NMF-based Comprehensive Latent Factor Learning with Multiview Data

Hua Zheng[1,3], Zhixuan Liang[2], Feng Tian[1*], Zhong Ming[3]

[1]Bournemouth University, UK. [2]The Hong Kong Polytechnic University, HK. [3]Shenzhen University, China

hzheng@bournemouth.ac.uk, 18094409g@connect.polyu.hk

Abstract

Multiview representations reveal the latent information of the data from different perspectives, consistency and complementarity. Unlike most multiview learning approaches, which focus only one perspective, in this project, we propose a novel unsupervised multiview learning algorithm, called comprehensive latent factor learning (CLFL), which jointly exploits both consistent and complementary information among multiple views. CLFL adopts a non-negative matrix factorization based formulation to learn the latent factors. It learns the weights of different views automatically which makes the representation more accurate. Experiment results on a synthetic and several real datasets demonstrate the effectiveness of our approach.

Introduction

In many real-world applications such as data analytics in video surveillance, image processing and natural language processing, data are collected from diverse domains or obtained from various feature extractors. They exhibit heterogeneous properties which are called multiview data.

Many existing multiview algorithms try to construct a latent subspace shared by multiple views. The multiview subspace learning (MSL) learns a unified feature representation from the subspace of all views, so all views share one representation. Non-negative matrix factorization (NMF) is one of the most popular and competitive subspace learning method.

Some works have been done in the area of NMF-based multiview learning. However, all the existing methods only focus on one type of perspective among multiple views. For example, Liu [2] focuses on the consistency while Wang [3] focuses on the complementarity.

To address this limitation, in this project, we propose a multiview learning algorithm, called comprehensive latent factor learning (CLFL), by jointly exploring both the perspectives of consistency and complementarity for multiview data.

Brief Review of NMF

In this section, we briefly introduce non-negative matrix factorization (NMF) [1]. Given an input non-negative data matrix $X = [x_1, x_2, ..., x_N] \in \mathbb{R}^{M\times N}$, each column of $\mathbf{X}$ is an instance vector. NMF aims to find two non-negative matrices $\mathbf{W} \in \mathbb{R}^{M\times K}$ and $\mathbf{H} \in \mathbb{R}^{N\times K}$ whose product can well approximate the original matrix $\mathbf{X}$.

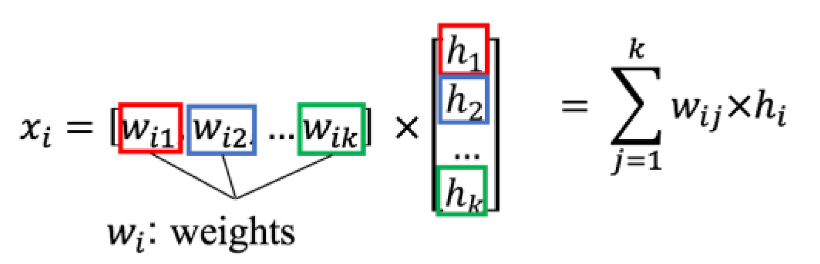


Figure 1: We can interpret x_i to be a weighted sum of some components, where each row in H is a component, and each row in W contains the weights of each component.

In particular, $\mathbf{H}$ can be considered as the new representation of data in terms of the basis $\mathbf{W}$. The cost function of standard NMF is defined as

$$\min \left\|\mathbf{X} - \mathbf{W}\mathbf{H}^T\right\|_F^2 \quad s.t. \mathbf{W}, \mathbf{H} \geq 0 \tag{1}$$

This standard NMF can be extended to multiview setting by adding the cost function of each single view together. Suppose a dataset has V views, the multivew learning objective function becomes:

$$\sum_{v=1}^{V} \min \left\|\mathbf{X}^{(v)} - \mathbf{W}^{(v)}\mathbf{H}^{(v)^T}\right\|_F^2 \quad s.t. \mathbf{W}, \mathbf{H} \geq 0 \tag{2}$$

Comprehensive Latent Factor Learning (CLFL)

Objective Function

We define the objective function as:

$$\sum_{v=1}^{V}\left\|\mathbf{X}^{(v)} - \mathbf{W}^{(v)}\mathbf{H}^{(v)^T}\right\|_F^2 + \sum_{v=1}^{V}(\alpha^{(v)})^\gamma \left\|\mathbf{H}^{(v)}\mathbf{Q}^{(v)} - \mathbf{H}^*\right\|_F^2$$
$$s.t. \mathbf{W}^{(v)} \geq 0, \mathbf{H}^{(v)} \geq 0, \mathbf{H}^* \geq 0, \sum_{v=1}^{V}\alpha^{(v)} = 1$$
$$\mathbf{Q}^{(v)} = Diag\left(\sum_{v=1}^{V}\mathbf{W}_{i,1}^{(v)}, \sum_{v=1}^{V}\mathbf{W}_{i,2}^{(v)}, ..., \sum_{v=1}^{V}\mathbf{W}_{i,K}^{(v)}\right) \tag{3}$$

$\mathbf{H}^*$ is referring to the common latent factor of all views. We use a single parameter γ to control the distribution of weight factors $\alpha^{(v)}$ in all V views, such that the important views will be assigned bigger weights. Also, in order to make different $\mathbf{H}^{(v)}$ comparable, we constrain $\left\|\mathbf{W}_{\cdot,k}^{(v)}\right\|_1 = 1$ by introducing auxiliary variables $\mathbf{Q}^{(v)}$ [2] to simplify the computation.

Optimization

The joint optimization function in (3) is not convex over all variables simultaneously. Thus, we propose an iterative optimization algorithm. For each view we have:

$$J = \left\|\mathbf{X}^{(v)} - \mathbf{W}^{(v)}\mathbf{H}^{(v)^T}\right\|_F^2 + (\alpha^{(v)})^\gamma \left\|\mathbf{H}^{(v)}\mathbf{Q}^{(v)} - \mathbf{H}^*\right\|_F^2 \tag{4}$$

To solve this optimization problem, we propose an iterative update procedure by fixing any three variables to compute the the other one. With these four steps, we alternatively update $\mathbf{W}^{(v)}$, $\mathbf{H}^{(v)}$, $\mathbf{H}^*$ as well as $\alpha^{(v)}$ and repeat the process interactively until the objective function is converged, then we can obtain the weight $\alpha^{(v)}$.

1. Compute the weight $\alpha^{(v)}$.

$$\alpha^{(v)} = \frac{(\gamma\mathbf{G}^{(v)})^{\frac{1}{1-\gamma}}}{\sum_{v=1}^{V}(\gamma\mathbf{G}^{(v)})^{\frac{1}{1-\gamma}}} \tag{5}$$

2. Calculate the view-specific latent factor H_s. We apply the approach proposed in [2] to obtain $\mathbf{H}^{(v)}$ and $\mathbf{H}^*$ as follows:

$$\mathbf{H}_s^{(v)} = \mathbf{H}^{(v)} - \mathbf{H}^* \tag{6}$$

Then all the specific latent factors are integrated by using the weights $1 - \alpha^{(v)}$ which were learned in step 1. The whole specific factor $\mathbf{H}_s$ can be obtained.

$$\mathbf{H}_s = \sum_{v=1}^{V}(1 - \alpha^{(v)})\mathbf{H}_s^{(v)} \tag{7}$$

3. By integrating the common latent factor and specific latent factor together with a single parameter β, a comprehensive latent factor representation was obtained as

$$\mathbf{H}_F = \beta\mathbf{H}^* + (1-\beta)\mathbf{H}_s \tag{8}$$

We use a flowchart to express our CLFL algorithm as follows:

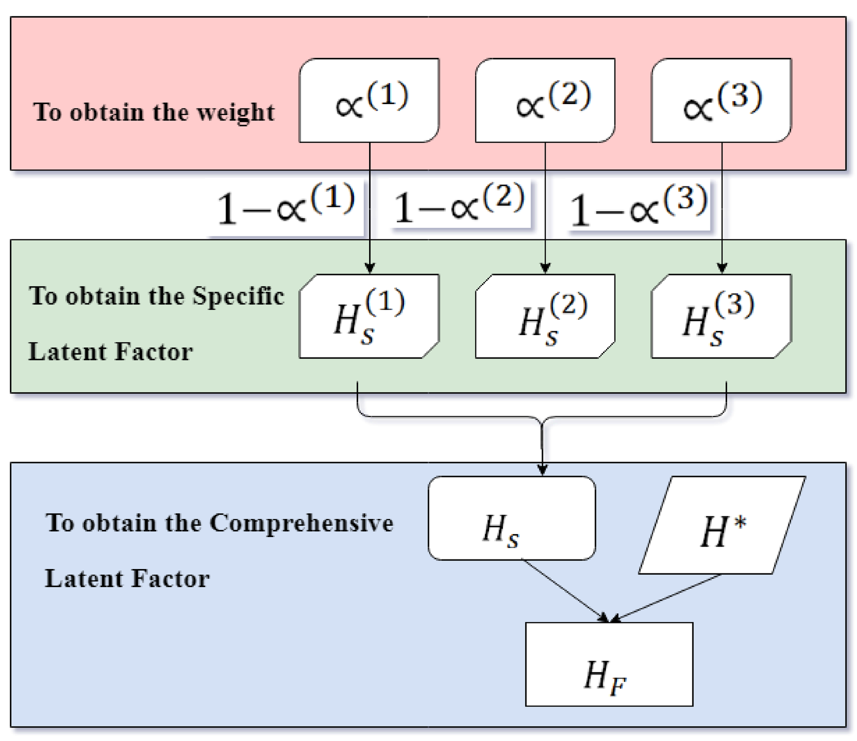


Figure 2: The process of CLFL

Experiment

Datesets

One synthetic and three real world datasets were used in the experiments. Below is a brief introduction.

- **Synthetic dataset:** It is generated from two clusters. Each cluster is composed of three Gaussian components that means an instance is represented as three views. For each cluster, five hundreds instances are randomly sampled.
- **3-Sources Text Dataset:** It is collected from three online news sources: BBC, Reuters, and The Guardian.
- **Reuters Multilingual dataset:** It contains feature characteristics of documents that are translated into 5 languages over 6 categories.
- **UCI Handwritten Digit dataset:** This handwritten digits (0-9) data is from the UCI repository.

These digits are represented in terms of the following six feature sets (files):

1. mfeat-fou: 76 Fourier coefficients of the character shapes;
2. mfeat-fac: 216 profile correlations;
3. mfeat-kar: 64 Karhunen-Love coefficients;
4. mfeat-pix: 240 pixel averages in 2 x 3 windows;
5. mfeat-zer: 47 Zernike moments;
6. mfeat-mor: 6 morphological features.

Figure 3: This dataset consists of features of handwritten numerals ('0'–'9') extracted from a collection of Dutch utility maps

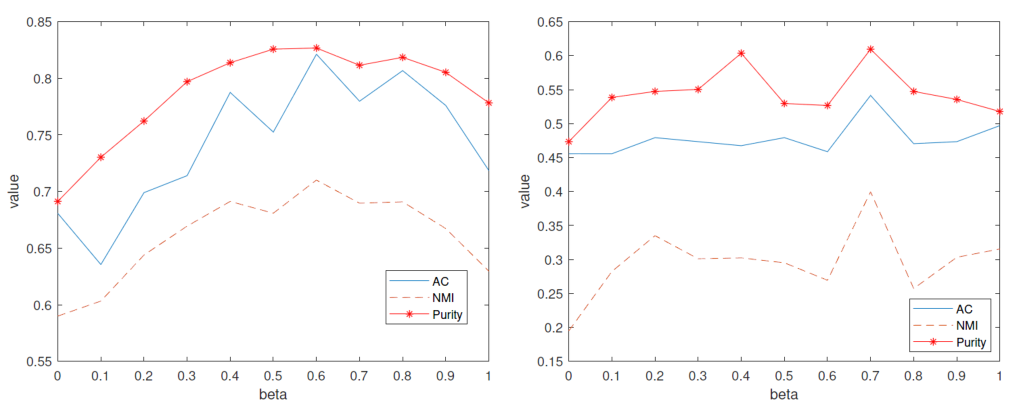
Clustering Results

The clustering results of different algorithms on four datasets are showed in Table 4. As we can see, CLFL outperforms all the other five algorithms in four different datasets in three metrics.

Methods	Metrics	Synthetic	3-Source	Reuters	digit
WSV	AC	0.5375	0.3876	0.1894	0.5385
	NMI	0.5023	0.2079	0.1618	0.5398
	Purity	0.6630	0.4586	0.1889	0.5570
BSV	AC	0.5815	0.4970	0.2711	0.6630
	NMI	0.5978	0.2937	0.1064	0.6252
	Purity	0.7640	0.5266	0.2806	0.6700
ConNMF	AC	0.6740	0.4290	0.1939	0.6865
	NMI	0.6535	0.2029	0.1471	0.6507
	Purity	0.7290	0.4734	0.2100	0.7340
ColNMF	AC	0.6550	0.4408	0.2711	0.6005
	NMI	0.5641	0.2163	0.1056	0.5376
	Purity	0.6520	0.5118	0.2917	0.7265
MultiNMF	AC	0.8465	0.6154	0.4483	0.8140
	NMI	0.7455	0.4300	0.3128	0.7336
	Purity	0.8515	0.6272	0.4500	0.8210
CLFL	AC	**0.8920**	**0.6450**	**0.4733**	**0.8740**
	NMI	**0.8008**	**0.4407**	**0.3170**	**0.7722**
	Purity	**0.8895**	**0.6391**	**0.4692**	**0.8780**

Figure 4: AC, NMI and Purity of different methods.

Also, Fig 3 shows how the performance of CLFL on the digit and 3-source datasets varies with the parameters β. We can see that, in the digit, when β is set nearly 0.6 it achieves the best performance and it will be around 0.7 for the 3-source. So, there is an optimal point between the specific latent factor and the common shared factor.



(a) digit (b) 3-source

Figure 5: Performance of CLFL with parameters β.

Conclusions

In this project, we have proposed a novel latent factor learning algorithm called CLFL. It discovers a comprehensive latent representation for multiview data, by exploiting the consistent and complementary information among different views, simultaneously. Also, CLFL learns the weights of all different views automatically. The specific latent factor and the common shared factor are integrated with a single parameter to control their weights for optimal representation of datasets. The clustering experimental results on four different datasets have demonstrated the effectiveness of our approach. In the future work, we will study the theory of choosing parameters and investigate to take local geometrical information into consideration to learn a better representation.

References

[1] Daniel D Lee and H Sebastian Seung. Learning the parts of objects by non-negative matrix factorization. *Nature*, 401(6755):788, 1999.

[2] Jialu Liu, Chi Wang, Jing Gao, and Jiawei Han. Multi-view clustering via joint nonnegative matrix factorization. In *Proceedings of the 2013 SIAM International Conference on Data Mining*, pages 252–260. SIAM, 2013.

[3] Jing Wang, Feng Tian, Hongchuan Yu, Chang Hong Liu, Kun Zhan, and Xiao Wang. Diverse non-negative matrix factorization for multiview data representation. *IEEE transactions on cybernetics*, 48(9):2620–2632, 2018.