Image smoothing via gradient sparsity and surface area minimization

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Introduction

Image smoothing is important in image processing. Among these image smoothing methods, the L_0 gradient minimization method is one of the most popular ones. However, the L_0 gradient minimization method suffers from the staircasing effect and over-sharpening issue, which highly degrade the quality of the smoothed image. To overcome these issues, we use not only the L_0 gradient term for finding edges, but also a surface area based term for the purpose of smoothing the inside of each region. An alternating minimization algorithm is suggested to efficiently solve the proposed model, where each subproblem has a closed-form solution. Leveraging the introduced surface area term, the proposed method can effectively alleviate the staircasing effect and the over-sharpening issue. The superiority of our method over the state-of-the-art methods is demonstrated by a series of experiments.

Highlights of our work

- We propose a new image smoothing model, which exploits the L_0 gradient term to preserve the edges and particularly, a surface area term to smooth the inside of each region. By the use of the surface area term, the staircasing effect and over-sharpening issue can be significantly alleviated.
- An alternating minimization method is adapted for effectively solving the proposed model, where each subproblem has an analytic expression. The efficiency of the proposed method is demonstrated by a series of numerical experiments via comparing with many state-of-the-art image smoothing methods.

Our model and algorithm

- $\min_I \|I - I_0\|_2^2 + \lambda(\sigma\phi(I) + \|\nabla I\|_0)$
  where I_0 is an input image, the gradient I_0 is calculated as difference between neighboring pixels along the x and y directions; $\phi(I)$ is the surface area of the mesh field.
- There are two regularization terms in the proposed model, and it can not be solved directly. We apply the classic quadratic penalty approach to solve it iteratively:
- $$\min_{I,U} f(I,U) := \|I - I_0\|^2 + \beta\|U - \nabla I\|^2 + \lambda\|U\|_0 + \lambda\sigma\phi(U)$$
- Given I, we have
  $$\bar{U} = \arg\min_U \beta\|U - \nabla I\|^2 + \lambda\sigma\phi(U) + \lambda\|U\|_0 \quad (1)$$
- Given U, we have
  $$\bar{I} = \arg\min_I \|I - I_0\|^2 + \beta\|U - \nabla I\|^2 \quad (2)$$

Algorithm 1 Image smoothing via gradient sparsity and surface area minimization

Input: input image I_0, regularization parameter λ, initial penalty parameter $\beta = \beta_0$, maximum penalty parameter $\beta_{\max}$, growth rate $\kappa > 1$, initial $k = 0$, $I^k = I_0$.

Output: smoothed image I

while $\beta < \beta_{\max}$

Given I^k, solve for U^{k+1} using (1);

Given U^{k+1}, solve for I^{k+1} according to (2);

$\beta \leftarrow \kappa\beta$, $k \leftarrow k + 1$.

Numerical results

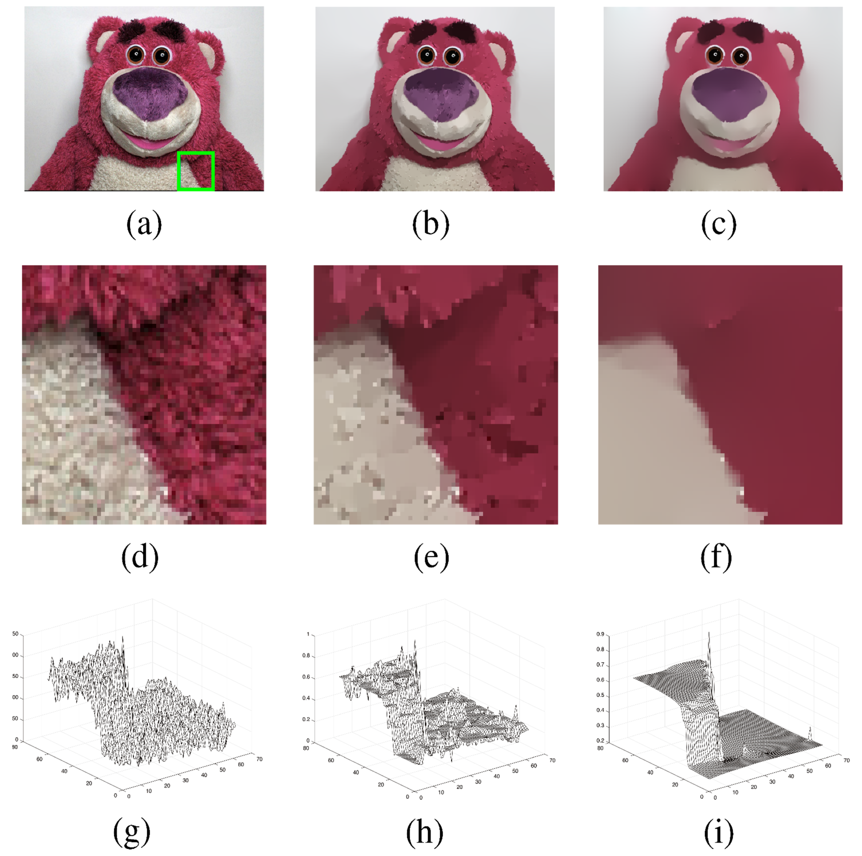

Fig.1 Our surface area smoothing results in comparison with Xu et al's method. (a) Input image; (b) Xu et al's result; (c) Our result; (d-f) zoom-in of the input image and the two results; (g-i) Graphs of the corresponding zoom-in parts (d-f) in gray-scale.

Fig.3 Difficulty of our method for smoothing texture image. (a) Input image; (b) Relative total variation ; (c) Xu et al; (d) Ours.

Fig. 2. Comparison of the smoothed results of Algorithm 1 and Xu et al with different parameters. The second and third rows are zoom-in parts of the first row. Our method can remove undesired artifacts compared with Xu et al's method under the same regularization parameter λ. When increasing λ for Xu et al's method, over-sharpening happens.