BEAMFORMER DESIGN UNDER TIME-CORRELATED INTERFERENCE AND ONLINE IMPLEMENTATION

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Outline

Background 1: EEG Inverse Problem

- EEG inverse problem

  Aim: Localize and reconstruct the brain activities with non-invasive measurements of induced electric potential outside of the skull.

  Difficulty: The activities of the sources are mutually correlated.

Background 2: Beamforming

The linearly constrained minimum variance (LCMV) beamformer is dominantly used.

- Minimum variance distortionless response (MVDR) beamformer [Van Veen 1997]
  Achieving the highest SINR among all linear beamformers when the brain activities are mutually uncorrelated.
  $\Longrightarrow$ non-optimal in the presence of the interfering signals correlated with the desired one.
- Nulling beamformer [S. S. Dalal 2006, H. B. Hui 2006]
  Cancelling the interfering activities, but amplificating the additive noise.

	correlated signals	noise
MVDR	×	○
Nulling	○	×
Proposed	○	○

Contributions

- Decompose the mean squared error (MSE) for the correlated case.
- Propose relaxed zero forcing (RZF) beamformer for solving EEG inverse problem in the presence of time correlated sources' activities.
  $\Longrightarrow$ Introduce a quadratic constraint that suppresses the effect of the correlation.
  $\longrightarrow$ Alleviate the tradeoffs between MVDR and nulling beamformers.
- Present an efficient online implementation of RZF based on dual-domain projections.
- Show the superior performance of the proposed beamformer by numerical experiments.

EEG Forward Model

EEG measurements at time instant k using n EEG sensors modeled as:

EEG forward model

$$\boldsymbol{y}(k) = \sum_{i=1}^{s} \underbrace{\boldsymbol{h}(\boldsymbol{\theta}_i) q_i(k)}_{\text{signal from } i\text{th source}} + \underbrace{\boldsymbol{n}(k)}_{\text{noise}} \in \mathbb{R}^n \quad (1)$$

- $q_i \in \mathbb{R}$: activity of ith source
- $\boldsymbol{h}(\boldsymbol{\theta}_i) \in \mathbb{R}^n$: leadfield vector
- $\boldsymbol{\theta}_i$: parameter for the position and orientation of the ith source

Assumptions

1. The positions and orientations of sources are known and fixed.
   $\Longrightarrow \boldsymbol{h}(\theta_i)$s are known.
2. $q_i(k)$s are mutually correlated but uncorrelated with the noise.

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- $q_1(k)$ is the activity of the desired source.
  $\Longrightarrow q_i(k)$ for $i = 2, 3, \cdots, s$ are the interfering activities.
- The fidelity of reconstruction is measured by the mean squared error (MSE).

Outline

MSE and Output Variance

$$\begin{aligned} J_{\mathrm{MSE}}(\boldsymbol{w}) &:= E[(\underbrace{\boldsymbol{w}^\top \boldsymbol{y}(k)}_{:=\hat{q}_1(k)} - q_1(k))^2] \\ &= \underbrace{E[(\boldsymbol{w}^\top \boldsymbol{y}(k))^2]}_{\text{output variance}} + \underbrace{E[q_1^2(k)]}_{\text{signal power}} - 2E[q_1^2(k)]\boldsymbol{w}^\top \boldsymbol{h}(\boldsymbol{\theta}_1) \\ &\quad \underbrace{-2\sum_{i=2}^{s} E[q_1(k)q_i(k)]\ \boldsymbol{w}^\top \boldsymbol{h}(\boldsymbol{\theta}_i)}_{\text{cross talk}} \end{aligned} \qquad (2)$$

- Uncorrelated case ($E[q_1(k)q_i(k)] = 0$)
  $\longrightarrow$ MVDR $\Longleftrightarrow$ minimum MSE (MMSE)
- Correlated case ($E[q_1(k)q_i(k)] \neq 0$)
  $\longrightarrow$ MVDR is significantly different from MMSE.

MSE and Output Variance

$$
\begin{aligned}
J_{\mathrm{MSE}}(\boldsymbol{w}) \;&:=\; E\Big[\Big(\underbrace{\boldsymbol{w}^{\mathsf{T}}\boldsymbol{y}(k)}_{:=\hat{q}_1(k)} - q_1(k)\Big)^2\Big] \\
&=\; \underbrace{E[(\boldsymbol{w}^{\mathsf{T}}\boldsymbol{y}(k))^2]}_{\text{output variance}} + \underbrace{E[q_1^2(k)]}_{\text{signal power}} - 2E[q_1^2(k)]\boldsymbol{w}^{\mathsf{T}}\boldsymbol{h}(\boldsymbol{\theta}_1) \\
&\quad \underbrace{-2\sum_{i=2}^{s} E[q_1(k)q_i(k)]\;\boldsymbol{w}^{\mathsf{T}}\boldsymbol{h}(\boldsymbol{\theta}_i)}_{\text{cross talk}} \qquad (2)
\end{aligned}
$$

Key idea: We introduce an additional constraint that suppresses the effect of the cross talk.

Relaxed Zero Forcing (RZF) Beamformer

$$\text{cross talk} = -2 \sum_{i=2}^{s} \underbrace{E[q_1(k) q_i(k)]}_{\text{unavailable}} \; \boldsymbol{w}^{\mathsf{T}} \boldsymbol{h}(\boldsymbol{\theta}_i)$$

Optimization problem

$$\begin{cases} \text{minimize} \;\; \mathbb{E}\left[\left(\boldsymbol{w}^T \boldsymbol{y}(k)\right)^2\right] & (3) \\ \text{subject to} \begin{cases} \boldsymbol{w}^T \boldsymbol{h}(\boldsymbol{\theta}_1) = 1 \\ \|\boldsymbol{H}_I^T \boldsymbol{w}\|^2 \leq \epsilon \quad (\epsilon \geq 0) \end{cases} & (4) \end{cases}$$

$$\boldsymbol{H}_I := [\boldsymbol{h}(\boldsymbol{\theta}_2), \; \boldsymbol{h}(\boldsymbol{\theta}_3), \; \cdots, \; \boldsymbol{h}(\boldsymbol{\theta}_s)]$$

Analytical solution: $\boldsymbol{w}_{\mathrm{RZF}} = \dfrac{\boldsymbol{R}_\epsilon^{-1} \boldsymbol{h}(\boldsymbol{\theta}_1)}{\boldsymbol{h}(\boldsymbol{\theta}_1)^T \boldsymbol{R}_\epsilon^{-1} \boldsymbol{h}(\boldsymbol{\theta}_1)}$,

where $\boldsymbol{R}_\epsilon := E[\boldsymbol{y}(k)\boldsymbol{y}(k)^T] + \tau_\epsilon \boldsymbol{H}_I \boldsymbol{H}_I^T \quad (\tau_\epsilon > 0)$.

Relaxed Zero Forcing (RZF) Beamformer

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where $\boldsymbol{R}_\epsilon := E[\boldsymbol{y}(k)\boldsymbol{y}(k)^T] + \tau_\epsilon \boldsymbol{H}_I \boldsymbol{H}_I^T \quad (\tau_\epsilon > 0).$

Dual-Domain Adaptive Algorithm (1/2)

An algorithm for implementing the RZF beamformer [Yukawa, Sung, Lee, TSP 2013].

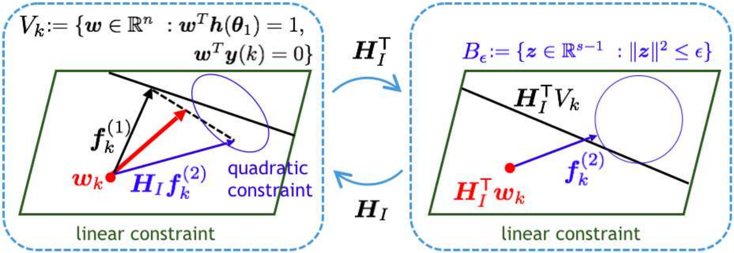
Algorithm

$$\boldsymbol{w}_{k+1} := \boldsymbol{w}_k + \lambda_k \mu_k \left(\alpha_k \boldsymbol{f}_k^{(1)} + (1-\alpha_k) \boldsymbol{H}_I \boldsymbol{f}_k^{(2)} \right), \ k \in \mathbb{N}, \quad (5)$$

where $\lambda_k \in (0,2)$ is the step size, $\alpha_k \in [0,1]$ and

$$\begin{aligned} \boldsymbol{f}_k^{(1)} &:= \operatorname{argmin}_{\boldsymbol{x} \in V_k} \|\boldsymbol{w}_k - \boldsymbol{x}\| \\ &\quad \text{for } V_k := \{\boldsymbol{w} \in \mathbb{R}^n : \boldsymbol{w}^T \boldsymbol{h}(\boldsymbol{\theta}_1) = 1, \ \boldsymbol{w}^T \boldsymbol{y}(k) = 0\}, \\ \boldsymbol{f}_k^{(2)} &:= \operatorname{argmin}_{\boldsymbol{x} \in B_\epsilon} \left\| \boldsymbol{H}_I^T \boldsymbol{w}_k - \boldsymbol{x} \right\| \\ &\quad \text{for } B_\epsilon := \{\boldsymbol{z} \in \mathbb{R}^{s-1} : \|\boldsymbol{z}\|^2 \le \epsilon\}. \end{aligned}$$

Dual-Domain Adaptive Algorithm (2/2)



A geometric interpretation of DDAA

Algorithm

$$\boldsymbol{w}_{k+1} := \boldsymbol{w}_k + \lambda_k \mu_k \left(\alpha_k \boldsymbol{f}_k^{(1)} + (1 - \alpha_k) \boldsymbol{H}_I \boldsymbol{f}_k^{(2)} \right), \ k \in \mathbb{N}, \tag{6}$$

Outline

Experimental Settings

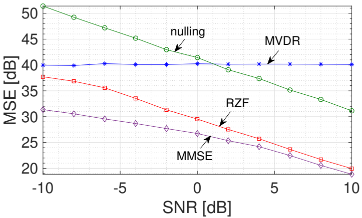
Simulate the case of reconstructing the source activity in interest from the EEG measurements. The settings of the experiments are as follows:

- Sensor space
  The EEG measurements are recorded with a HydroCel Geodesic Sensor Net utilizing 128 channels as the EEG cap layout. FieldTrip (FT) toolbox is used to aid generation of volume conduction model (VCM) and leadfields.

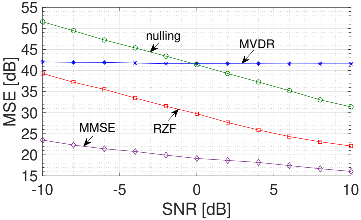
- Source space
  Activities of $s = 37$ sources are generated.
  - The desired activity $q_1(k)$ is generated by an autoreggressive model of order 6 (all the coefficients for each order are set to 0.2).
  - The interfering activities are generated as $q_i(k) = \gamma q_1(k) + \eta n_i(k)$, $\gamma > 0,\ \eta > 0,\ i = 2,\ 3,\ \cdots,\ s$, where $n_i(k)$ follows independently and identically distributed (i.i.d.) standard normal distribution.

MSE under Different SNR Conditions

- Comparision based on the analytical solutions.



(a) low correlation ($\rho = 0.3$)



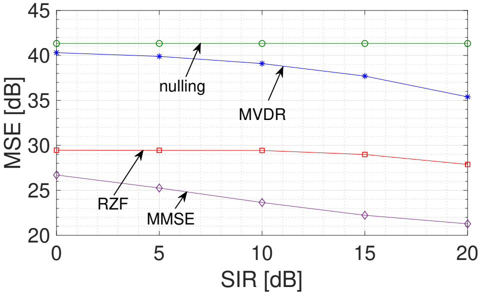
(b) high correlation ($\rho = 0.9$)

SIR $= 0$ dB

RZF achieves better performance than MVDR and nulling.

MSE under Different SIR Conditions

- Comparision based on the analytical solutions.

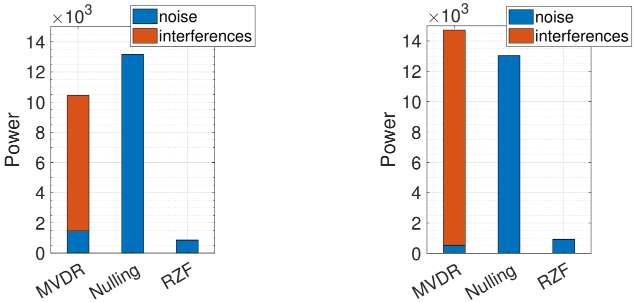


SNR $= 0$ dB and low correlation ($\rho = 0.3$)

RZF achieves better performance.

Power of Noise/Interference Leakage

- Comparision based on the analytical solutions.



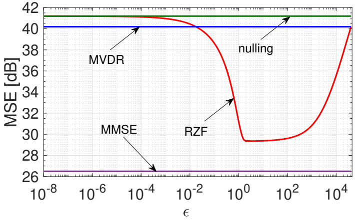
(a) low correlation ($\rho = 0.3$) (b) high correlation ($\rho = 0.9$)

Power of the noise/interference leakage (under SNR $=$ SIR $= 0$ dB)

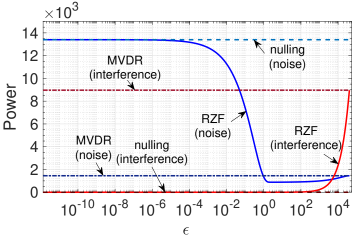
The proposed beamformer attains excellent tradeoff.

Insensitivity to the Choice of ϵ

- Comparision based on the analytical solutions.



(a) MSE performance



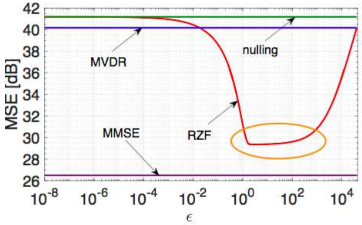
(b) Power of leakage

SNR $=$ SIR $= 0$ dB and low correlation ($\rho = 0.3$)

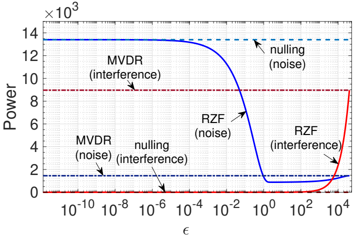
RZF is reasonably insensitive to the choice of ϵ.

Insensitivity to the Choice of ϵ

- Comparision based on the analytical solutions.



(a) MSE performance

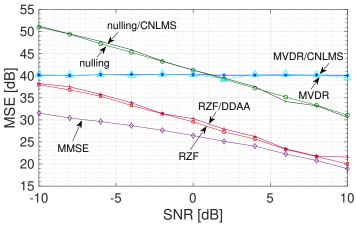


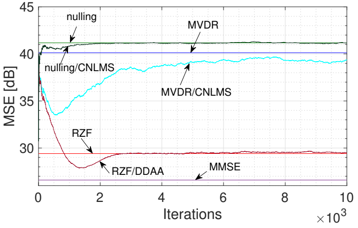
(b) Power of leakage

SNR $=$ SIR $= 0$ dB and low correlation ($\rho = 0.3$)

RZF is reasonably insensitive to the choice of ϵ.

Online Implementation





(a) steady-state performance (b) learning curves under SNR $= 0$ dB

SIR $= 0$ dB and low correlation ($\rho = 0.3$)

The RZF is successfully implemented by DDAA.

Outline

Conclusion

Summary

- Present the RZF beamformer which minimizes the output variance under the constraints of bounded interference leakage and undistorted target signal.
- Present DDAA for an adaptive implementation of the proposed RZF beamformer.
- Show the RZF significantly outperformed the MVDR and nulling beamformers by numerical experiments.

Appendix

- $\boldsymbol{w}_{RZF}$ and ϵ of RZF

From the Karush-Kuhn-Tucker conditions, if the values of $\boldsymbol{w}_{RZF}$ are given, then we can caluculate the corresponded ϵ by

$$\left\| \boldsymbol{H}_I^T \boldsymbol{w}_{RZF} \right\|^2 = \epsilon.$$