A History-based Stopping Criterion in Recursive Bayesian State Estimation

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Motivation

- Recursive Bayesian state estimation (RBSE) procedure is not always precise and fast due to noisy observations.
- Set a high confidence threshold is a conventional solution, but it is not efficient in terms of budget/time.
- A stopping criterion is proposed based on the posterior distribution defined over the state and a history-based objective which predict the

METHOD

Using the MAP estimation, system picks an estimate with highest posterior probability value conditioned on the evidence ε, which is conventionally constrained with a confidence level τ and is expressed as the following:

$$\hat{\sigma} = \arg\max_{\sigma \in \mathcal{A}} p(\sigma|\mathcal{H}_s)$$
$$\text{s.t. } p(\sigma|\mathcal{H}_s) \geq \tau$$

where σ denotes the target state, $\mathcal{H}_s = \{\varepsilon_{1:s}, \Phi_{1:s}, \mathcal{H}_0\}$ denotes the task history, and Φ denotes the query subset.

Assuming all evidence comes from two separate unimodal probability distributions and noisy observations are i.i.d., we can rewrite the posterior as,

$$p(\sigma|\boldsymbol{\varepsilon}_{1:S}, \Phi_{1:S}, \mathcal{H}_0) = p(\sigma|\mathcal{H}_0) \prod_{s=1}^{S} \frac{p(\boldsymbol{\varepsilon}_s|\sigma, \Phi_s)}{p(\boldsymbol{\varepsilon}_s|\Phi_s)}$$

A new term called ***Momentum*** is proposed as follows;

$$m_s(a) = p(a|\mathcal{H}_{s-1}) \left(\log p(a|\mathcal{H}_s) - \log p(a|\mathcal{H}_{s-1})\right)$$

and the average momentum over sequences is calculated as:

$$M_S(a) = \frac{1}{S} \sum_{s=1}^{S} m_s(a)$$

The proposed stopping criterion for MAP estimation can be expressed as:

$$\hat{\sigma} = \arg\max_{\sigma \in \mathcal{A}} p(\sigma|\mathcal{H}_s)$$
$$\text{s.t. } p(\sigma|\mathcal{H}_s) + \alpha M_s(\sigma) \geq \tau$$

where α is a hyperparameter.

Proposition 1. *Given $a, b \in \mathcal{D}$ where $a \neq b$ and $\alpha \geq 0$, if $\exists\, \mathcal{H}_{s-1}$ s.t. $p(a|\mathcal{H}_{s-1}) < p(b|\mathcal{H}_{s-1})$, then;*

$$p\left(p(a|\mathcal{H}_s) + \alpha M_s(a) > p(b|\mathcal{H}_s) + \alpha M_s(b)\right) \geq p\left(p(a|\mathcal{H}_s) > p(b|\mathcal{H}_s)\right)$$

EXPERIMENTS & RESULTS

A language-model-assisted EEG based Brain-computer Interface (BCI) typing system called RSVP Keyboard has been used as a testbed.

We have observed statistically significant changes in the average number of sequences with $p < 0.002$ using Wilcoxon signed-rank test.

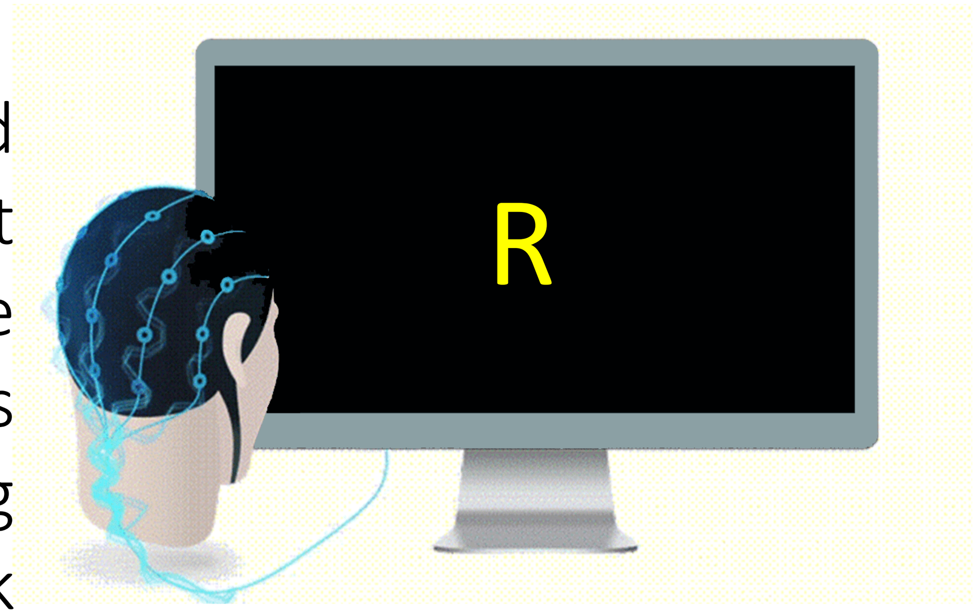

Figure 1. Schematic of RSVP Keyboard System.

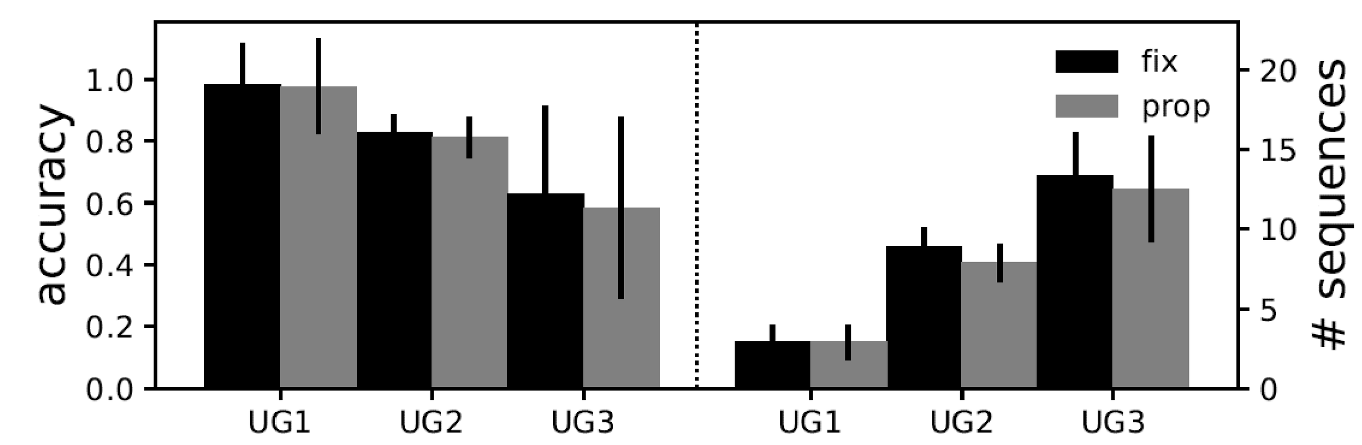

Figure 2. The average accuracy and average number of sequences spent for estimation. UG1, UG2, UG3 represent user groups with AUC belongs to [%90; %100], [%78; %90], and [%50; %78] interval, respectively. It shows that number of sequences increases and accuracy decreases with respect to the user performance decrements.

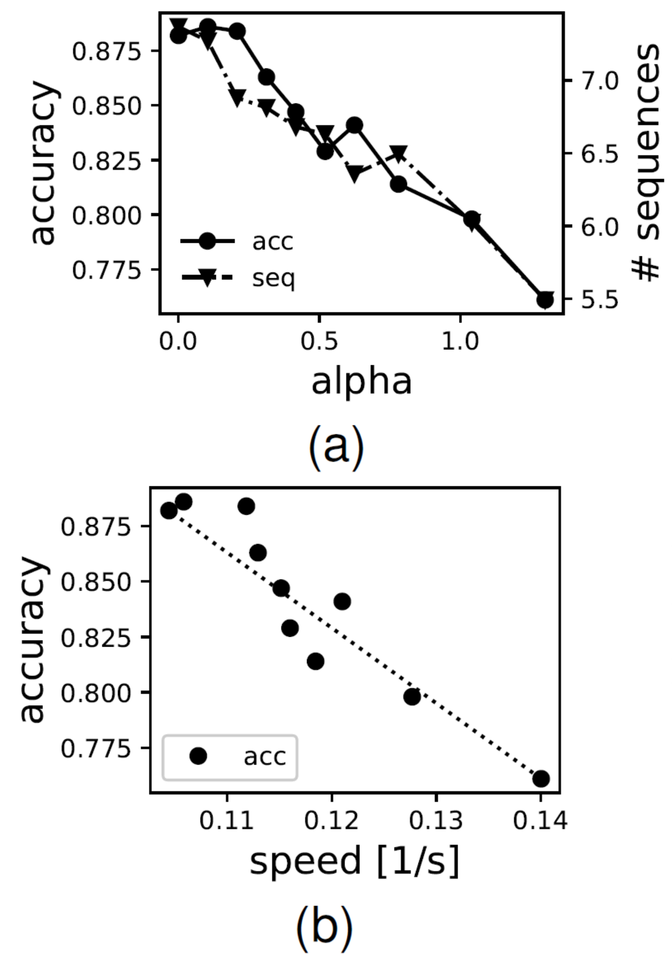

Figure 3. Average results for a user with AUC= %82:5. Speed is determined by 1 over number sequences, where each sequence takes 1.3s in our application. Given the figures it is desired to pick the point which has the largest distance to the line and on the right hand side to achieve better accuracy.

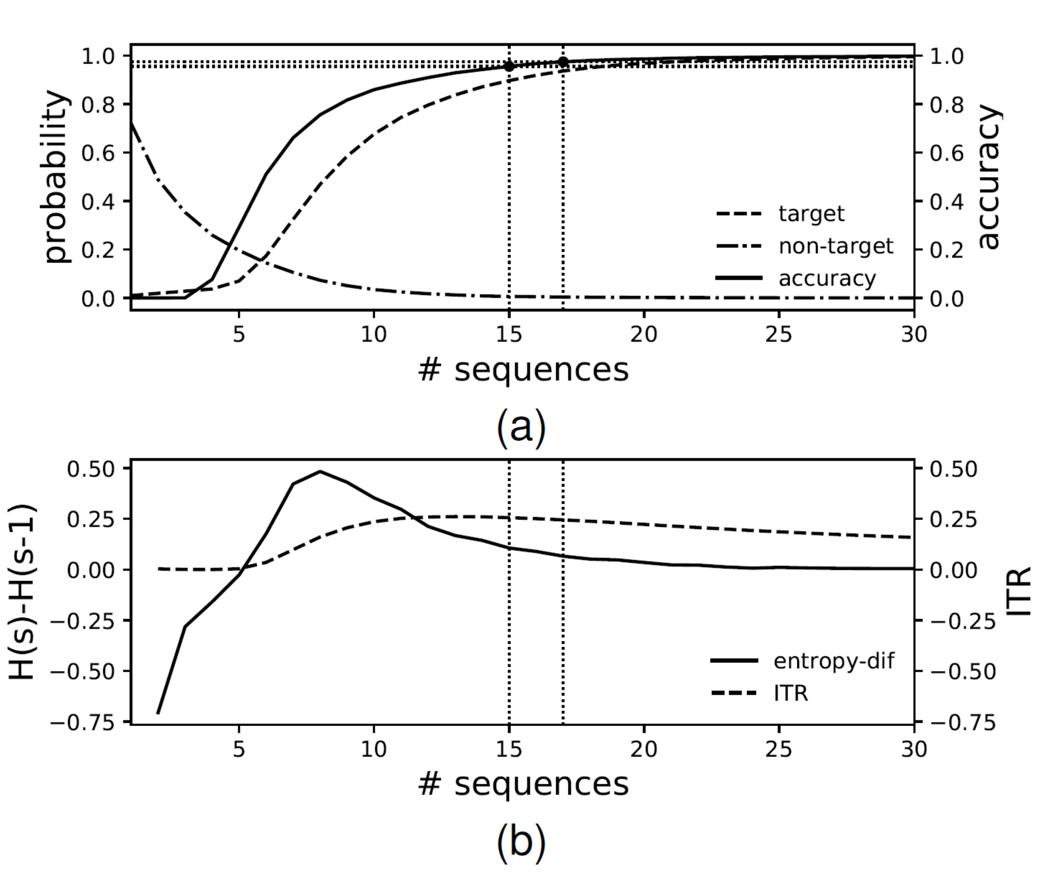

Figure 4. (a) Probability of the state (target letter) and a competing candidate which has the highest prior probability (non-target) over all possible 28 choices in the dictionary. Accuracy is computed over 1000 Monte-Carlo simulations. The system saves 2 sequences while sacrificing %1.1 average accuracy. (b) Information transfer rate (ITR) and entropy changes during RBSE. When posterior probability is saturated, the changes (in terms of entropy and ITR) are negligible.

CONCLUSION

- A stopping objective is proposed as a linear weighted combination of the posterior and the average changes of log of the posterior displacement, i.e. *Momentum*.
- Using the proposed objective, we gain better accuracy/speed ratio.
- We analytically showed that the RBSE framework under the proposed stopping criterion gives us higher probability than the conventional fix method.

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