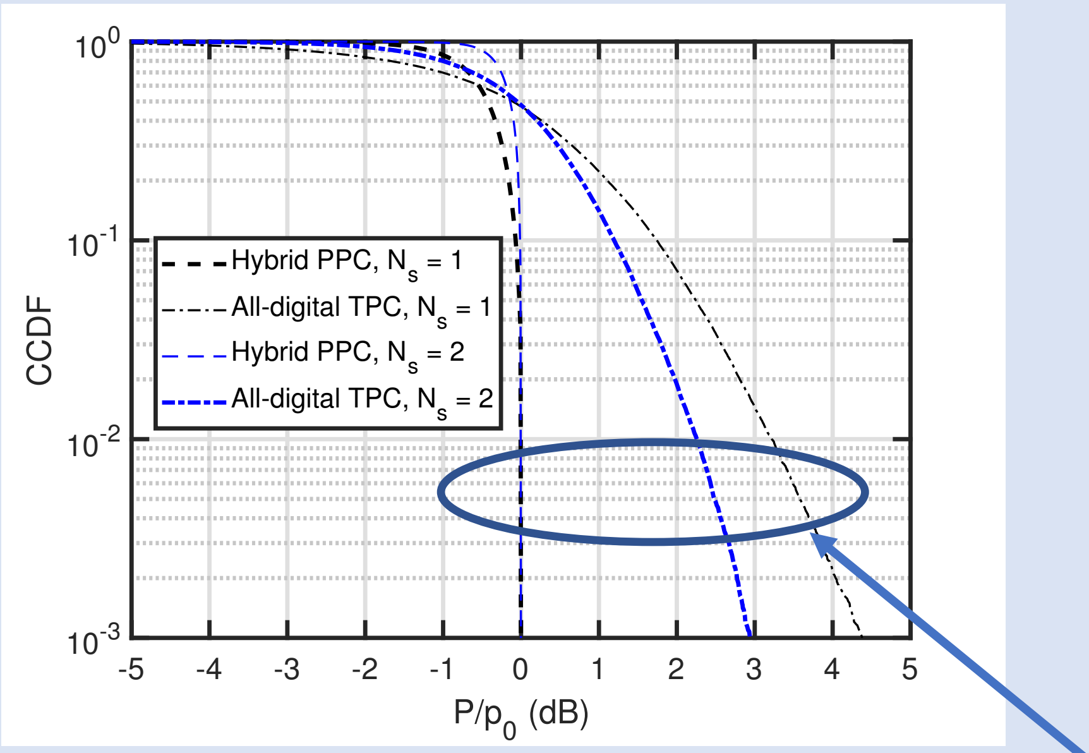

**Fig. 3**: Complementary Cumulative Distribution Function (CCDF) obtained with the proposed PPC and TPC designs. The number of transmit and receive antennas is $N_t = 64$ and $N_r = 16$, and the number of RF chains is $L_t = 4$ and $L_r = 2$.

Per-antenna constraints fulfilled in every case

Reasonably good performance of PPC design

Power backoff of 3-4 dB needed for TPC design

[1] J. Rodriguez-Fernandez, R. Lopez-Valcarce, and N. Gonzalez-Prelcic, “Hybrid precoding and combining for frequency-selective mmWave MIMO systems with per-antenna power constraints”, available at arxiv, 2018.

[2] G. H. Golub and C. F. van Loan, “Matrix computations, 3rd ed.” Johns Hopkins Univ. Press, 1996.