Multiresolution Time-of-Arrival Estimation from Multiband Radio Channel Measurements

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Introduction

- In radio communications, the time-of-arrival (TOA) estimation starts with the estimation of the underlying non bandlimited multipath channel from bandlimited observations of communication signals.
- Modeling the channel impulse response (CIR) as a sparse sequence of Diracs pulses, the TOA estimation becomes a problem of parametric spectral inference from observed bandlimited signals.
- To increase resolution without arriving at unrealistic sampling rates, we consider a multiband sampling approach, and propose: (i) **a practical multibranch receiver for the acquisition** and (ii) **an algorithm for multiresolution TOA estimation** based on the ESPRIT algorithm.

Multiband Sampling of the Radio Channel

- **The multipath radio channel model** assuming K propagation paths is

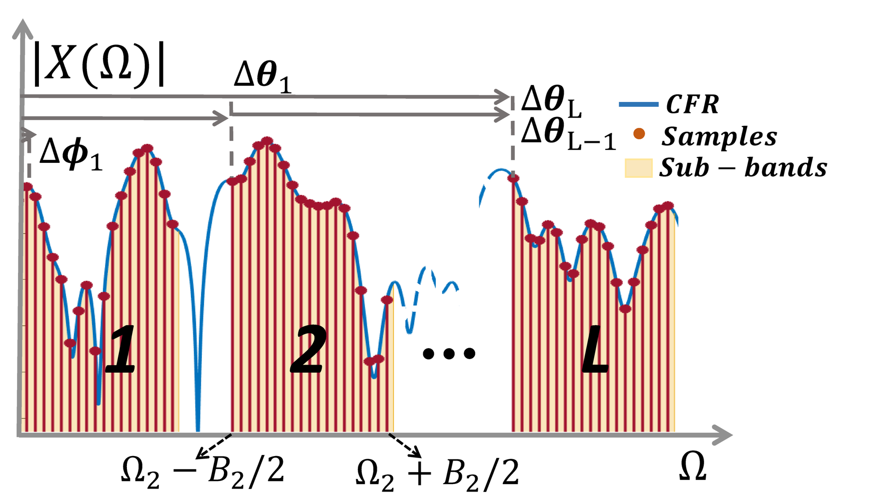
$$\widetilde{h}(t) = \sum_{k=1}^{K} \widetilde{\alpha}_k \delta(t - \tau_k) \overset{\text{CTFT}}{\longleftrightarrow} \widetilde{H}(\Omega) = \sum_{k=1}^{K} \widetilde{\alpha}_k e^{-j\Omega\tau_k} \quad (1)$$

where $\widetilde{\alpha}_k \in \mathbb{R}$ and $\tau_k \in \mathbb{R}_+$ represent the gain and time-delay of the kth resolvable path.

- **Multiband sampling** assumes probing of the channel (1) by a wide-band sensing signal $\widetilde{s}(t)$ defined by CTFT

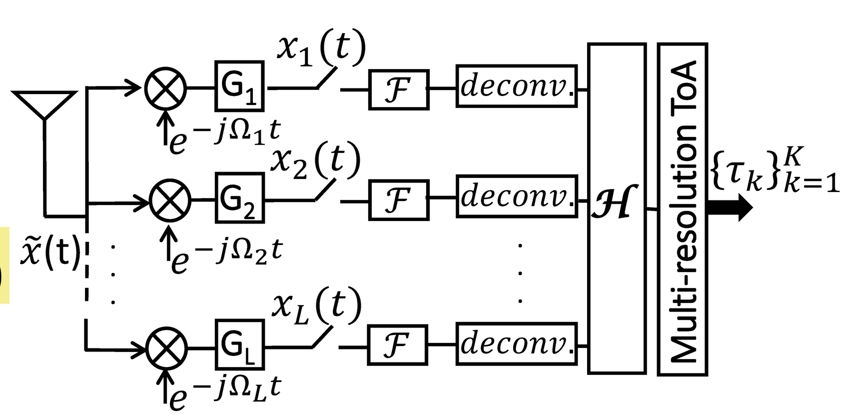
$$\widetilde{S}(\Omega) = \widetilde{S}_i(\Omega), \quad \Omega \in \mathcal{W}_i, i \in [1, L]$$

  - $\mathcal{W}_i = [\Omega_i - B_i/2, \Omega_i + B_i/2]$,
  - Ω_i is the center frequency,
  - B_i is the bandwidth of the ith sub-band.



- **The multibranch receiver** down-converts the received signal to baseband and performs lowpass filtering and sampling of

$$X_i(\Omega) = G_i(\Omega)H_i(\Omega)S_i(\Omega) + N_i(\Omega)$$



  - $G_i(\Omega)$ is the frequency response of the ith receiver chain,
  - $H_i(\Omega)$ and $S_i(\Omega)$ are the baseband equivalents of $\widetilde{H}(\Omega + \Omega_i)$ and $\widetilde{S}_i(\Omega)$,
  - $N_i(\Omega)$ is the bandlimited white Gaussian noise.

- **Discrete-time data model:** Assuming that $x_i(t)$ has a finite duration T, and it satisfies conditions for Nyquist sampling, the data model in the frequency domain is

$$\boldsymbol{x}_i = \boldsymbol{h}_i \odot \boldsymbol{g}_i \odot \boldsymbol{s}_i + \boldsymbol{n}_i \quad (2)$$

where $\boldsymbol{x}_i$, $\boldsymbol{g}_i$, $\boldsymbol{h}_i$, $\boldsymbol{s}_i$, and $\boldsymbol{n}_i$ are collecting N samples of $X_i[n]$, $G_i[n]$, $H_i[n]$, $S_i[n]$ and $N_i[n]$, respectively. The elements of $\boldsymbol{h}_i$ are

$$H_i[n] = \widetilde{H}(n\Omega_t + \Omega_i) = \sum_{k=1}^{K} \widetilde{\alpha}_k e^{-j\Omega_i\tau_k} e^{-jn\Omega_t\tau_k}, \quad n = 0, \cdots, N-1$$

where $\Omega_t = \frac{1}{N}\Omega_s = \frac{2\pi}{T}$.

- After deconvolution, the channel model can be written as

$$\boldsymbol{h}_i = \boldsymbol{M}\boldsymbol{\Theta}_i\boldsymbol{\alpha} + \boldsymbol{n}'_i, \quad i = 1, \cdots, L \quad (3)$$

where

$$\boldsymbol{M} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \Phi_1 & \Phi_2 & \cdots & \Phi_K \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_1^{N-1} & \Phi_2^{N-1} & \cdots & \Phi_K^{N-1} \end{bmatrix}, \quad \boldsymbol{\Theta}_i = \begin{bmatrix} \Theta_{i,1} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \Theta_{i,K} \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \widetilde{\alpha}_1 \\ \vdots \\ \widetilde{\alpha}_K \end{bmatrix},$$

$\Phi_k = e^{-j\phi_k}$, $\phi_k = \Omega_t\tau_k$, $\Theta_{i,k} = e^{-j\theta_{i,k}}$, $\theta_{i,k} = \Omega_i\tau_k$, and $\boldsymbol{n}'_i$ represents zero mean additive white Gaussian noise.

Multiresolution TOA Estimation

- The Hankel matrices of size $P \times Q$ are constructed from single $\boldsymbol{h}_i$ as

$$\boldsymbol{\mathcal{H}}_i = \begin{bmatrix} H_i[0] & H_i[1] & \cdots & H_i[Q] \\ H_i[1] & H_i[2] & \cdots & H_i[Q+1] \\ \vdots & \vdots & \ddots & \vdots \\ H_i[P-1] & H_i[P] & \cdots & H_i[N-1] \end{bmatrix} \quad (4)$$

where $P = N - Q - 1$, Q is a design parameter and we require $P > K$ and $Q \geq K$.

- To achieve high resolution estimation of the $\{\tau_k\}_{k=1}^{K}$ we form a block Hankel matrix

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} \boldsymbol{H}_1 \\ \boldsymbol{H}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}' \\ \boldsymbol{M}'\boldsymbol{\Theta} \end{bmatrix} \boldsymbol{\Theta}_1 \boldsymbol{A} + \boldsymbol{N} \quad (5)$$

where $\boldsymbol{M}'$ is submatrix of $\boldsymbol{M}$, $\boldsymbol{\Theta} = \boldsymbol{\Theta}_L\boldsymbol{\Theta}_1^{-1}$, $\boldsymbol{A} = [\boldsymbol{\alpha}, \boldsymbol{\Phi\alpha}, \boldsymbol{\Phi}^2\boldsymbol{\alpha}, \cdots, \boldsymbol{\Phi}^{Q-1}\boldsymbol{\alpha}]$ and $\boldsymbol{\Phi} = diag(\Phi_1, \Phi_2, \ldots \Phi_K)$.

- Exploiting the **double shift invariance structure** of $\boldsymbol{\mathcal{H}}$ similar as in the case of MI-ESPRIT: unambiguous estimates of $\{\tau_k\}_{k=1}^{K}$ are obtained from $\boldsymbol{\Phi}$, while high resolution but ambiguous estimates are obtained from $\boldsymbol{\Theta}$.
- $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$ are estimated by finding Least Squares (LS) approximate solutions to

$$\boldsymbol{U}_{\Phi 1} = \boldsymbol{J}_{\Phi 1}^{(1)}\boldsymbol{U} = \begin{bmatrix} \boldsymbol{M}'' \\ \boldsymbol{M}''\boldsymbol{\Theta} \end{bmatrix} \boldsymbol{\Theta}_1\boldsymbol{T}^{-1}, \qquad \boldsymbol{U}_{\Theta 1} = \boldsymbol{J}_{\Theta 1}\boldsymbol{U} = \boldsymbol{M}'\boldsymbol{T}^{-1},$$
$$\boldsymbol{U}_{\Phi 2} = \boldsymbol{J}_{\Phi 2}^{(1)}\boldsymbol{U} = \begin{bmatrix} \boldsymbol{M}'' \\ \boldsymbol{M}''\boldsymbol{\Theta} \end{bmatrix} \boldsymbol{\Phi\Theta}_1\boldsymbol{T}^{-1}, \qquad \boldsymbol{U}_{\Theta 2} = \boldsymbol{J}_{\Theta 2}\boldsymbol{U} = \boldsymbol{M}'\boldsymbol{\Theta}\boldsymbol{T}^{-1}, \quad (6)$$

where $\boldsymbol{U}$ is the K dimensional orthonormal basis of the column span of $\boldsymbol{\mathcal{H}}$, $\boldsymbol{M}''$ is a submatrix of $\boldsymbol{M}'$, and the selection matrices are

$$\boldsymbol{J}_{\Phi 1}^{(r)} = \boldsymbol{I}_2 \otimes [\boldsymbol{I}_{P-r} \quad \boldsymbol{0}_{P-r,r}], \qquad \boldsymbol{J}_{\Theta 1} = [1 \quad 0] \otimes \boldsymbol{I}_P,$$
$$\boldsymbol{J}_{\Phi 2}^{(r)} = \boldsymbol{I}_2 \otimes [\boldsymbol{0}_{P-r,r} \quad \boldsymbol{I}_{P-r}], \qquad \boldsymbol{J}_{\Theta 2} = [0 \quad 1] \otimes \boldsymbol{I}_P.$$

- The LS solutions of (6) satisfy

$$\boldsymbol{\Psi} := \boldsymbol{U}_{\Phi 1}^{\dagger}\boldsymbol{U}_{\Phi 2} = \boldsymbol{T\Phi T}^{-1},$$
$$\boldsymbol{\Upsilon} := \boldsymbol{U}_{\Theta 1}^{\dagger}\boldsymbol{U}_{\Theta 2} = \boldsymbol{T\Theta T}^{-1}.$$

- To pair estimates of $\{\tau_k\}_{k=1}^{K}$ from $\boldsymbol{\Phi}$ and $\boldsymbol{\Theta}$, we find $\boldsymbol{T}$ that is jointly diagonalizing $\boldsymbol{\Psi}$ and $\boldsymbol{\Upsilon}$ and estimate

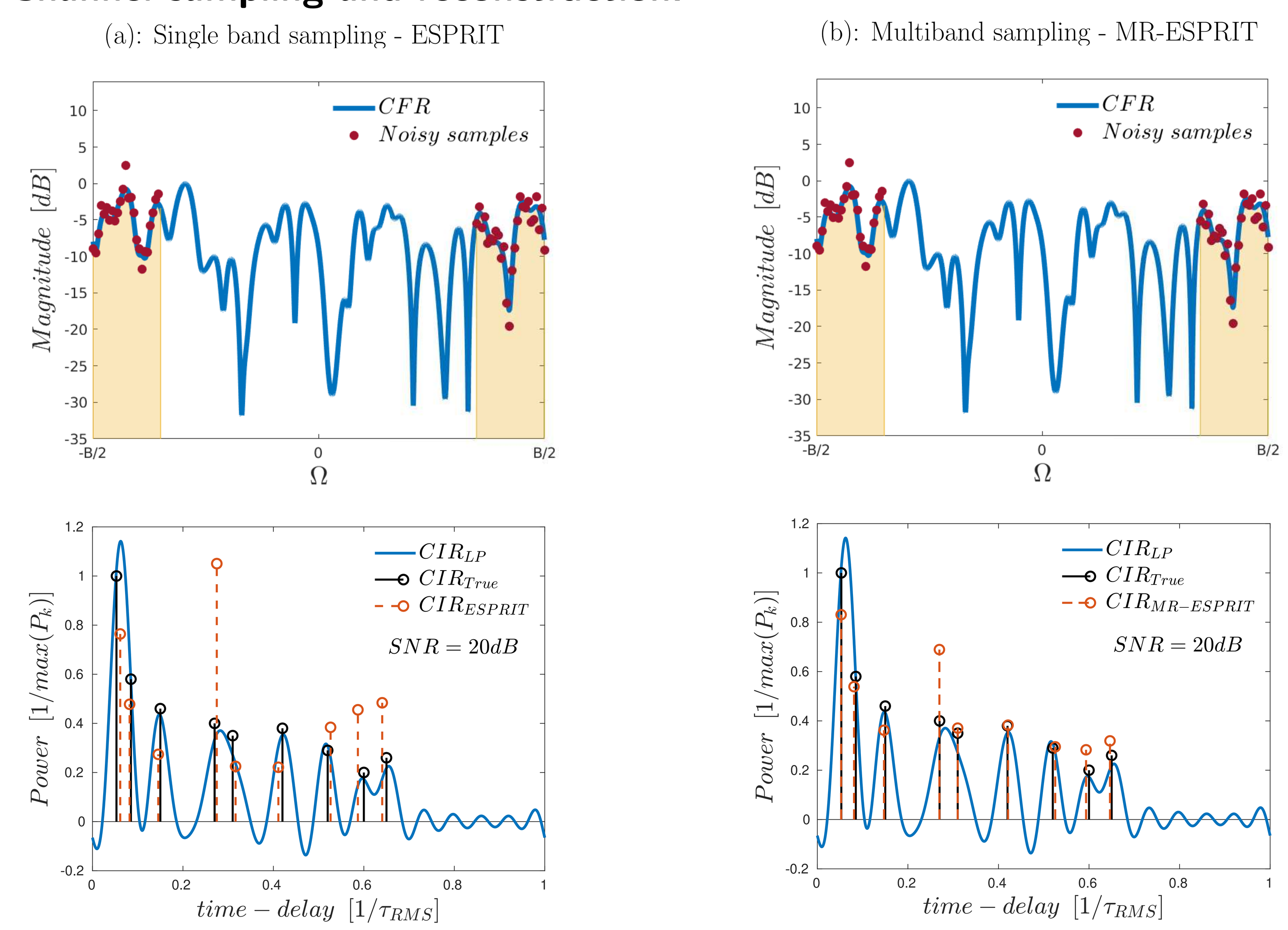
$$\tau_k = \Omega_t^{-1}\phi_k = (\Omega_2 - \Omega_1)^{-1}(\theta_k + 2\pi n_k). \quad (7)$$

The final estimate of τ_k is obtained based on θ_k, by finding the best fitting integer satisfying (7), which is $n_k = \text{round}\left\{1/2\pi\left(\Omega_t^{-1}(\Omega_2 - \Omega_1)\phi_k - \theta_k\right)\right\}$.

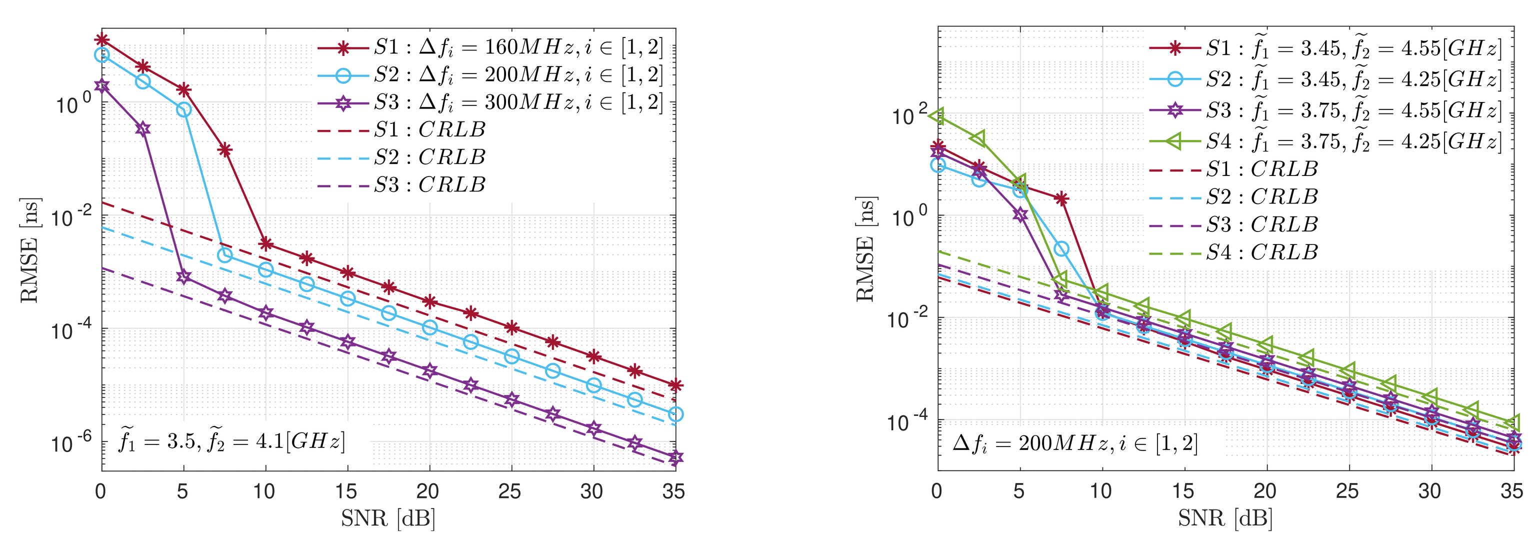
Numerical evaluation

- A standard outdoor UWB channel model with eight dominant multipath components (MPCs) is considered with $\tau_{RMS} = 40ns$.
- The continuous time is modeled using a 3 GHz grid, where the channel tap delays are spaced at 333.33 ps.
- The Root Mean Square Error (RMSE) is used as a metric for evaluation, which is obtained over 10^4 independent Monte Carlo runs.

Channel sampling and reconstruction:



Performance of the algorithm:



Conclusions

- The block Hankel matrix formed from multiband radio channel samples has a multiple shift invariance structure.
- The invariance structure of a single sub-band provides coarse parameter estimates, while the invariance structure of the lowest against the highest frequency sub-band provides high-resolution, but phase wrapped estimates.
- The multiresolution TOA estimation from multiband channel samples increases the resolution of TOA estimates while it reduces the spectral occupancy and sampling costs.

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