Aalto University

Multicarrier radar-communications waveform design for RF-convergence and coexistence

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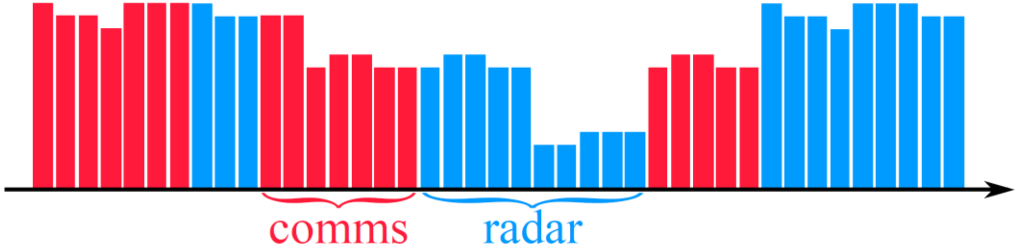
RF Convergence

- **Using the same platform for multiple purposes**
  - Radar and communications tasks
- **Different approaches:**
  - Tasks are performed sequentially (time division)
  - Tasks are performed simultaneously by exploiting different DoFs:
    - *Frequency*
    - *Antenna elements*
    - *Radiation patterns (comms or radar in sidelobes)*
  - Integrated waveforms (one waveform for both tasks, e.g. embedding data symbols into the radar waveform → very common)

Proposed approach

- **Use multicarrier waveforms for which interleaved subcarriers or subsets of subcarriers can be assigned to different tasks**



- **Objectives:**
  - ➢ Devise a strategy to assign subcarriers to either task
  - ➢ Optimize the power use for each task

Performance metric

- **Mutual Information (MI) is chosen as the performance metric for our objectives**
- **Is MI a suitable metric?**
  - ➢ For comms it is directly related to the capacity

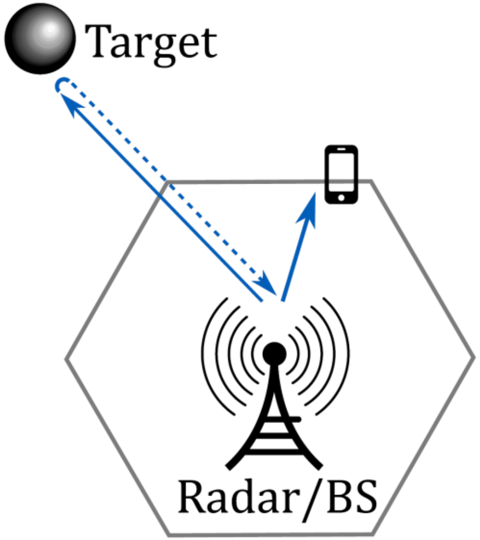
    $$C = \max I(X;Y), \quad X - \text{Channel input}, Y - \text{Channel output}$$

  - ➢ For radar MI maximization has been connected to minimum mean square error (MMSE) and was shown to also provide waveforms with good detection properties

    $$\max I(Y;H), \quad Y - \text{Received signal}, H - \text{Target impulse response}$$

System model

- A dual-use radar-communications OFDM waveform is considered
- Waveform is reflected off the target and received by the communications user
- Waveform can be modeled as follows:



$$\mathbf{x} = \mathbf{F}^H[\mathbf{W}\mathbf{r} + (\mathbf{I} - \mathbf{W})\mathbf{c}],$$

$\mathbf{F}^H$ – IDFT matrix

$\mathbf{W}$ – $N \times N$ diagonal subcarrier selection matrix (0s and 1s)

$\mathbf{r}$ – Radar transmitted symbols

$\mathbf{c}$ – Communications transmitted symbols

Compound objective function

- A compound MI based objective function is formulated for subcarrier assignment and optimum power allocation

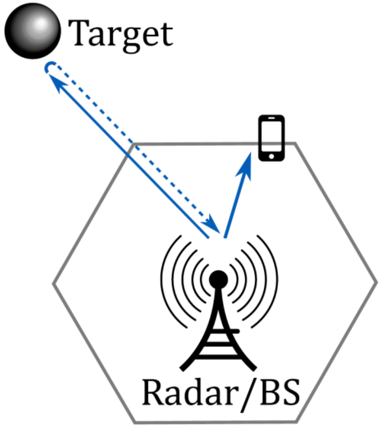
$$\underbrace{I(\mathbf{y}_r;\mathbf{h}_r)}_{\text{radar MI}} + \underbrace{I(\mathbf{y}_c;\mathbf{x})}_{\text{comms MI}},$$

$\mathbf{y}_r$ – Signal at the radar receiver

$\mathbf{h}_r$ – Target impulse response

$\mathbf{y}_c$ – Signal at the communications receiver

$\mathbf{x}$ – Transmitted dual-use waveform



$$\frac{1}{2}\left[\sum_{k=0}^{N-1}\log\left(1+\frac{\mathbf{w}^2[k]|\mathbf{r}[k]|^2\boldsymbol{\sigma}^2_{h_r}[k]}{\mathbf{u}^2[k]|\mathbf{c}[k]|^2\boldsymbol{\sigma}^2_{h_r}[k]+\sigma^2_n}\right)+\sum_{k=0}^{N-1}\log\left(1+\frac{\mathbf{u}^2[k]|\mathbf{c}[k]|^2\boldsymbol{\sigma}^2_{h_c}[k]}{\mathbf{w}^2[k]|\mathbf{r}[k]|^2\boldsymbol{\sigma}^2_{h_c}[k]+\sigma^2_m}\right)\right]$$

Compound objective function

- **For any kth subcarrier only w or u can be non-zero, thus the objective function can be simplified as follows:**

$$\frac{1}{2}\left[\sum_{k=0}^{N-1}\log\left(1+\frac{\mathbf{w}[k]|\mathbf{r}[k]|^2\boldsymbol{\sigma}^2_{h_r}[k]}{\sigma_n^2}\right)+\sum_{k=0}^{N-1}\log\left(1+\frac{\mathbf{u}[k]|\mathbf{c}[k]|^2\boldsymbol{\sigma}^2_{h_c}[k]}{\sigma_m^2}\right)\right],$$

$\mathbf{w}[k]$ — Radar weight on kth subcarrier - $\{0,1\}$

$\mathbf{u}[k]$ — Comms weight on kth subcarrier - $\{0,1\}$

$\boldsymbol{\sigma}^2_{h_r}[k]$ — Radar channel gain on kth subcarrier

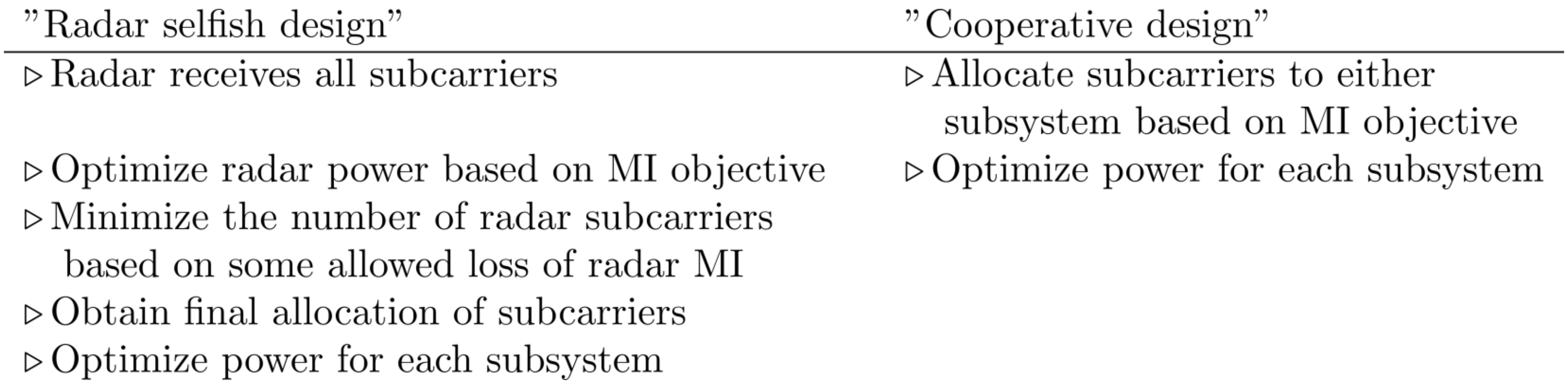
$\boldsymbol{\sigma}^2_{h_c}[k]$ — Comms channel gain on kth subcarrier

σ_n^2 — Noise power @ radar receiver

σ_m^2 — Noise power @ comms receiver

Proposed design algorithms

- **Two design algorithms are proposed:**
  - “Radar selfish design”
  - “Cooperative design”
- **A brief description of the two algorithms:**



”Radar selfish design”	”Cooperative design”
▷ Radar receives all subcarriers	▷ Allocate subcarriers to either subsystem based on MI objective
▷ Optimize radar power based on MI objective	▷ Optimize power for each subsystem
▷ Minimize the number of radar subcarriers based on some allowed loss of radar MI	
▷ Obtain final allocation of subcarriers	
▷ Optimize power for each subsystem	

Radar selfish design

- **Radar power allocation optimization is formulated as:**

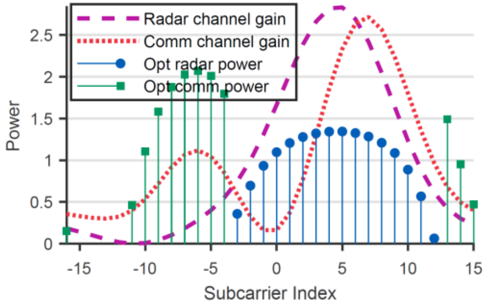
$$\begin{aligned} \underset{\{|\mathbf{r}[k]|^2\}}{\text{maximize}} \quad & \sum_{k=0}^{N-1} \log\left(1 + \frac{\mathbf{w}[k]|\mathbf{r}[k]|^2 \boldsymbol{\sigma}^2_{h_r}[k]}{\sigma_n^2}\right) && \leftarrow \text{Radar MI} \\ \text{subject to} \quad & \sum_{k=0}^{N-1} \mathbf{w}[k]|\mathbf{r}[k]|^2 \leq P_{\mathrm{T}} && \leftarrow \text{Total radar power budget} \end{aligned}$$

- **Comms power allocation optimization is formulated as:**

$$\begin{aligned} \underset{\{|\mathbf{c}[k]|^2\}}{\text{maximize}} \quad & \sum_{k=0}^{N-1} \log\left(1 + \frac{\mathbf{u}[k]|\mathbf{c}[k]|^2 \boldsymbol{\sigma}^2_{h_c}[k]}{\sigma_m^2}\right) && \leftarrow \text{Comms MI} \\ \text{subject to} \quad & \sum_{k=0}^{N-1} \mathbf{u}[k]|\mathbf{c}[k]|^2 \leq P_{\mathrm{T}} && \leftarrow \text{Total comms power budget} \end{aligned}$$

Radar selfish design

- Both optimization problems can be solved exactly and their solutions are water filling solutions
- Subcarriers with higher channel gain and low noise and interference power receive more power
- Example of power allocation for both subsystems before minimizing the number of radar subcarriers



Radar selfish design

- **Minimizing the number of radar subcarriers is done so that comms can receive more subcarriers**
- **This problem is non-convex (l_0-norm minimization)**
- **The best convex approximation is used (l_1-norm minimization)**

Radar MI

From previous step

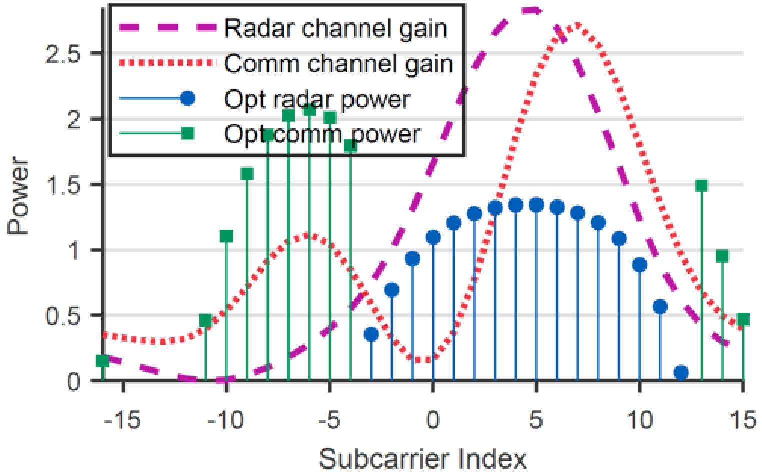
$$\begin{aligned} \underset{\{\mathbf{w}[k]\}}{\text{minimize}} \quad & \sum_{k=0}^{N-1} \mathbf{w}[k] \\ \text{subject to} \quad & \boxed{\sum_{k=0}^{N-1} \log\left(1 + \frac{\mathbf{w}[k]\boxed{|\mathbf{r}[k]|^2}\boldsymbol{\sigma}^2_{h_r}[k]}{\sigma_n^2}\right)} \geq t \\ & 0 \leq \mathbf{w}[k] \leq 1, \forall k = 0 \ldots N-1 \end{aligned}$$

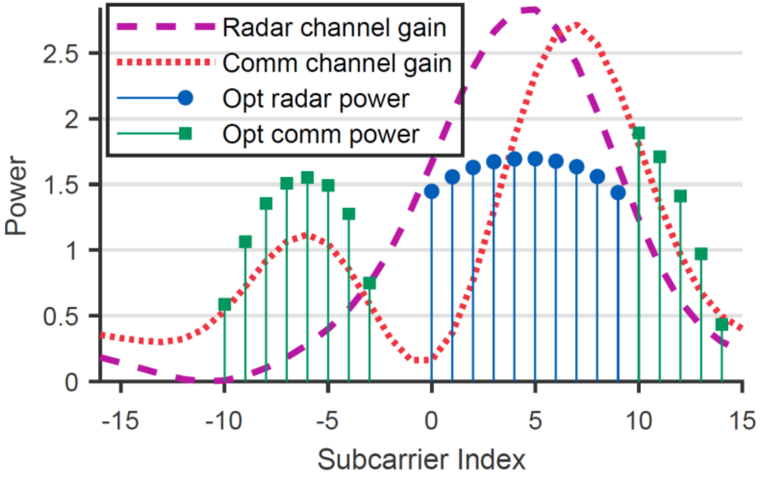
t is chosen $5-25\%$ smaller than the initial maximum MI

- **A rounding to 0 or 1 is used to obtain the actual w**
- **Iterate until no change in w**

Radar selfish design

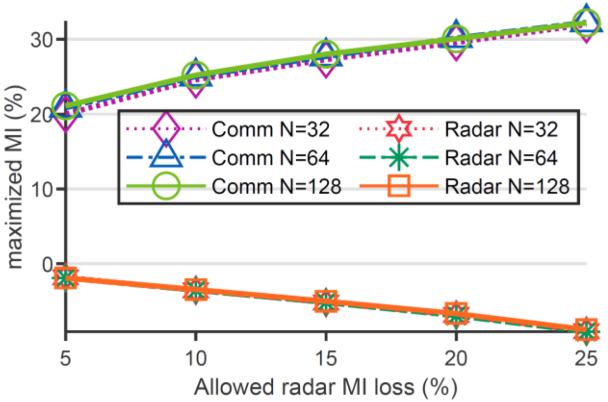
- **An example of subcarrier allocation and power optimization for both subsystems at the initial and final steps**





Radar selfish design

- **This design involves a trade-off between MI loss for radar and MI gain for the communications subsystems**
- **The average MI change from first to last step of the algorithm for different number of subcarriers (500 channel realizations)**



shows that *it pays off to allow a small decrease in radar maximized MI for a larger comms maximized MI → higher capacity*

Cooperative design

- Subcarriers are assigned to the radar or the comms subsystem based on maximizing the compound objective
- The objective can be further simplified to:

$$\frac{1}{2}\left[\sum_{k=0}^{N-1}\log\left(1+\frac{\mathbf{w}[k]\boldsymbol{\sigma}^2_{h_r}[k]}{\sigma^2_n}+\frac{\mathbf{u}[k]\boldsymbol{\sigma}^2_{h_c}[k]}{\sigma^2_m}\right)\right]$$

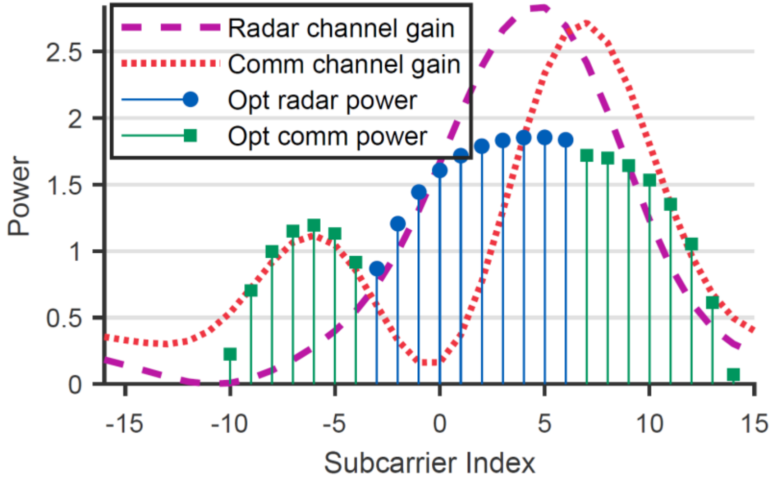
- It turns out the optimum w and u are given by:

$$\begin{cases} \text{If } \dfrac{\boldsymbol{\sigma}^2_{h_c}[k]}{\sigma^2_m} > \dfrac{\boldsymbol{\sigma}^2_{h_r}[k]}{\sigma^2_n} & \text{then } \mathbf{w}[k]=0, \mathbf{u}[k]=1 \\ \text{If } \dfrac{\boldsymbol{\sigma}^2_{h_c}[k]}{\sigma^2_m} \leq \dfrac{\boldsymbol{\sigma}^2_{h_r}[k]}{\sigma^2_n} & \text{then } \mathbf{w}[k]=1, \mathbf{u}[k]=0 \end{cases}$$

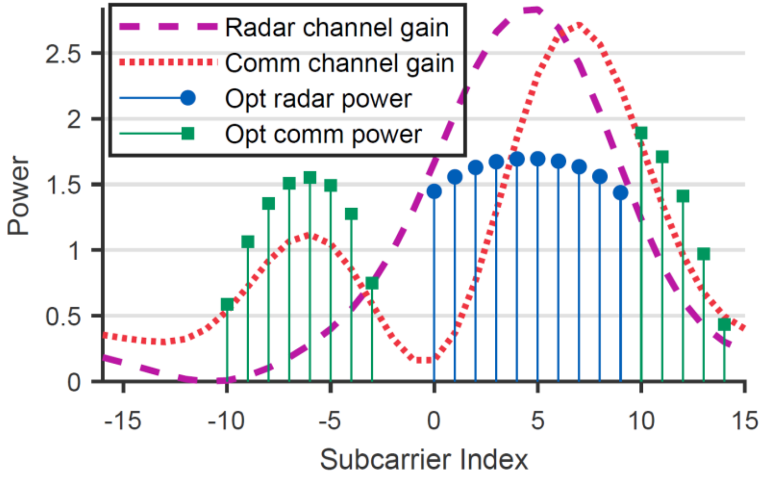
- Subcarriers go to the subsystems that experience larger “channel gain to noise” ratio

Cooperative design

- **Example of final power allocation for both subsystems, which is different than for the first design algorithm**



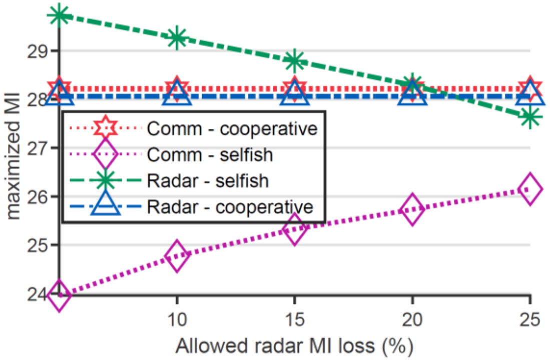
Cooperative design



Radar selfish design

Radar selfish vs cooperative design

- **Comparing the maximized MI achieved using both strategies**
  - ➢ Cooperative design is favorable to the comms subsystem as expected
  - ➢ Radar selfish design is favorable to the radar subsystem as long as there is not too much MI loss allowed



Future work

- **Plenty of extensions can be considered in the future:**
  - Generalizing the objective function with different ratios for the MI terms
  - Considering different objective functions for the radar (CRB, P_D)
  - A more concrete analysis for the performance of each subsystem
  - Addressing the PAPR issue of multicarrier waveforms
  - And others …

Thank you