Terrain-Scattered Jammer Suppression in MIMO Radar Using Space-(Fast) Time Adaptive Processing

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Introduction

- **Terrain-scattered/diffuse jammer suppression** is one of the most important issues in radar signal processing [1, 2].
- Significantly increased **degrees of freedom** enable superiorities of MIMO radar over phased-array (PA) radar [3].
- **New opportunities** of clutter/jammer suppression have been shown in MIMO radar in recent years [3, 4].
- **Space-time adative processing (STAP)** techniques play an important role in radar signal processing, especially for clutter/jammer suppression [5].

Contributions

- Problem of terrain-scattered jammer suppression using **space-(fast) time adaptive processing (SFTAP)** is studied in MIMO radar framework.
- **Correlation function** of jamming components after matched filtering (MF) at the receiving end of MIMO radar is derived.
- A minimum variance distortionless response **(MVDR) type SFTAP design** which considers waveform-introduced range sidelobes and cold clutter stationarity over different pulse intervals is proposed.
- **Closed-form solution** to the MVDR type design and a **relaxed SFTAP design** are provided.

Background

- Jamming signals take the form of high-power transmission that aims at impairing the receive system.
- Terrain-scattered jamming occurs when the high-power jammer transmits its energy to ground, and it reflects the energy in a dispersive manner.
- Pure mutual orthogonality of multiple waveforms does not exist, which leads to necessity of studying the effect of MF on the received jamming signals.
- MIMO radar faces the challenge of significantly increased computational burden, therefore, developing computationally affordable STAP techniques is important.

Signal Model

- **Target signal**:

$$\mathbf{y}_{\mathrm{t}}(\zeta,\tau)=\sqrt{\frac{E}{M}}\alpha_{\mathrm{t}}D_{\mathrm{t}}(\tau)\big(\mathbf{R}_{\boldsymbol{\phi}}^{\mathrm{T}}(\zeta)\mathbf{a}(\theta_{\mathrm{t}})\big)\otimes\mathbf{b}(\theta_{\mathrm{t}}).$$

- **Clutter signal**:

$$\mathbf{y}_{\mathrm{c}}(\zeta,\tau)=\sqrt{\frac{E}{M}}\sum_{i=1}^{N_c}\xi_i D_i(\tau)\big(\mathbf{R}_{\boldsymbol{\phi}}^{\mathrm{T}}(\zeta)\mathbf{a}(\theta_i)\big)\otimes\mathbf{b}(\theta_i).$$

- **Jamming signal**:

$$\mathbf{y}_{\mathrm{j}}(\zeta,\tau)=\sum_{j=1}^{J}\sum_{p=1}^{P}\beta_{j,p}\boldsymbol{\eta}_{j,p}(\zeta,\tau)\otimes\mathbf{b}(\vartheta_{j,p}).$$

- **Entire signal**: $\mathbf{y}_{\mathrm{j}}(\zeta,\tau)\triangleq\mathbf{y}_{\mathrm{t}}(\zeta,\tau)+\mathbf{y}_{\mathrm{c}}(\zeta,\tau)+\mathbf{y}_{\mathrm{j}}(\zeta,\tau)+\mathbf{y}_{\mathrm{n}}(\zeta,\tau).$

Signal Model (Cont'd)

- **Parameters**:

  E: Transmit energy; M: Number of transmit antennas.

  ζ, τ: Fast-time and slow-time indices, respectively.

  α_{t}, ξ_i: Reflection coefficients of target and the ith clutter patch.

  θ_{t}, θ_i: Spatial directions of target and the ith clutter patch.

  $D_{\mathrm{t}}(\tau)$, $D_i(\tau)$: Doppler shift of target and the ith clutter patch.

  $\mathbf{a}(\theta)$, $\mathbf{b}(\theta)$: $M \times 1$ transmit and $N \times 1$ receive steering vectors.

  $\mathbf{R}_{\boldsymbol{\phi}}(\zeta)$: Correlation matrix of emitted waveforms denoted by $\boldsymbol{\phi}$.

  N_c: Number of clutter patches; $\otimes$: Kronecker product.

  J: Number of jamming sources; P: Number of diffuse multipath.

  $\beta_{j,p}$, $\vartheta_{j,p}$: Magnitude and spatial angle of jamming signal associated with the jth jammer and the pth propagation path.

  $\boldsymbol{\eta}_{j,p}(\zeta, \tau)$: Match-filtered jamming signal associated with the jth jammer and the pth propagation path; $(\cdot)^{\mathrm{T}}$: Transpose.

Jamming Characteristics

- Consider the commonly used **barrage noise jamming** signals $s_j(t,\tau)$, $j=1,\ldots,J$.
- Jamming signals are **mutually independent** and **stationary white** random processes.
- **Correlation** between original jamming signals:

$$\mathbb{E}\left\{s_j(t,\tau)s_{j'}^*(t',\tau')\right\} = S_{\mathrm{j}}(f_c)\delta_{jj'}\delta(t-t')\delta_{\tau\tau'}$$

$(\cdot)^*$: Conjugate transpose operator.
t, t': Fast-time indices; τ, τ': Slow-time indices.
$(\cdot)_j$ (or $(\cdot)_{j'}$): W.r.t. jth (or j'th) jamming signal.
$S_{\mathrm{j}}(f_c)$: Jamming power spectral density at carrier frequency f_c.
$\delta(\cdot)$, $\delta_{j,j'}$ (also $\delta_{\tau\tau'}$): Dirac and Kronecker delta functions, respectively.

Correlation Analysis

- Perform correlation analysis on the MF vector $\boldsymbol{\eta}_{j,p}(\zeta,\tau)$.
- Explicit expression: $\boldsymbol{\eta}_{j,p}(\zeta,\tau) \triangleq \int_{T_p} s_j(t-\zeta_0-\zeta_p,\tau)\boldsymbol{\phi}^*(t-\zeta)\mathrm{d}t$.
- $\boldsymbol{\eta}_{j,p}(\zeta,\tau)$ is the only term that determines the correlation properties of jamming components.
- The $M \times M$ correlation matrix of $\boldsymbol{\eta}_{j,p}(\zeta,\tau)$:

Correlation matrix

$$\begin{aligned}\mathbf{R}^{\boldsymbol{\eta}}_{j,p,j',p'}(\zeta,\zeta',\tau,\tau') &\triangleq \mathbb{E}\{\boldsymbol{\eta}_{j,p}(\zeta,\tau)\boldsymbol{\eta}^{\mathrm{H}}_{j',p'}(\zeta',\tau')\} \\ &= \mathbb{E}\Big\{\iint_{T_p} s_j(t-\zeta_0-\zeta_p,\tau)s^*_{j'}(u-\zeta_0-\zeta_{p'},\tau') \\ &\qquad \times \boldsymbol{\phi}^*(t-\zeta)\boldsymbol{\phi}^{\mathrm{T}}(u-\zeta')\mathrm{d}t\mathrm{d}u\Big\} \\ &= S_{\mathrm{j}}(f_c)\delta_{jj'}\delta_{\tau\tau'}\mathbf{R}^{\mathrm{T}}_{\boldsymbol{\phi}}(\zeta_p-\zeta_{p'}+\zeta'-\zeta+\zeta_0).\end{aligned}$$

Correlation Analysis (Cont'd)

- $\mathbf{R}^{\eta}_{j,p,j',p'}$ is guaranteed to be nonzero once $\zeta_p - \zeta_{p'} + \zeta' - \zeta = 0$.
- The $MN \times MN$ correlation matrix of the jamming signal:

Jamming correlation matrix

$$\mathbf{R}_{\mathrm{j}}(\zeta, \zeta', \tau, \tau') \triangleq \mathbb{E}\{\mathbf{y}_{\mathrm{j}}(\zeta, \tau)\mathbf{y}_{\mathrm{j}}^{\mathrm{H}}(\zeta', \tau')\}$$

$$= \sum_{j=1}^{J}\sum_{j'=1}^{J}\sum_{p=1}^{P}\sum_{p'=1}^{P} \beta_{j,p}\beta^{*}_{j',p'}\mathbf{R}^{\eta}_{j,p,j',p'}(\zeta, \zeta', \tau, \tau') \otimes \left(\mathbf{b}(\vartheta_{j,p})\mathbf{b}^{\mathrm{H}}(\vartheta_{j',p'})\right)$$

$$= S_{\mathrm{j}}(f_c)\delta_{\tau\tau'} \sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{p'=1}^{P} \beta_{j,p}\beta^{*}_{j,p'}\mathbf{R}^{\mathrm{T}}_{\phi}(\zeta_p - \zeta_{p'} + \zeta' - \zeta + \zeta_0)$$

$$\otimes \left(\mathbf{b}(\vartheta_{j,p})\mathbf{b}^{\mathrm{H}}(\vartheta_{j,p'})\right).$$

SFTAP Design

- Stack the available Q taps of data vectors associated with the τth pulse into an $MNQ \times 1$ virtual data vector $\mathbf{y}(\tau)$:

$$\begin{aligned}\mathbf{y}(\tau) &\triangleq \left[\mathbf{y}^{\mathrm{T}}(\zeta_0, \tau), \ldots, \mathbf{y}^{\mathrm{T}}(\zeta_0 + Q - 1, \tau)\right]^{\mathrm{T}} \\ &= \mathbf{y}_{\mathrm{t}}(\tau) + \mathbf{y}_{\mathrm{c}}(\tau) + \mathbf{y}_{\mathrm{j}}(\tau) + \mathbf{y}_{\mathrm{n}}(\tau).\end{aligned}$$

- The $MNQ \times MNQ$ target-free covariance matrix of $\mathbf{y}(\tau)$:

$$\begin{aligned}\mathbf{R}_{\mathbf{y}}(\tau) &\triangleq \mathbb{E}\{\mathbf{y}_{\mathrm{c}}(\tau)\mathbf{y}_{\mathrm{c}}^{\mathrm{H}}(\tau)\} + \mathbb{E}\{\mathbf{y}_{\mathrm{j}}(\tau)\mathbf{y}_{\mathrm{j}}^{\mathrm{H}}(\tau)\} + \mathbb{E}\{\mathbf{y}_{\mathrm{n}}(\tau)\mathbf{y}_{\mathrm{n}}^{\mathrm{H}}(\tau)\} \\ &= \mathbf{R}_{\mathrm{c}}(\tau) + \mathbf{R}_{\mathrm{j}} + \mathbf{R}_{\mathrm{n}} \triangleq \mathbf{R}_{\mathrm{c}}(\tau) + \mathbf{R}_{\mathrm{jn}}.\end{aligned}$$

- For the τth pulse, SFTAP aims at finding an adaptive filter which **minimizes** the output interference power without attenuating target and meanwhile **maximizes** the output signal-to-jammer-plus-noise ratio (SJNR).

MVDR SFTAP Design

- **Key issue**: Stationarity of cold clutter over different pulse intervals after SFTAP should be maintained.
- Proposed SFTAP design:

MVDR SFTAP Design

$$\min_{\mathbf{w}(\tau)} \quad \mathbf{w}^{\mathrm{H}}(\tau)\mathbf{R}_{\mathrm{jn}}\mathbf{w}(\tau) \tag{1a}$$

$$\text{s.t.} \quad \mathbf{w}^{\mathrm{H}}(\tau)\mathbf{s}_{\mathrm{t}}(\theta_{\mathrm{t}}) = 1 \tag{1b}$$

$$\frac{\mathbf{w}^{\mathrm{H}}(\tau)\mathbf{R}_{\mathrm{c}}(\tau)\mathbf{w}(\tau)}{\mathbf{w}^{\mathrm{H}}(0)\mathbf{R}_{\mathrm{c}}(\tau)\mathbf{w}(0)} = 1 \tag{1c}$$

$$\mathbf{w}^{\mathrm{H}}(\tau)\tilde{\mathbf{u}}(\zeta_0, \theta_t) = 0 \tag{1d}$$

$\mathbf{s}_{\mathrm{t}}(\theta_{\mathrm{t}})$: $MNQ \times 1$ target steering vector; $\mathbf{w}(0)$: Weight vector for the first pulse; $\tilde{\mathbf{u}}(\zeta_0, \theta_t) \triangleq [0, \mathbf{u}^{\mathrm{T}}(\zeta_0 + 1, \theta_{\mathrm{t}}), \ldots, \mathbf{u}^{\mathrm{T}}(\zeta_0 + Q - 1, \theta_{\mathrm{t}})]^{\mathrm{T}}$ with $\mathbf{u}(\zeta, \theta_{\mathrm{t}}) \triangleq \left(\mathbf{R}_{\boldsymbol{\phi}}^{\mathrm{T}}(\zeta)\mathbf{a}(\theta_{\mathrm{t}})\right) \otimes \mathbf{b}(\theta_{\mathrm{t}})$.

MVDR SFTAP Design (Cont'd)

- Design of (1) deals with SFTAP problem for each transmitted pulse since Doppler information of clutter signals changes over slow-time domain.
- Constraint (1c) ensures the cold clutter stationarity over different pulse intervals; (1d) accounts for attenuating sidelobes at range bins other than target direction.
- Closed-form solution to (1):

Closed-Form Solution

$$\mathbf{w}(\tau) = (\mathbf{R}_{\mathrm{jn}} + \lambda \mathbf{R}_{\mathrm{c}}(\tau))^{-1} \mathbf{v}(\zeta_0, \theta_t) \Big(\mathbf{v}^{\mathrm{H}}(\zeta_0, \theta_t) \times (\mathbf{R}_{\mathrm{jn}} + \lambda \mathbf{R}_{\mathrm{c}}(\tau))^{-1} \mathbf{v}(\zeta_0, \theta_{\mathrm{t}}) \Big)^{-1} \mathbf{e}$$

$\mathbf{v}(\zeta_0, \theta_t) \triangleq [\mathbf{s}_{\mathrm{t}}(\theta_{\mathrm{t}}), \tilde{\mathbf{u}}(\zeta_0, \theta_t)]$, $\mathbf{e} \triangleq [1, 0]^{\mathrm{T}}$, and λ is determined by $\lambda_{\min}\{\mathbf{R}_{\mathrm{c}}^{-1/2}(\tau)\mathbf{R}_{\mathrm{jn}}\mathbf{R}_{\mathrm{c}}^{-1/2}(\tau)/\big(\mathbf{w}^{\mathrm{H}}(0)\mathbf{R}_{\mathrm{c}}(\tau)\mathbf{w}(0)\big)\}$.

Relaxed SFTAP Design

- Closed-form solution exists when subspace of adaptive weights defined by constraints of (1) is nonempty. In practice, constraints (1c) and (1d) can be relaxed.
- Proposed **relaxed SFTAP** design:

MVDR SFTAP Design

$$\min_{\mathbf{w}(\tau)} \quad \mathbf{w}^{\mathrm{H}}(\tau)\mathbf{R}_{\mathrm{jn}}\mathbf{w}(\tau) \tag{2a}$$

$$\text{s.t.} \quad \mathbf{w}^{\mathrm{H}}(\tau)\mathbf{s}_{\mathrm{t}}(\theta_{\mathrm{t}}) = 1 \tag{2b}$$

$$\|\mathbf{w}^{\mathrm{H}}(\tau)\mathbf{R}_{\mathrm{c}}^{1/2}(\tau) - \mathbf{w}^{\mathrm{H}}(0)\mathbf{R}_{\mathrm{c}}^{1/2}(\tau)\| \leq \epsilon \tag{2c}$$

$$|\mathbf{w}^{\mathrm{H}}(\tau)\tilde{\mathbf{u}}(\zeta_0, \theta_{\mathrm{t}})| \leq \gamma \tag{2d}$$

$\epsilon \geq 0$: Bounding the output clutter distortion caused by $\mathbf{w}(\tau)$; $\gamma \geq 0$: Characterizing the worst range sidelobes towards target direction. For given γ, the **feasibility** of (2) is guaranteed if $\epsilon \geq \epsilon_{\min}$ (minimum output clutter distortion w.r.t. (2b) and (2d)).

Simulations

- $M = 8$ transmit and $N = 8$ receive antennas spaced half wavelength apart from each other.
- Transmit energy $E = M$.
- 4 sets of unimodular waveforms: Polyphase-coded (PC), CA, CAN, and WeCAN-based waveforms.
- One CPI contains 10 pulses.
- $P = 19$ diffuse multipath uniformly distributed within $[-9^\circ, 9^\circ]$, in the presence of $J = 1$ jamming source.
- Target parameter: $\theta_t = 0^\circ$.
- SNR $= 0$ dB, CNR$= 30$ dB, and JNR $= 30$ dB.

Simulation Results (Cont'd)

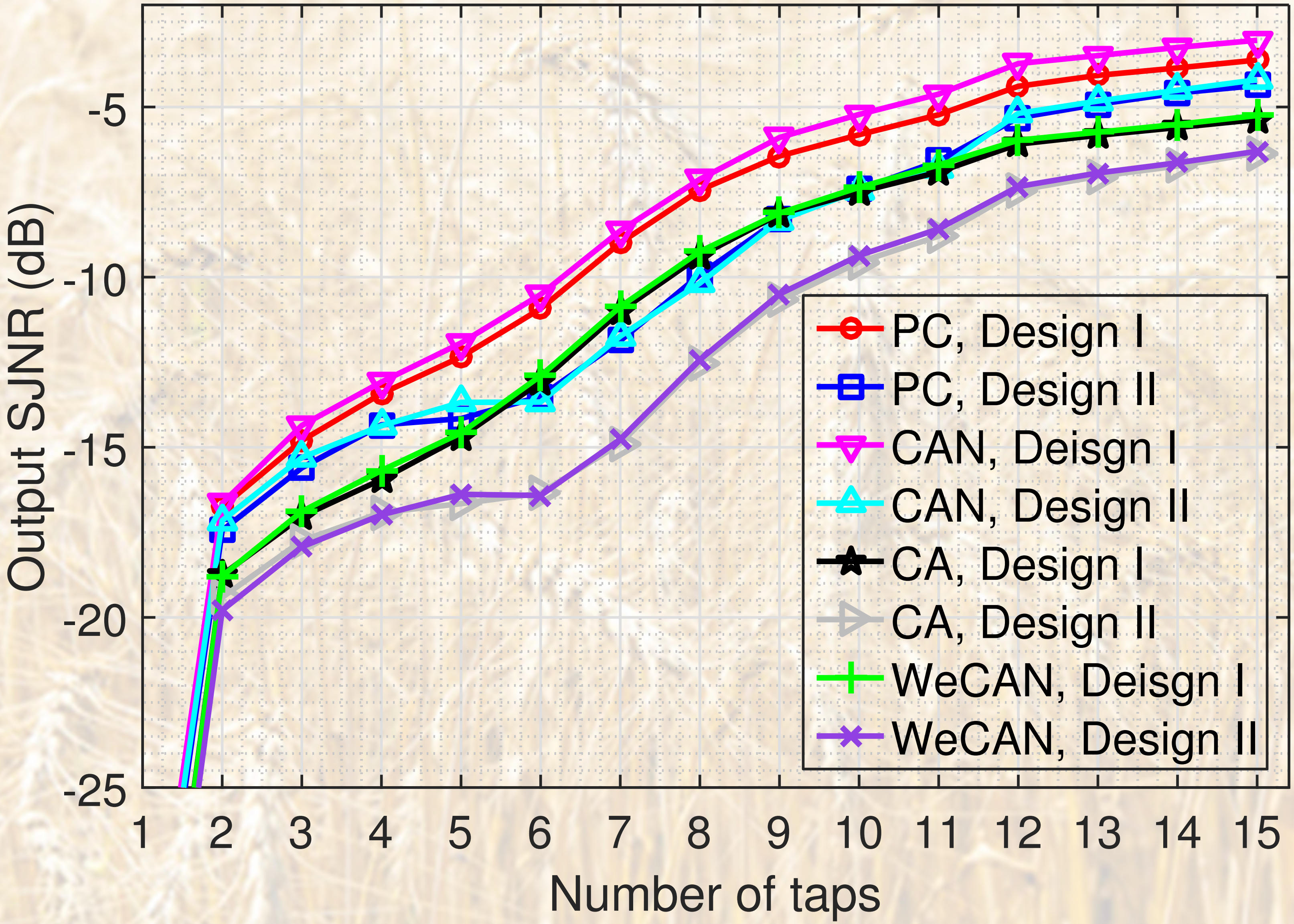

Figure 1: SJNR performance versus employed data taps.

Simulation Results (Cont'd)

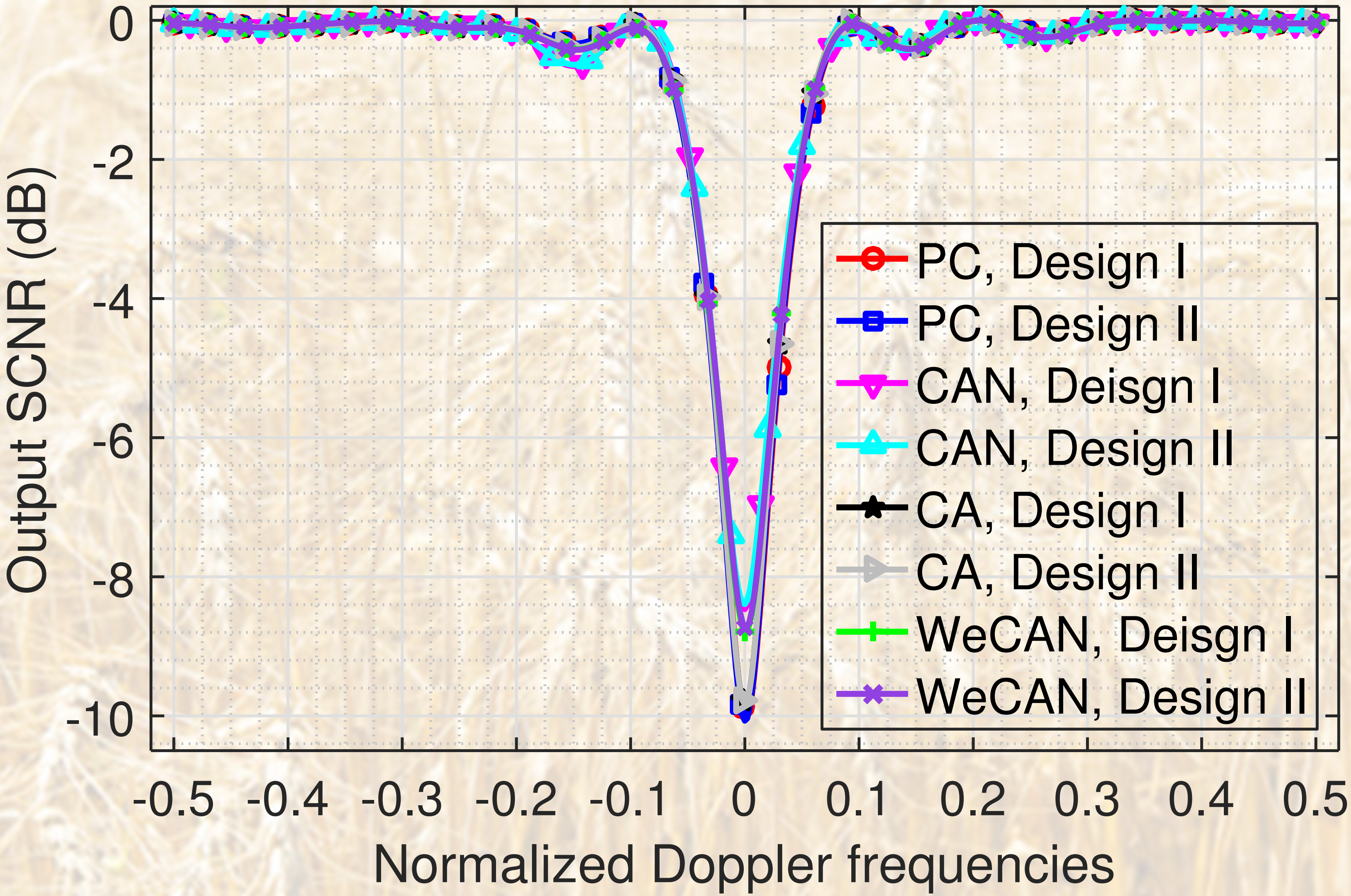

Figure 2: SCNR performance versus normalized Doppler frequencies.

Conclusions

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- Problem of **terrain-scattered jammer suppression** using **SFTAP** has been addressed for MIMO radar.
- The **effect of matched filtering** at the receiving end on barrage noise type jamming has been derived by establishing connections with waveform correlation matrix.
- Proposed **MVDR type SFTAP** and **relaxed SFTAP** designs have been shown able to reduce waveform-introduced range sidelobes and maintain cold clutter stationarity over different pulse intervals.
- **Closed-form solution** to the MVDR type SFTAP design has been obtained.

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