Online Estimation and Smoothing of a Target Trajectory in Mixed Stationary/Moving Conditions

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In a nutshell

- **Reconstruction** of a target trajectory from noisy position measurements arises in many modern applications to track objects, people, or vehicles/robots
- Conventional tracking and smoothing approaches assume that targets move according to a given kinematic model and make only limited maneuvers
- **Design** and **assessment** of a maximum likelihood trajectory estimation algorithm for targets in arbitrary mixed stationary/moving conditions
- Online smoothing of estimated trajectory by exploiting the theory of Bézier curves

Problem formulation

- Target starts with *unknown* initial position $\boldsymbol{p}_0$ and velocity $\boldsymbol{v}$
- Piecewise-linear kinematic model to approximate arbitrary trajectories over sliding observation windows $[t_0, t_K]$, $K \geq 1$

$$\boldsymbol{p}_k = \boldsymbol{p}_{k-1} + \delta_k \boldsymbol{v} T \qquad k = 1, \dots, K$$

with T measurement interval and δ_k position increment wrt $(k-1)$-th position

- Assuming target is stationary up to an *unknown* time instant $k = j$

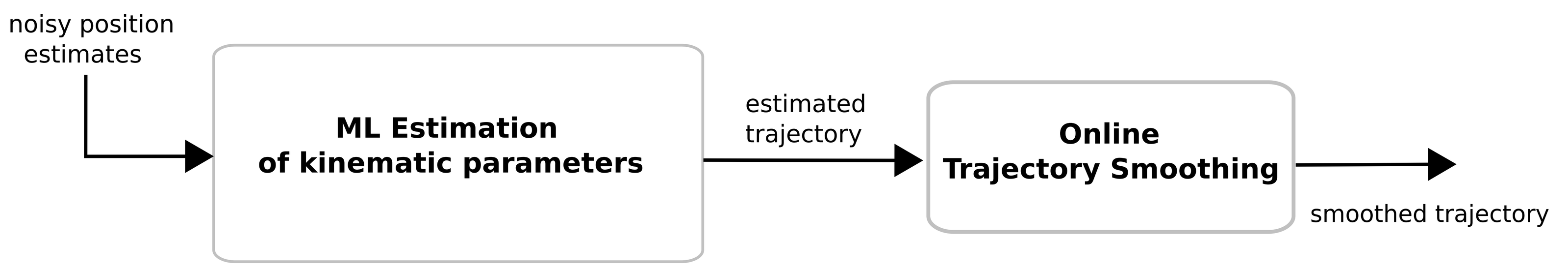
$$\boldsymbol{p}_k = \boldsymbol{p}_0 + \boldsymbol{v} T \boldsymbol{\delta}^\top(j) \boldsymbol{a}_k$$

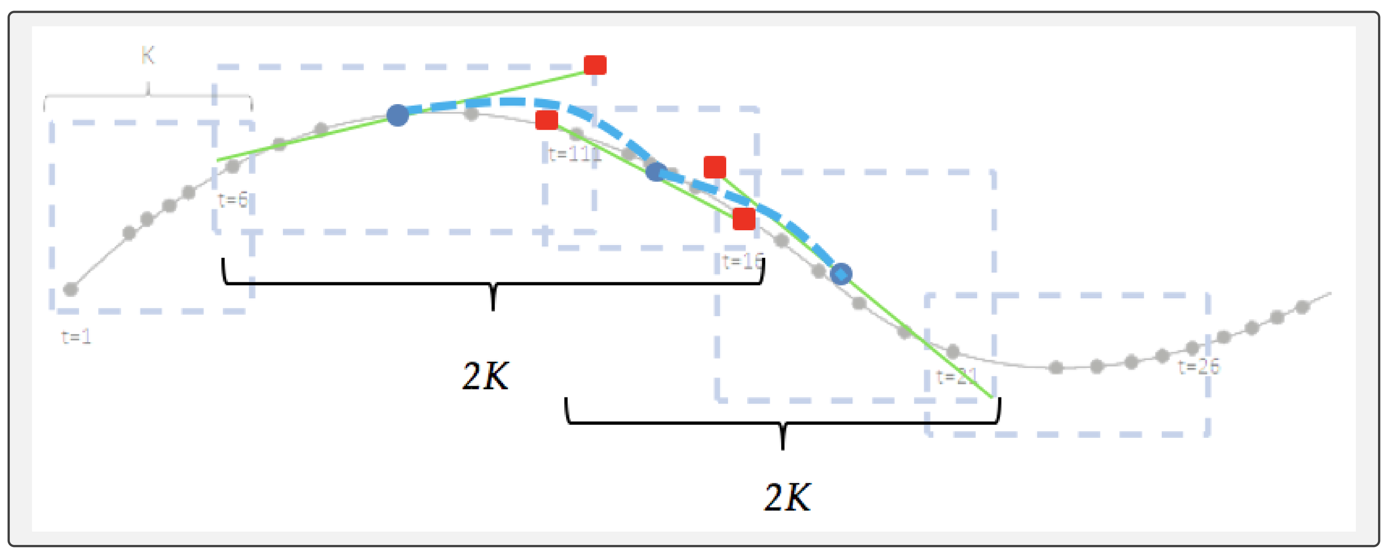
where $\boldsymbol{\delta}(j) = \begin{bmatrix} \boldsymbol{0}_{j-1} \\ \boldsymbol{1}_{K-j+1} \end{bmatrix}$ gives trajectory structure and $\boldsymbol{a}_k = \begin{bmatrix} \boldsymbol{1}_k \\ \boldsymbol{0}_{K-k} \end{bmatrix}$

- **Problem**: estimate $\boldsymbol{p}_0$, $\boldsymbol{v}$, j and reconstruct target trajectory over time

Proposed two-step approach

1. ML estimation of unknown $\boldsymbol{p}_0$, $\boldsymbol{v}$ and time instant j when target starts to move
2. Trajectory reconstruction based on the theory of Bézier curves for online smoothing





Sliding-window approach with marked midpoints (blue circles) and control points of the Bézier curves (red squares)

1. ML Estimation of kinematic parameters

- $\boldsymbol{Y} = [\boldsymbol{y}_1\ \boldsymbol{y}_2\ \cdots\ \boldsymbol{y}_K]$ available noisy position estimates in current processing window
- assuming Gaussian-distributed noise $\boldsymbol{n}_k \sim \mathcal{N}(\boldsymbol{0}_d, \sigma^2 \boldsymbol{I}_d)$ in d-dimensional space

$$\boldsymbol{y}_k = \boldsymbol{\Theta} \boldsymbol{w}_k + \boldsymbol{n}_k$$

with $\boldsymbol{\Theta} = [\boldsymbol{p}_0\ \boldsymbol{v}]$ unknown kinematic parameters and $\boldsymbol{w}_k = \begin{bmatrix} 1 \\ T\boldsymbol{\delta}^\top \boldsymbol{a}_k \end{bmatrix}$

Theorem 1. *The ML estimates of the unknown kinematic parameters are*

$$[\hat{\boldsymbol{p}}_0\ \hat{\boldsymbol{v}}] = \boldsymbol{Y}\boldsymbol{B}(\hat{\jmath})$$

where $\boldsymbol{B}(\hat{\jmath}) = \frac{1}{\gamma(\hat{\jmath})} \begin{bmatrix} T\left(\frac{2K-2\hat{\jmath}+3}{3} \boldsymbol{1}_{1\times K} - \boldsymbol{\alpha}^\top(\hat{\jmath})\right) \\ \frac{2K}{(K-\hat{\jmath}+1)(K-\hat{\jmath}+2)} \boldsymbol{\alpha}^\top(\hat{\jmath}) - \boldsymbol{1}_{1\times K} \end{bmatrix}^\top$, $\boldsymbol{\alpha}(j) = [\boldsymbol{0}_{j-1}^\top\ 1\ 2\ \cdots\ (K-j+1)]^\top$

and $\gamma(\hat{\jmath}) = \frac{T}{6}\left[K(4K - 4\hat{\jmath} + 6) - 3(K - \hat{\jmath} + 1)(K - \hat{\jmath} + 2)\right]$

$$\hat{\jmath} = \underset{j \in \{1,\dots,K\}}{\arg\min} \sum_{k=1}^{K} \|\boldsymbol{y}_k - \boldsymbol{Y}\boldsymbol{H}(j)\boldsymbol{u}_k(j)\|^2$$

with $\boldsymbol{H}(j) = [\boldsymbol{1}_K\ T\boldsymbol{\alpha}(j)]$ *and* $\boldsymbol{u}_k(j) = \frac{1}{\gamma(j)} \begin{bmatrix} \frac{T(2K-2j+3)}{3} - T\boldsymbol{\delta}^\top(j)\boldsymbol{a}_k \\ \frac{2K\boldsymbol{\delta}^\top(j)\boldsymbol{a}_k}{(K-j+1)(K-j+2)} - 1 \end{bmatrix}$

Result: $[\hat{\boldsymbol{p}}_0\ \hat{\boldsymbol{v}}]$ available in closed-form, $\hat{\jmath}$ simple search among K values!

2. Trajectory smoothing via Bézier curves

- Problem: resolve time ambiguity at overlap points and reconstruct continuous trajectory
- **Proposed approach**: join mid-points through Bézier curves with *control points*

$$B(t) = \sum_{i=0}^{n} P_i B_i^n(t) = \sum_{i=0}^{n} P_i \binom{n}{i} t^i (1-t)^{n-i}, \ t \in [0,1] \quad (n-1 \text{ control points})$$

Example: Cubic Bézier curve

$$B(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3, \quad t \in [0,1]$$

(P_1, P_2): control points; (P_0, P_3): fixed points

- Bézier curve sampling: minimum-distance association with ML estimate trajectory points

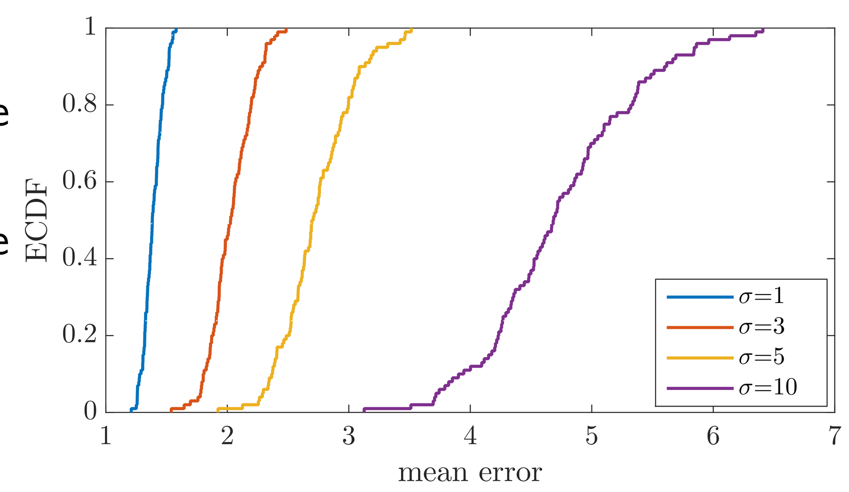
$$(B(t) - P)B'(t) = 0 \qquad (\perp \text{ to tangent line})$$

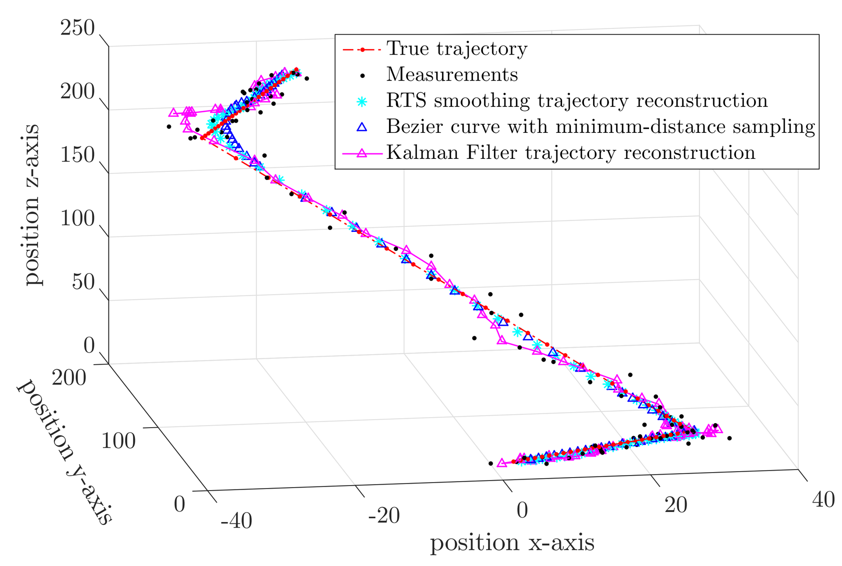
where derivative $B'(t) = n \sum_{i=1}^{n} (P_{i+1} - P_i) B_i^{n-1}$

Performance assessment

Simulation Results

- ✓ accuracy of 1-2 m for small to moderate localization errors
- ✓ good performance even in case of very large position errors ($\sigma = 10$)

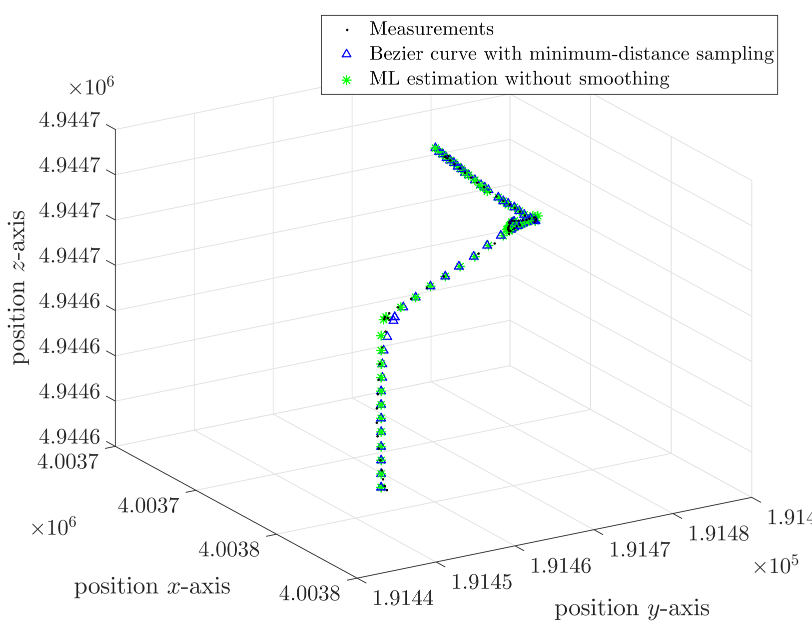




- ✓ proposed Bézier reconstruction outperforms RTS smoother, with less measurements
- ✓ KF less accurate due to limited number of observations
- ✓ accuracy improvement of 21% wrt RTS and 58% wrt KF

Real Experiments with Quadcopters

- ✓ excellent ability to follow quadcopter's complex helical trajectory
- ✓ correct handling of points where drone was hovering (quasi-stationary condition)
- ✓ Bézier smoothing improves RMSE, especially close to aggressive maneuvers



Conclusions and future work

- Proposed approach provides better accuracy than KF, while introducing only a small delay in the online processing
- Proposed approach also outperforms RTS, despite only a fraction of the whole data is used for smoothing in each window
- Possible extension to more complex patterns of moving/stationary conditions in the processing window, possibly under more general observations error model
- Application to AOA-based localization in presence of multipath propagation [6]

References

[1] A. Fascista, G. Ciccarese, A. Coluccia and G. Ricci, "Angle of Arrival-Based Cooperative Positioning for Smart Vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 19, no. 9, pp. 2880-2892, Sept. 2018

[2] A. Coluccia and F. Ricciato, "Maximum Likelihood trajectory estimation of a mobile node from RSS measurements," *9th Annual Conference on Wireless On-Demand Network Systems and Services (WONS)*, pp. 151-158, Jan. 2012

[3] S. Särkkä, *Bayesian Filtering and Smoothing*. Cambridge University Press, 2013

[4] Y. Bar-Shalom, T. Kirubarajan, and X. Li, *Estimation with Applications to Tracking and Navigation*. John Wiley & Sons, 2001

[5] H. E. Rauch, C. T. Striebel, and F. Tung, "Maximum Likelihood estimates of linear dynamic systems," *AIAA Journal*, vol. 3, no. 8, pp. 1445-1450, 1965

[6] A. Fascista, A. Coluccia, and G. Ricci, "Mobile Positioning in Multipath Environments: A Pseudo Maximum Likelihood approach", *arXiv:1808.00857*, 2019

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