Tensor-Factorization-Based 3D Single Image Super-Resolution with Semi-Blind Point Spread Function Estimation

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Objective

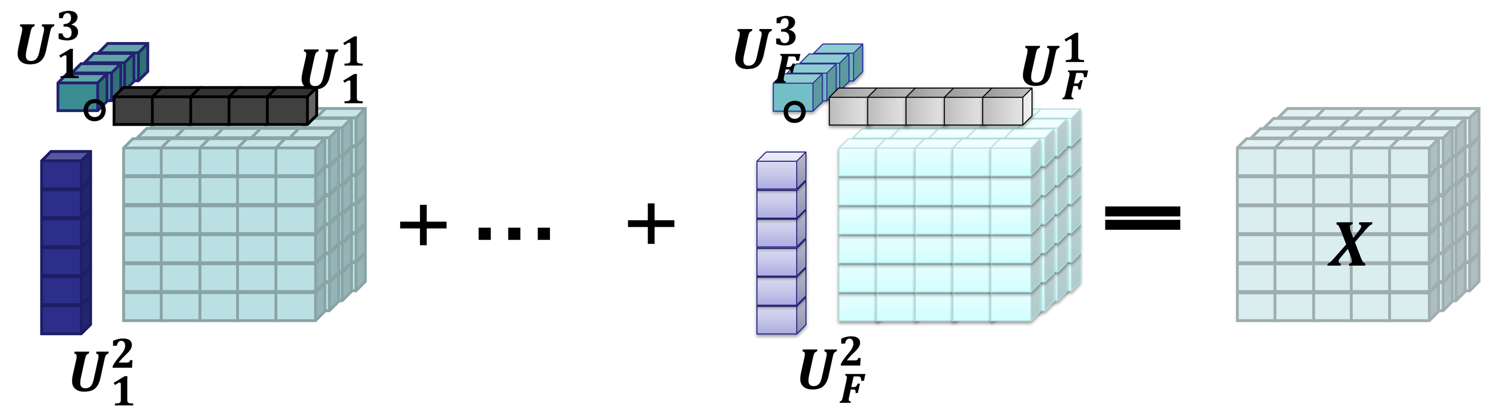
- Proposing a fast volumetric single image super-resolution (SISR) technique with a semi-blind estimation of the Point Spread Function (PSF).

Tensor factorization

Any tensor can be written as the sum of F simple tensors (the outer product of 1D arrays)

$$\boldsymbol{X} = \sum_{f=1}^{F} U^1(:,f) \circ U^2(:,f) \circ U^3(:,f) = [\![U^1, U^2, U^3]\!], \quad (1)$$

where U^1, U^2, U^3 are sets of the 1D arrays. When F is minimal, the decomposition (CPD) is unique under mild conditions. [1]



The SISR problem

Image degradation: the LR image (**Y**) is a down-sampled (D, by factor r) and blurred (H) HR image (**X**) with some additive white noise **N**. Using CPD for **X** it is

$$\mathbf{Y} = [\![D_1 H_1 U^1, D_2 H_2 U^2, D_3 H_3 U^3]\!] + \mathbf{N} \quad (2)$$

The non-convex **minimization problem** is then

$$\min_{\overline{U},\overline{\sigma}} \|\mathbf{Y} - [\![D_1 H_1(\sigma_1) U^1, D_2 H_2(\sigma_2) U^2, D_3 H_3(\sigma_3) U^3]\!]\|_F^2 \quad (3)$$

Image estimation

$$\min_{U_1} \frac{1}{2} \left\|\mathbf{Y}^{(1)} - D_1 H_1 U^1 (D_3 H_3 U^3 \odot D_2 H_2 U^2)^T\right\|_F^2$$
$$\min_{U_2} \frac{1}{2} \left\|\mathbf{Y}^{(2)} - D_2 H_2 U^2 (D_3 H_3 U^3 \odot D_1 H_1 U^1)^T\right\|_F^2 \quad (4)$$
$$\min_{U_3} \frac{1}{2} \left\|\mathbf{Y}^{(3)} - D_3 H_3 U^3 (D_2 H_2 U^2 \odot D_1 H_1 U^1)^T\right\|_F^2$$

- **Y** is unfolded, $\odot$ is the Kathri-Rao product
- Solution: pseudoinversion with Tikhonov regularization [2]

PSF estimation

$$\min_{\overline{\sigma}} \left\|\mathbf{Y} - \mathcal{F}^{-1}\left(\mathcal{F}\tilde{h}(\overline{\sigma}) \cdot \mathcal{F}\mathbf{X}\right)\right\|_F^2 + i(\overline{\sigma}) \quad (5)$$

- blurring can be written as $\mathcal{F}^{-1}\left(\mathcal{F}\tilde{h} \cdot \mathcal{F}\mathbf{X}\right)$, where $\overline{h}$ is the zero-padded, circularly shifted Gaussian kernel
- $\overline{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ are assumed to be in predefined intervals $\overline{a}$, characterized by the indicator function $i(\overline{\sigma})$
- the proximal function of (5) can be solved with gradient descent [3]

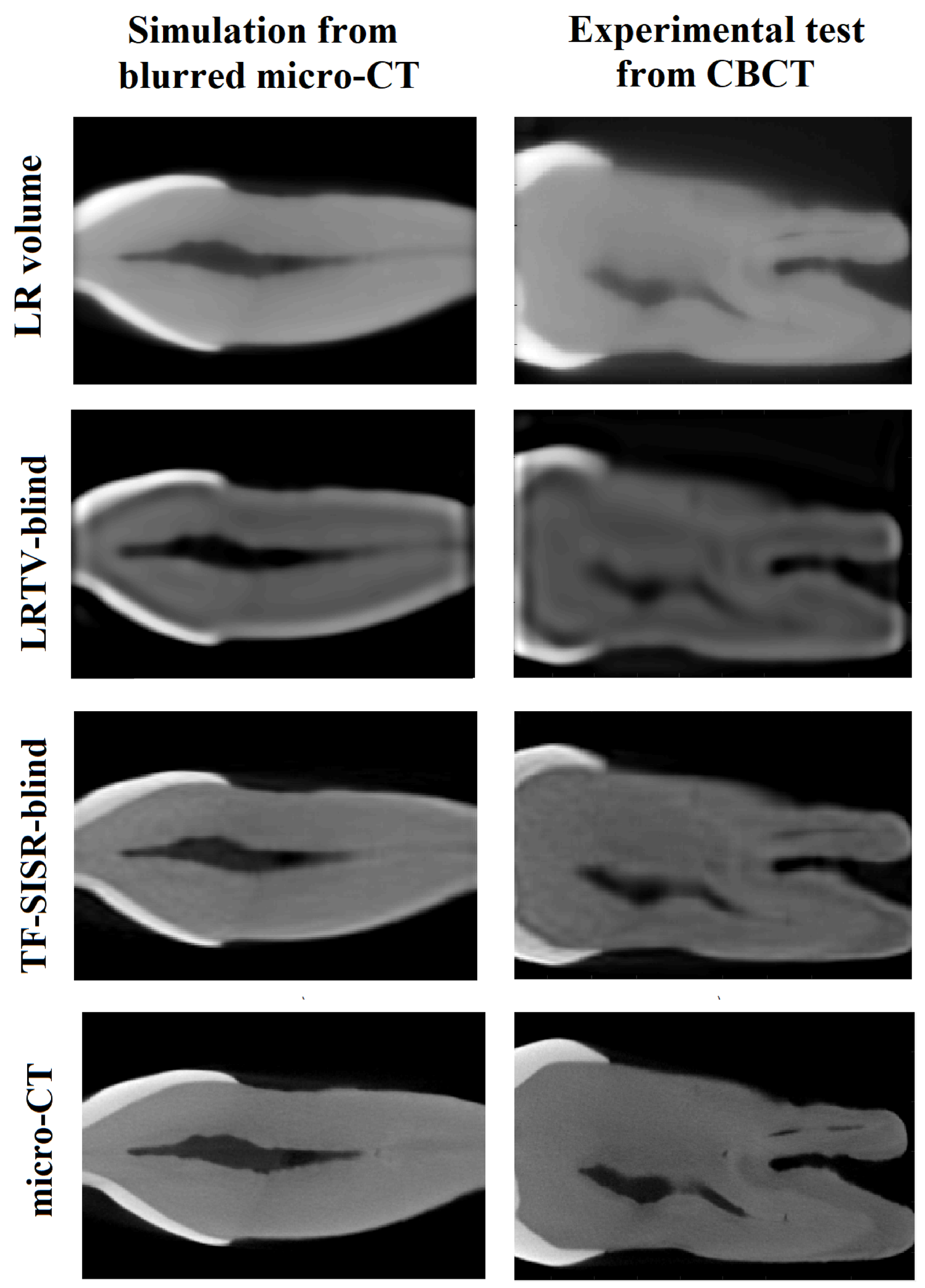
The algorithm

Input: $\mathbf{Y}, r, F, \overline{a}, \epsilon, N, M$
1) **for** i = 0:N **do**
2) $H_1, H_2, H_3 \leftarrow \overline{\sigma}$
3) **for** j = 0:M **do**
4) update U_1, U_2, U_3 sequentially using (4)
5) $X \leftarrow \overline{U}$
6) **while** residual > ϵ
7) update $\overline{\sigma}$ using (5)
8) **Output**: X, the estimated HR image

Results

- Testing on dental CT volumes against a low-rank total variation (LRTV) method [4] combined with our PSF-optimization
- Quantitative results:

		Sim.	Exp.
PSNR	LR-HR	22 dB	19 dB
	LRTV	24 dB	26 dB
	TF-SSIR	27 dB	30 dB
Time	LRTV	9087 s	11832s
	TF-SISR	298 s	354 s



Conclusion

- processing the 3D volumes in less than 5 min
- image quality is at least as good as the state of the art

References

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[3] N. Zhao, Q. Wei, A. Basarab, D. Kouamé, JY Tourneret, "Blind deconvolution of medical ultrasound images using a parametric model for the point spread function," in Ultrasonics Symposium (IUS), 2016 IEEE International. IEEE, 2016, pp. 1–4.

[4] F. Shi, J. Cheng, L. Wang, PT Yap, D. Shen, "LRTV: MR image super-resolution with low-rank and total variation regularizations," IEEE Trans. Med. Imag., vol. 34, no. 12, pp. 2459–2466, 2015.