An Iterative Time Domain Denoising Method

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May 14, 2019

Introduction

- The classic signal restoration problem is to determine the ideal signal $x(n)$ from the detected signal $y(n)$ given by the additive model
$$y(n) = p(n) * x(n) + \eta(n) \tag{1}$$
where $p(n)$ denotes the impulse response of the data acquisition system, the asterisk $(*)$ represents linear convolution, and $\eta(n)$ is a random process.
- The noise samples $\eta(n)$ are generally assumed to be white and Gaussian distributed.
- The noise is typically assumed to be independent of $x(n)$.

Some Previous Methods to Remove Noise

- averaging filters
- adaptive median filters
- Weiner filter
- Wavelet Thresholding
- Empirical Mode Decomposition Interval Thresholding (EMT-IT)

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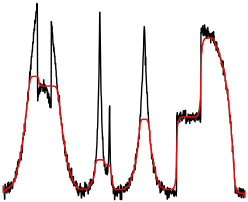
Iterative Mean Filters



- Black is original signal with additive noise.
- Red is the result of an iterative averaging filter, the Squeeze Box Filter (SBF)
- Pros
  - Accurate when the signal is smooth.
- Cons
  - Diminishes sharp edges, peaks, and valleys.

SBF with Thresholding (SBFT)

1) Set iteration indices $i, j = 0$ and $y_{i,j}(n) = y(n)$.
2) Set iteration limits $\lambda_1, \lambda_2 > 0$, thresholds $T_{i,1}, T_{i,2} \geq 0$ for $i = 0, 1, 2, \ldots, \lambda_2$, and convergence criteria $\epsilon > 0$.
3) Each iteration j (j starts at one) begins by determining the set of locations of local maxima (peaks) and local minima (valleys). The locations of these extrema are defined by the set

$$\mathcal{N}_E = \{n \mid y_{i,j-1}(n) \text{ meets condition 1 or 2 }\}$$

Condition 1: $y_{i,j-1}(n) > y_{i,j-1}(n-1)$ and $y_{i,j-1}(n) > y_{i,j-1}(n+1)$
Condition 2: $y_{i,j-1}(n) < y_{i,j-1}(n+l)$ and $y_{i,j-1}(n) < y_{i,j-1}(n+1)$

4) Without using the local extrema values, samples within an odd length L window centered at $y_{i,j-1}(n)$ are used to determine the local mean. These extrema maybe replaced with the local mean values. That is for $n \in \mathcal{N}_E$ the local mean is computed as:

$$\bar{y}_{i,j-1}(n) = \frac{1}{L-1}\left(\left(\sum_{l=-\lfloor \frac{L}{2} \rfloor}^{\lfloor \frac{L}{2} \rfloor} y_{i,j-1}(n+l)\right) - y_{i,j-1}(n)\right)$$

where $\lfloor \cdot \rfloor$ is the greatest integer function.

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5) The minimum and maximum values within the length L window centered at $y_{i-1}(n)$ are determined

$$m = \min\left(\left\{y_{i,j-1}(n+l) \;\middle|\; l = 0, \pm 1, \pm 2, \pm \left\lfloor \frac{L}{2} \right\rfloor \right\}\right)$$

and

$$M = \max\left(\left\{y_{i,j-1}(n+l) \;\middle|\; l = 0, \pm 1, \pm 2, \pm \left\lfloor \frac{L}{2} \right\rfloor \right\}\right).$$

6) The outlier maybe replace according to

$$y_{i,j}(n) = \begin{cases} y_{i,j-1}(n) & \text{if } |M - m| \geq T_{i,1} \text{ or} \\ & \qquad |\bar{y}_{i,j-1}(n) - y_{i,j-1}(n)| \geq T_{i,2} \\ \bar{y}_{i,j-1}(n) & \text{otherwise.} \end{cases}$$

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7a) If $j < \lambda_1$ and convergence in the Cauchy sense is not attained, that is

$$\sum_{n=0}^{N-1} |y_{i,j-1}(n) - y_{i,j}(n)| > \epsilon, \qquad (2)$$

then j is incremented by one and another iteration, starting from Step 3, is performed.

7b) If $j = \lambda_1$ or Cauchy convergence, is attained, then when $i < \lambda_2$, i is incremented by one, $j = 0$, and

$$y_{i,j}(n) = y_{i-1,\lambda_1}(n) * h(n)$$

where $h(n)$ is a low pass filter. The process continues starting at Step 3.

8) The algorithm stops when $i = \lambda_2$. An approximation of the noise free signal is produced as

$$\widehat{y}(n) = y_{\lambda_2,\lambda_1}(n).$$

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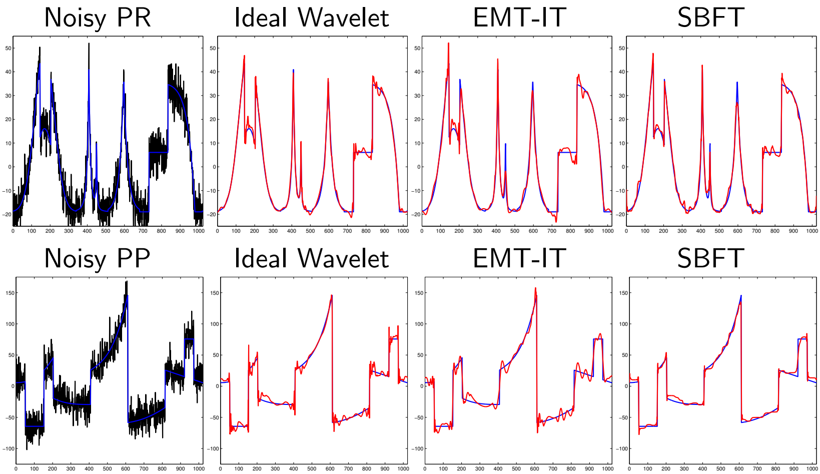
Experiment

- Compared SBFT with
  - Ideal wavelet, wavelet denoising with hard and soft thresholding from Wavelab 850 found at https://statweb.stanford.edu/∼wavelab/
  - EMD-IT from Y. Kopsinis and S. McLaughlin, Development of EMD-based denoising methods inspired by wavelet thresholding, *IEEE Trans. Signal Processing*, vol. 57, no. 4, pp. 18471860, April 2009.
- Wavelab 850 test signals used
  - Piece-Regular (PR) with additive Gaussian noise of $\sigma = 5$.
  - Piece-Polynomial (PP) with additive Gaussian noise of $\sigma = 5$.
  - Blocks (BL) with additive Gaussian noise of $\sigma = 1$.
  - Doppler (DP) with additive Gaussian noise of $\sigma = 0.1$.
- SBFT parameters

	PR	PP	BL	DP
λ_1	3	45	30	2
$T_{0,1}$	21	70	4	0.34
$T_{0,2}$	18	60	2	0.35
$T_{1,1}$	12	40	2.4	0.29
$T_{1,2}$	5	25	1.4	0.13

Results



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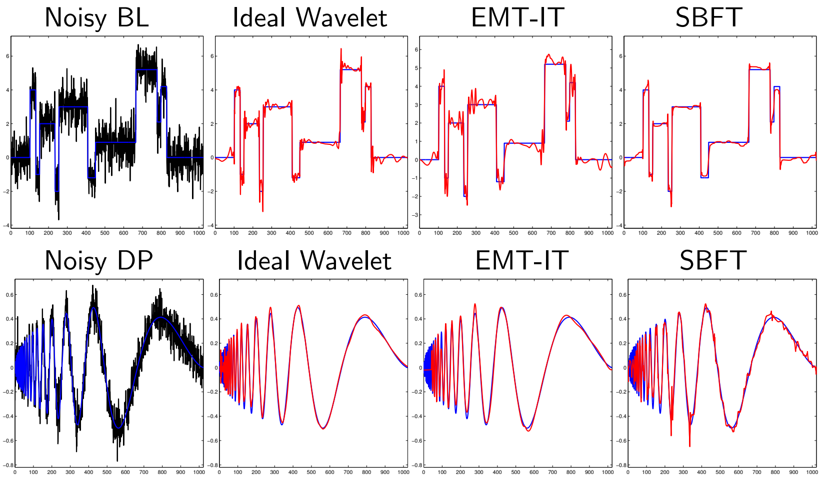
Results


Noisy BL
Ideal Wavelet
EMT-IT
SBFT
Noisy DP
Ideal Wavelet
EMT-IT
SBFT

Signal to Noise Ratio (SNR)

- $\text{SNR}\{\widehat{\mathbf{y}}\} = 20\log_{10}\left(\frac{\|\mathbf{x}\|}{\|\mathbf{x}-\widehat{\mathbf{y}}\|}\right)$ dB
  where $\mathbf{x}$ is the noise free signal and $\widehat{\mathbf{y}}$ is the reconstructed signal.

	WaveLab Signal			
Method	PR	PP	BL	DP
unprocessed	11.30	10.49	10.05	9.65
wavelet-ST	12.24	16.13	9.61	11.42
wavelet-HT	18.05	12.61	14.89	18.31
EMD-IT	18.26	16.24	16.48	18.51
SBFT	20.50	18.53	**19.64**	16.37
Ideal Wavelet	**22.83**	**18.71**	19.59	**20.58**

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Conclusion

- The SBFT incorporates a thresholding operation into the iterative averaging SBF algorithm to preserve a signal's peaks and valleys, while the additive noise is being reduced.
- The results show the proposed SBFT is capable of in excess of 2 dB SNR improvement over the wavelet-ST, wavelet-HT, and EMD-IT methods on certain signals.
- In the PP restoration the SBFT nearly attained the same SNR as the ideal wavelet method.
- In the BL example the SNR of SBFT exceeds the SNR of the ideal wavelet method and provided over 3 dB improvement over the others.
- When restoring the band limited DP signal, the SBFT did not perform on par with wavelet-HT and EMD-IT where the subband decomposition may have been advantageous.
- The evidence provided shows that the SBFT may be a more robust denoising method than subband decomposition based methods with certain signals.

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Thank you!!!

Questions?